Metastable Q-balls

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Abstract. We will breifly review application of Euclidean path integral technique for the study of quantum decay of the bound system in field theory with global $U(1)$-invariance. As an illustration of the method, we numerically compute the decay rate of metastable Q-ball to the leading semiclassical order and present interpolating formula for the whole region of metastability.

1 Introduction

Theories with bosonic fields can provide extended lumps. These objects in models with single complex field are usually referred as Q-balls [1] or nontopological solitons in theories with additional fields [2]. Terminology indicates the crucial role of conserved nontopological charge Q which is determined by global $U(1)$-invariance. Localization of charge and density provided by nonlinear interaction and wealthy models usually considered only in small coupling regime. In this case properties of quantum state can be approximately described by solutions of classical fields equations with corresponding charge Q and energy E. Stability of configuration is not provided by topology of vacuum and we will assume that in the minimum of potential $V(0) = 0$ for value of complex field

$$\varphi = 0. \tag{1}$$

In quantum theory above the vacuum (1) complex field naturally results to bosons and antibosons with mass m. It is natural to compare the energy of two configurations with the same charge Q, i.e. soliton energy E and energy of free bosons mQ[1]. In the case

$$E > mQ$$

the decay of soliton to free particles is kinematically possible. Examples with similar $E(Q)$ dependence were provided by original model [3] of nontopological solitons and it is reasonable to calculate corresponding decay rate. Similar question considered in [4] for the modeling of trapped Bose-Einstein Condensate (BEC).

Although the metastable vacuum decay is thoroughly considered in [5], additional global $U(1)$-invariance provides some subtleties (see, for example [6]). There are also technical problems with exponential dependence of complex fields after Euclidean continuation. In the paper [7] we generalize the Coleman method and numerically calculate decay rate for a

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[1]We should stress here that we work in the small coupling regime and this comparison is legible.

model with single complex field. The chosen scalar potential presented in Fig. 1 and the leading term of V on smallest coupling g can be expressed in the form

$$\frac{1}{g^2}U(g|\varphi|/m).$$

Moreover, it can be used for description of dynamics of complex field in the Friedberg-Lee-

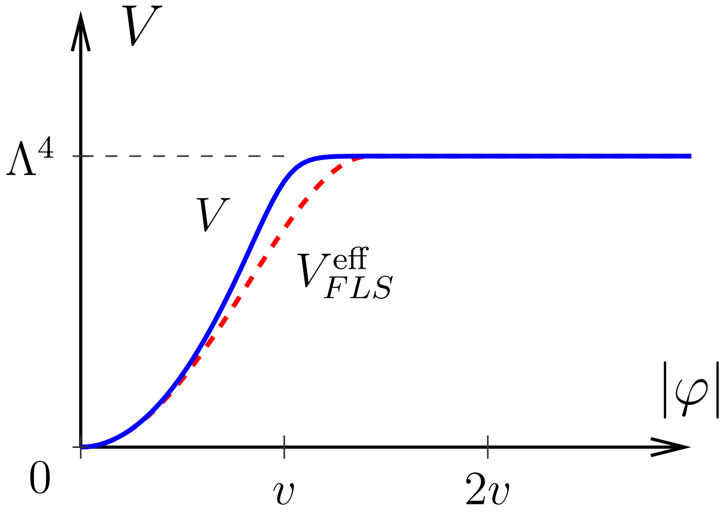

Figure 1. Scalar potential for numerical calculation.

Sirlin (FRS) model [3].

In this paper we will briefly review classical properties of Q-balls in the next section. In Sec. 3 we will discuss Euclidean equations of motion and main numerical results of [7].

2 Classical Q-balls

For a start, we briefly review the properties of small Q-balls. To be concrete, consider the mode of complex scalar field φ with potential

$$V = -\frac{m^2 v^2}{b}\log\left(\frac{e^{-\varphi\bar{\varphi}/v^2} + e^{-b}}{1 + e^{-b}}\right), \qquad b = 8 \tag{2}$$

shown in Fig. 1. The scalar bosons in this model have mass $\approx m$ in vacuum $\varphi = 0$ and become almost massless at large φ: $V \to \Lambda^4 \equiv m^2 v^2 \log(e^b + 1)/b$ as $|\varphi| \to +\infty$. This corresponds to short-range attractive interaction between the bosons.

