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The post-Newtonian limit of hybrid f(R)-gravity

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Abstract. The post-Newtonian formalism is developed for hybrid f(R)-gravity. The model is tested in the weak field regime of the Solar System. Using scalar-tensor representation of hybrid f(R)-gravity we impose restrictions on the parameters of the theory and show that hybrid f(R)-gravity is not ruled out by the observations in the weak field limit.

1 Introduction

The General Relativity (GR) is the universally recognized theory of gravity. It successfully describes a huge range of scales and gravitational regimes. Together with Standard model, they represent two pillars of modern physics. Unfortunately, some phenomena cannot be explained completely in frameworks of these two approaches. Already in 1930s the problem of galactic rotation curves arose [1]. Moreover, the accelerated expansion of the Universe (i.e., "dark energy") has been found from cosmological observations recently [2] and we still can not merge gravity with quantum mechanics. So it becomes obvious that the GR should be modified. One way to expand GR is adding the corrections in terms of the Ricci scalar R to the Einstein-Hilbert action [3, 4]. Such approach underlies f(R)-gravity.

The family of f(R)-gravity theories is divided into two classes: metric f(R)-gravity and Palatini f(R)-gravity [5]. In the metric f(R)-gravity the Ricci scalar R depends only on metric $g_{\mu\nu}$, whereas in the Palatini approach the Ricci scalar $\mathscr{R}$ depends on the metric and on the independent affine connection $\hat{\Gamma}^{\alpha}_{\beta\gamma}$. Both of two classes have some unsolved problems at the moment. The metric f(R)-gravity can explain well the accelerated expansion of the Universe, but fails in the Solar System or vice versa [5, 6]. The Palatini f(R)-gravity perfectly describes both of the Solar System and accelerated expansion but such models may lead to undesired gradient instabilities as has been shown by studies of cosmological perturbations [7].

The hybrid metric-Palatini f(R)-gravity is a new approach in f(R)-gravity which unites metric and Palatini theories but lacks their shortcomings [8]. The main task of hybrid f(R)-gravity is explanation of Dark Energy and Dark Matter by changing only the geometry of a space-time without the introduction of new unknown particles in the theory. Some important results have already been achieved [9]. This theory allow to reproduce the rotation curve of galaxies [10] and the galactic cluster dynamics without assuming huge amounts of dark matter [9]. Also an interesting aspect of this theory is the possibility to generate long-range

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forces without entering into conflict with local tests of gravity and without invoking any kind of screening mechanism (which would however require that at the present time the cosmological evolution reduces to the GR)[9].

However, any theory should be verifiable. And gravitational model should satisfy the observational data in the Solar System. The main tool for testing theories of gravity in the Solar System is the so-called post-Newtonian formalism (PPN) [11].

Parameterized post-Newtonian formalism was originally developed to compare various metric theories with each other and GR [11]. The post-Newtonian limit is established in the framework of the asymptotically flat space-time background and small velocities. Motion of matter would obey the hydrodynamics equations for the perfect fluid. Distinctions between GR and other theories of gravity are reflected via the set of 10 post-Newtonian parameters. Each parameter is responsible for its effect. Also all these parameters have been measured in different experiments. So comparing the PPN parameters which are predicted by the theory with experimental values of these parameters it is possible to impose restrictions on parameters of the model in the weak field regime.

The structure of the paper is the following. In Section 2 we describe briefly the hybrid f(R)-gravity; in the Section 3 we represent the main ideas of the post-Newtonian formalism; the Section 4 is devoted to developing of PPN formalism for hybrid f(R)-gravity models; and in the Section 5 we impose restrictions on the hybrid f(R)-gravity using the Solar System data. The Section 6 contains conclusions.

Throughout this paper the Greek indices $(\mu, \nu, ...)$ run over $0,1,2,3$ in the signature $(-,+,+,+)$. All calculations are performed in the Natural units system $\hbar = k_b = c = 1$, but the speed of light is present in the equations for easy understanding of the text.

2 Hybrid f(R)-gravity

The hybrid f(R)-gravity is the theory which unites two approaches of f(R)-gravity: metric and Palatini. The gravitational action of hybrid f(R)-gravity also consists of two parts [8]:

$$S_{grav} = S_{metric} + S_{Palatini} = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g}[R + f(\mathscr{R})], \tag{1}$$

where c is the speed of light, G is the Newtonian gravitational constant, $g_{\mu\nu}$ is the metric, R and $\mathscr{R}$ are the metric and the Palatini curvatures respectively.