Notably, the model possesses global conserved charge

$$Q = i\int d^3\boldsymbol{x}\,(\varphi\partial_t\bar{\varphi} - \bar{\varphi}\partial_t\varphi)\ , \tag{3}$$

related to phase rotation symmetry $\varphi \to e^{i\alpha}\varphi$, $\bar{\varphi} \to e^{-i\alpha}\bar{\varphi}$ in the action. Conservation of this quantity is vital [1] for the existence and stability of Q-balls — nontopological solitons with nonzero charge Q.

The simplest way to find nontopological solitons in the model is to substitute stationary spherically-symmetric ansatz

$$\varphi_Q(\boldsymbol{x},\, t) = \chi_Q(r)\, \mathrm{e}^{i\omega t}\,, \qquad r = |\boldsymbol{x}|, \tag{4}$$

into the classical field equations and numerically solve the resulting ordinary differential equation for real $\chi_Q(r)$ with regularity conditions $\partial_r \chi_Q(0) = \chi_Q(\infty) = 0$. Alternatively, one can use piecewise parabolic function [8] to approximate potential and obtain a family of localized solutions. Crucial for our study regime $E > mQ$ can be found for small Q-balls as it presented in Fig. 2.

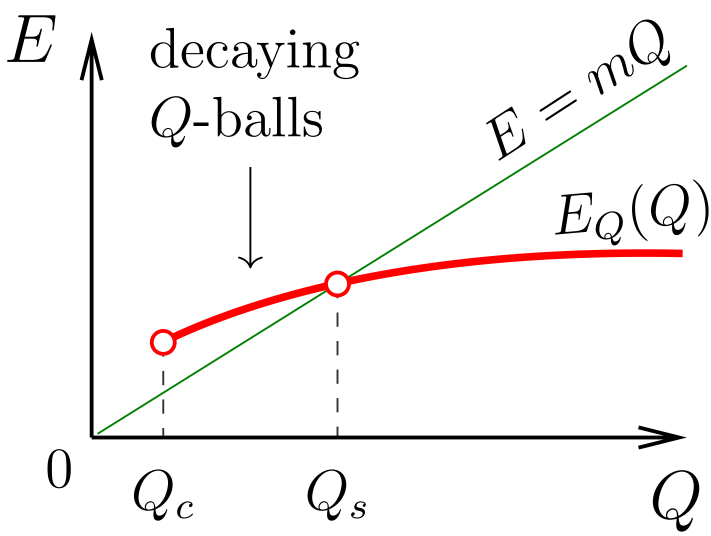

Figure 2. Metastable Q-balls for $Q_c < Q < Q_s$.

It should be noted that for the same charge Q one can obtain two solutions with different profiles, see Fig. 3. We will refer solution with smaller energy as Q-ball. There is additional branch of unstable solutions with larger energy and more widely profile — Q-clouds[2]. As pointed out in [10] Q-clouds can be interpreted as the critical bubbles between Q-balls and states with free particles.

Instability of Q-clouds corresponds to relativistic generalization of Vakhitov-Kolokolov criterion [11], $\partial Q/\partial\omega > 0$. In Fig. 4 we presented crucial for this point dependence $Q(\omega)$. The single mode of instability can be found in model with analytical solution [8, 10] or numerically [7]. Thus, Q-clouds mark the barrier between classically stable Q-balls and the state with Q free particles.

3 Euclidean method and results for the decay rate

For the false vacuum decay in models with real scalar fields there is powerful Euclidean technique [5]. This process is described by the "bounce" — an $O(4)$ invariant real Euclidean

[2]This type of solutions were originally studied in [9]

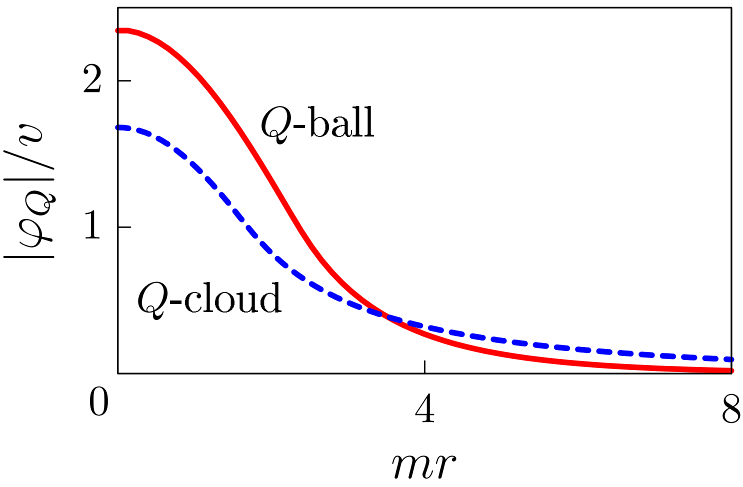

Figure 3. Profiles of classical solutions for the same charge Q.

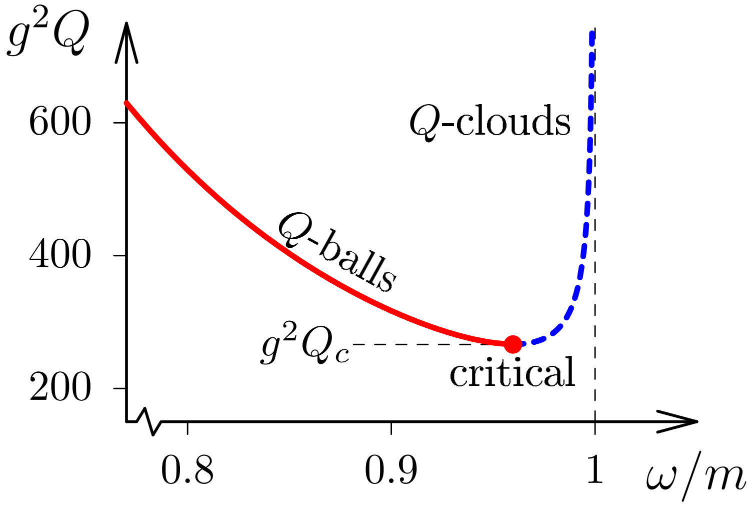

Figure 4. Dependence $Q(\omega)$ for Q-balls and Q-clouds. One can see that $\partial Q/\partial\omega > 0$ for Q-clouds.

solution $\phi_{cl}(\boldsymbol{x}^2 + \tau^2)$ coinciding with the false vacuum at $\tau \to -\infty$ and arriving to the catchment area of the true vacuum at $\tau = 0$. If continued to Minkowski time $t \equiv -i\tau$, this solution describes real field configuration, the bubble of true vacuum, evolving in the final state. The rate of false vacuum decay is exponentially suppressed, where the leading exponent is given by the Euclidean action of the bounce $S_E[\phi_{cl}]$.

We expect to find similar, though properly modified, procedure for computing the rate of Q-ball decay. We should find semiclassical solution $\varphi_{cl}(\boldsymbol{x}, \tau)$, $\bar{\varphi}_{cl}(\boldsymbol{x}, \tau)$ satisfying Euclidean field equations in the model,

$$(\partial_\tau^2 + \nabla_{\boldsymbol{x}}^2)\varphi_{cl} = V'\varphi_{cl} \,, \qquad (\partial_\tau^2 + \nabla_{\boldsymbol{x}}^2)\bar{\varphi}_{cl} = V'\bar{\varphi}_{cl} \,, \tag{5}$$