Drawing an analogy between the scalar-tensor theories and the higher order theories of gravity, it is possible to represent hybrid f(R)-gravity as a scalar-tensor theory [9]:

$$S_{grav} = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} \left[(1+\phi)R + \frac{3}{2\phi}\partial_\mu \phi \partial^\mu \phi - V(\phi)\right], \tag{2}$$

where ϕ ia the scalar field and $V(\phi)$ is the scalar potential. In such case, the field equations have the second order [9]:

$$(1+\phi)R_{\mu\nu} = \frac{8\pi G}{c^4}(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T) + \frac{1}{2}g_{\mu\nu}(V(\phi) + \nabla_\alpha \nabla^\alpha \phi) + \nabla_\mu \nabla_\nu \phi - \frac{3}{2\phi}\partial_\mu \partial_\nu \phi$$

$$\nabla_\mu \nabla^\mu \phi - \frac{1}{2\phi}\partial_\mu \phi \partial^\mu \phi - \frac{\phi[2V(\phi) - (1+\phi)V_\phi]}{3} = -\frac{8\pi G}{3c^4}\phi T \tag{3}$$

In some sense, considering a theory like $R + f(\mathscr{R})$ means that all the positive results of GR are represented by the Einstein-Hilbert part of the action R, while the further "gravitational budget" is endowed in the metric-affine $f(\mathscr{R})$ component [9].

3 Post-Newtonian formalism

PPN is very effective testbed for the viability of gravitational theories in the weak field limit. According to the post-Newtonian approach the metric of any gravitational theory can be represented as a set of combinations of PPN parameters and PPN potentials. PPN potentials are universal for all theories whereas all distinctions are contained in PPN parameters [11].

The general post-Newtonian metric has the following form [11]:

$$
\begin{aligned}
g_{00} = & - 1 + 2\frac{1}{c^2}U - 2\beta\frac{1}{c^4}U^2 + (2\gamma + 1 + \alpha_3 + \zeta_1 - 2\xi)\frac{1}{c^4}\Phi_1 \\
& - 2(2\beta - 3\gamma - 1 - \zeta_2 - \xi)\frac{1}{c^4}\Phi_2 + 2(1+\zeta_3)\frac{1}{c^4}\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\frac{1}{c^4}\Phi_4 \\
& - 2\xi\frac{1}{c^4}\Phi_W - \frac{1}{c^4}(\zeta_1 - 2\xi)\mathscr{A} - \frac{1}{c^4}(\alpha_1 - \alpha_2 - \alpha_3)w^2U - \frac{1}{c^4}\alpha_2 w^a w^b U_{ab} \\
& + \frac{1}{c^4}(2\alpha_3 - \alpha_1)w^a U_a, \\
g_{0j} = & - [4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi]\frac{1}{2c^3}U_j - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)\frac{1}{c^3}W_j \\
& + \frac{1}{2c^3}[\alpha_1 - 2\alpha_2]w_j U - \alpha_2 w^i U_{ij}, \\
g_{ij} = & \left(1 + 2\gamma\frac{1}{c^2}U\right)\delta_{ij},
\end{aligned}
\tag{4}
$$

where

$$
\begin{aligned}
& U = \int G_{eff}\frac{\rho'}{|\vec{r}-\vec{r'}|}d^3r',\ \Phi_1 = \int G_{eff}\frac{\rho' v'^2}{|\vec{r}-\vec{r'}|}d^3r',\ \Phi_2 = \int G_{eff}^2\frac{\rho' U'}{|\vec{r}-\vec{r'}|}d^3r', \\
& \Phi_3 = \int G_{eff}\frac{\rho'\Pi'}{|\vec{r}-\vec{r'}|}d^3r',\ \Phi_4 = \int G_{eff}\frac{p'}{|\vec{r}-\vec{r'}|}d^3r',\ \ \mathscr{A} = \int G_{eff}\frac{\rho'[\vec{v'}(\vec{r}-\vec{r'})]^2}{|\vec{r}-\vec{r'}|^3}d^3r', \\
& \Phi_W = \int G_{eff}^2\rho'\rho''\frac{(r-r')_j}{|\vec{r}-\vec{r'}|^3}\left[\frac{(r'-r'')^j}{|\vec{r}-\vec{r''}|} - \frac{(r-r'')^j}{|\vec{r'}-\vec{r''}|}\right]d^3r'd^3r'', \\
& U_j = \int G_{eff}\frac{\rho' v'_j}{|\vec{r}-\vec{r'}|}d^3r',\ W_i = \int G_{eff}\frac{\rho'\vec{v'}(\vec{r}-\vec{r'})(r-r')_i}{|\vec{r}-\vec{r'}|^3}d^3r'
\end{aligned}
\tag{5}
$$

are post-Newtonian potentials; post-Newtonian parameters are described in the Table 1 [11]. Here w^i is the velocity of the PPN coordinate frame relative to a universal preferred frame; G_{eff} is the effective gravitational constant; ρ is the density of the rest mass, measured in a local freely falling reference frame, accompanying gravitating matter; Π is internal energy per unit mass; p is pressure measured as ρ; $v^i = \frac{dr^i}{dt}$ is coordinate velocity of matter.