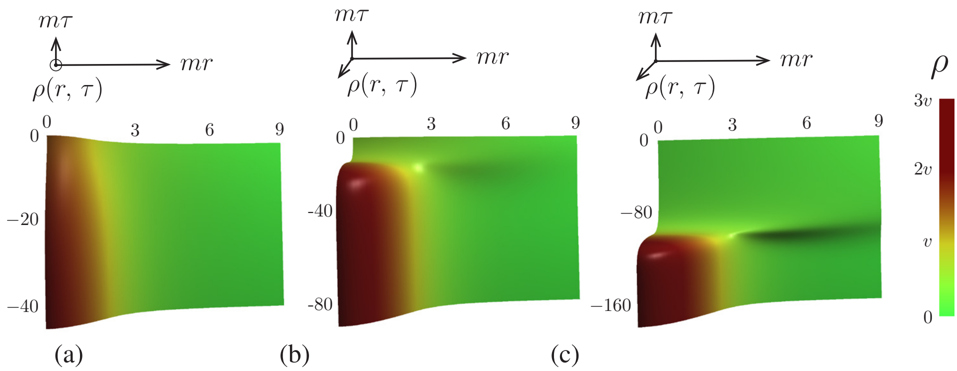

Figure 5. Semiclassical solutions $\rho(r,\tau) \equiv (\varphi\bar{\varphi})^{1/2}$ describing decay of Q-balls with $Q/Q_c \approx 1.05$ (a), 1.33 (b), and 1.56 (c).

where V' is a derivative of $V(\bar{\varphi}\varphi)$ with respect to its argument. The boundary condition is more subtle issue, see [7] for careful derivation. It is natural to expect that this solution coincides with the Q-ball (4) at the start of the process,

$$\varphi_{cl} \to \mathrm{e}^{\omega\tau+\eta_0/2}\chi_Q(r)\,, \qquad \bar{\varphi}_{cl} \to \mathrm{e}^{-\omega\tau-\eta_0/2}\chi_Q(r) \qquad \text{as} \qquad \tau \to -\infty\,, \tag{6}$$

where we introduced phase shift η_0 of the Q-ball which cannot be excluded in general. It can be represented in more appropriate for equations (5) form

$$\varphi_{cl} = \mathrm{e}^{-\omega\beta-\eta_0}\,\bar{\varphi}_{cl}\,, \qquad \partial_\tau\varphi_{cl} = -\mathrm{e}^{-\omega\beta-\eta_0}\partial_\tau\bar{\varphi}_{cl} \qquad\qquad \text{at} \qquad \tau = -\frac{\beta}{2} \to -\infty\,,$$

which is consistent with the asymptotics (6). We should stress here that the functions φ_{cl} and $\bar{\varphi}_{cl}$ in Eq. (5) are not complex conjugate to each other. This situation is usual for the saddle point approximation. Our derivation suggests that instead of being mutually conjugate, φ_{cl} and $\bar{\varphi}_{cl}$ can be considered as independent *real* functions of $\boldsymbol{x}$ and τ. This gives real Euclidean action,

$$S_E \equiv -iS = \int_{-\beta/2}^{\beta/2} d\tau d^3\boldsymbol{x}\,[\partial_\tau\varphi\partial_\tau\bar{\varphi} + \nabla_{\boldsymbol{x}}\varphi\nabla_{\boldsymbol{x}}\bar{\varphi} + V(\varphi\bar{\varphi})]\,, \tag{7}$$

where β will be sent to infinity in the end of the calculation. Our second boundary condition

$$\varphi_{cl} = \bar{\varphi}_{cl}\,, \qquad \partial_\tau\varphi_{cl} = -\partial_\tau\bar{\varphi}_{cl} \qquad\qquad \text{at} \quad \tau = 0 \tag{8}$$

ensures that the solution describes classical evolution of waves/particles in the final state after continuation to real time $t \equiv i\tau$. Indeed, Eq. (8) implies that the functions φ_{cl} and $\bar{\varphi}_{cl}$ are real and satisfy $\varphi_{cl}(\boldsymbol{x},\tau) = \bar{\varphi}_{cl}(\boldsymbol{x},-\tau)$ on the Euclidean time axis. This makes them conjugate to each other at real t (imaginary τ). Note that the solution can be continued to $\tau > 0$ using $\tau \to -\tau$ symmetry. Numerically solving Eqs. (5) with corresponding boundary conditions, one obtains a family of real solutions φ_{cl}, $\bar{\varphi}_{cl}$ interpolating between the Q-ball (4) and arriving to the sector of true vacuum at $\tau = 0$. In Euclidean time semiclassical solution should to interpolate between localized soliton and homogeneous condensate as presented in Fig. 5 for different charges.

Following to [12] one can make back transformation to Minkowski time to obtain the illustration of the decay, see Fig. 6. The total disintegration of localized configuration can be interpreted as collective tunneling to free particles.

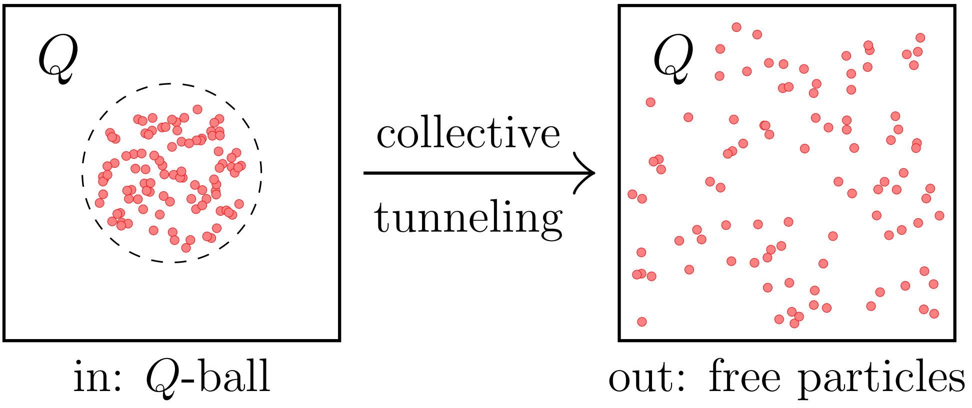

Figure 6. In final configuration of Q-ball decay $\varphi < v$, i.e. the state is in the catchment of the vacuum.

Saddle-point approximation automatically provides result for the decay rate in the form

$$\Gamma_Q = A_Q \, e^{-F_Q}$$

The main contribution to the suppression exponent F_Q comes from classically forbidden region between Q-ball configuration and free particles at $\tau = 0$. Our result for the suppression exponent with its asymptotic and fitting function is presented in Fig. 7, quantitative expressions of them are:

$$F_Q \to d_1 + d_2 \log\left(1 - Q/Q_s\right) \ , \tag{9}$$

$$F_Q \approx (Q - Q_c)\left[c_1 + c_2 \log\left(1 - Q/Q_s\right)\right] \ , \tag{10}$$

where $c_1 = -0.28$, $c_2 = -2.6$.

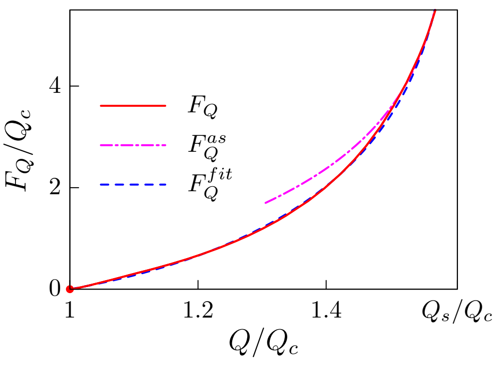

Figure 7. Suppression exponent F_Q for the rate of Q-ball decay, its $Q \to Q_s$ asymptototic F_Q^{as} and fitting function F_Q^{fit}, Eqs. (9) and (10).

4 Conclusions

We developed general semiclassical method to calculate the decay rate $\Gamma_Q = A_Q \, \mathrm{e}^{-F_Q}$ of metastable Q-balls at $Q \gg 1$. The method can be applied in arbitrary models at the cost of numerically obtaining certain Euclidean solutions $\varphi_{cl}(\boldsymbol{x}, \tau)$ that enter the semiclassical expressions for the exponent F_Q and prefactor A_Q of the rate. We generalized the method to finite-temperature processes.

To illustrate the method, we performed explicit numerical calculations in the model. Namely, we computed the exponent. Notably, the model we use is close to a certain limit of the celebrated Friedberg-Lee-Sirlin model [3] which describes complex and real fields with renormalizable potential.

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