4 Hybrid f(R)-gravity in the weak field limit

The hybrid f(R)-gravity can be represented as massive scalar-tensor theory [9]. It is known that such models does not allow a straightforward implementation of the post-Newtonian formalism because the coefficients of modified post-Newtonian potentials in the post-Newtonian metric are not constants, but they have a spatial dependence [12]. However, there are two limits where PPN approximation can be applied to massive scalar-tensor theories: very massive ($m_\varphi >> 1/r$) and very light scalar field ($m_\varphi << 1/r$) [12]. We investigate only the second possibility in our consideration.

Table 1. Post-Newtonian parameters

PPN parameter	Physical meaning	Experimental value
γ	space-curvature produced by unit rest mass	$1 \pm 2.3 \times 10^{-5}$
β	"nonlinearity" in the superposition law for gravity	$1 \pm 8 \times 10^{-5}$
ξ	existence of preferred location effects	4×10^{-9}
α_1 α_2 α_3	existence of preferred frame effects	10^{-4} 7×10^{-5} 2×10^{-9}
ζ_1 ζ_2 ζ_3 ζ_4	the failure of conservation laws of energy, momentum and angular momentum	4×10^{-20} 2×10^{-2} 4×10^{-5} 10^{-8}

First of all we need to represent the metric and the scalar field as a series of perturbations around a background values:

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu}, \\ \phi &= \phi_0 + \varphi, \end{aligned} \tag{6}$$

where $\eta_{\mu\nu}$ is the Minkowskian metric, ϕ_0 is the asymptotic value of the scalar field far away from the local system (and should be given by the cosmological background solution), $h_{\mu\nu}$ and φ are perturbations of the metric and of the scalar field respectively.

According to (6) the field equations (3) in the weak field limit take the following form:

$$\begin{aligned} \Delta\left(h_{00} - \frac{\varphi}{1+\phi_0}\right) &= -\frac{8\pi G}{c^4(1+\phi_0)}\left[\rho(c^2+\Pi) + 2\rho v^2 + 3p - \rho h_{00} - \frac{\varphi}{1+\phi_0}\rho\right] \\ &\quad + h_{00,00} + h_{ab}h_{00,ab} - (\nabla h_{00})^2 - \frac{\varphi}{1+\phi_0}\Delta h_{00} - \frac{\varphi_{,00}}{1+\phi_0} \\ &\quad - \frac{1}{1+\phi_0}h_{ab}\varphi_{ab} - \frac{1}{(1+\phi_0)^2}(\nabla\varphi)^2 - \frac{1}{1+\phi_0}h_{00}\Delta\varphi + \frac{V'\varphi}{1+\phi_0} \\ &\quad + \frac{V''\varphi^2}{2(1+\phi_0)}, \end{aligned} \tag{7}$$

$$\Delta h_{0j} = \frac{16\pi G}{c^3(1+\phi_0)}\rho v_i \tag{8}$$

$$\Delta\left(h_{ij} + \delta_{ij}\frac{\varphi}{1+\phi_0}\right) = -\frac{8\pi G}{(1+\phi_0)c^2}\rho\delta_{ij}, \tag{9}$$

$$\begin{aligned} (\Delta - m_\varphi^2)\varphi &= \frac{8\pi G\phi_0}{3c^4}[\rho(c^2+\Pi) - 3p] + \frac{8\pi G}{3c^2}\rho\varphi + \varphi_{,00} + h_{ab}\varphi_{,ab} \\ &+ \frac{3\phi_0+1}{2\phi_0(1+\phi_0)}(\nabla\varphi)^2 - \frac{\varphi^2}{3}\left[\frac{V'''\phi_0(\phi_0+1)}{2} + V''(\phi_0+1) - V'\right], \end{aligned} \tag{10}$$

where we expand the scalar potential $V(\phi)$ in the Taylor series as $V(\phi) = V_0 + V'\varphi + \frac{V''\varphi}{2!} + \frac{V'''\varphi}{3!}$ and denote $m_\varphi^2 = [2V_0 - V' - (1+\phi_0)\phi_0 V'']/3$ as a scalar field mass. The zeroth-order term $\frac{\phi_0}{3}[2V_0 - (1+\phi_0)V']$ which should appear in the scalar field equation (10) can be absorbed into a coordinate redefinition and we neglect it. Also the term V_0 is responsible for effects of DE and further we will not take this term into account.

To obtain the field equations in the weak field limit (7) we use the Nutku gauge conditions [13]:

$$h^{\alpha}_{\beta,\alpha} - \frac{1}{2}\delta^{\alpha}_{\beta} h^{\mu}_{\mu,\alpha} = \frac{\varphi_{,\beta}}{1+\phi_0}. \tag{11}$$

In the first PN order ($\sim 1/c^2$) field equation (10) takes the form:

$$\left(\nabla^2 - m_{\varphi}^2\right)\varphi^{(2)} = \frac{8\pi G\phi_0}{3c^4}\rho. \tag{12}$$

The solution of this screened Poisson equation is

$$\varphi^{(2)} = -\frac{2G\phi_0}{3c^4}\mathscr{U}e^{-m_{\varphi}r}, \tag{13}$$

where $\mathscr{U} = M/r$ is a source potential, M is the mass of the source and index (2) denotes the first PN order.

The equations (7) and (9) in the first PN order also reduces:

$$\nabla^2\left(h^{(2)}_{00} - \frac{\varphi^{(2)}}{1+\phi_0}\right) = -\frac{8\pi G}{(1+\phi_0)c^2}\rho, \tag{14}$$

and

$$\nabla^2\left(h^{(2)}_{ij} + \delta_{ij}\frac{\varphi^{(2)}}{1+\phi_0}\right) = -\frac{8\pi G}{(1+\phi_0)c^2}\rho\,\delta_{ij}, \tag{15}$$

and their solutions are

$$h^{(2)}_{00} = \frac{2G}{(1+\phi_0)c^2}\mathscr{U}\left(1 - \frac{\phi_0}{3}e^{-m_{\varphi}r}\right), \tag{16}$$

and

$$h^{(2)}_{ij} = \frac{2G}{(1+\phi_0)c^2}\delta_{ij}\mathscr{U}\left(1 + \frac{\phi_0}{3}e^{-m_{\varphi}r}\right). \tag{17}$$

respectively.

Comparing expressions (16), (17) with the metric (4), values of the PPN-parameters can be obtained [9]

$$G_{eff} = \frac{G}{(1+\phi_0)}\left(1 - \frac{\phi_0}{3}e^{-m_{\varphi}r}\right), \tag{18}$$

$$\gamma = \frac{1+\phi_0 e^{-m_{\varphi}r}/3}{1-\phi_0 e^{-m_{\varphi}r}/3}. \tag{19}$$

The remaining part of the equations (7,10) give the contribution to the next post-Newtonian order ($\sim 1/c^4$). For the scalar field the solution is following:

$$\begin{aligned}
\varphi^{(4)} = \quad & \frac{(3\phi_0+1)G^2\phi_0}{9(1+\phi_0)c^4}\mathscr{U}^2 e^{-2m_\varphi r} - \frac{2G\phi_0}{3c^4}\int d^3r'\Pi'\rho'\frac{e^{-m_\varphi|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \\
+ \quad & \frac{2G\phi_0}{c^4}\int d^3r' p'\frac{e^{-m_\varphi|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} - \frac{4G^2\phi_0}{3(1+\phi_0)c^4}\int d^3r' U'\rho'\frac{e^{-m_\varphi|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \\
- \quad & \frac{2(3\phi_0-1)G^2\phi_0}{9(1+\phi_0)c^4}\int d^3r' U'\rho' e^{-m_\varphi r'}\frac{e^{-m_\varphi|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \\
+ \quad & \frac{G^2\phi_0 m_\varphi^2}{3\pi(1+\phi_0)c^4}\int d^3r' U'^2 e^{-m_\varphi r'}\frac{e^{-m_\varphi|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \\
+ \quad & \frac{(7\phi_0+1)G^2\phi_0 m_\varphi^2}{36\pi(1+\phi_0)c^4}\int d^3r' U'^2 e^{-2m_\varphi r'}\frac{e^{-m_\varphi|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \\
- \quad & \frac{G^2\phi_0^2}{27\pi c^4}\left[V' - (1+\phi_0)V'' - \frac{(1+\phi_0)\phi_0}{2}V'''\right]\int d^3r'\frac{U'^2 e^{-2m_\varphi r'} e^{-m_\varphi|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \\
+ \quad & \frac{G\phi_0}{3c^2}\partial_t\partial_t\int d^3r'\rho'\frac{e^{-m_\varphi|\vec{r}-\vec{r}'|}}{m_\varphi} \qquad (20)
\end{aligned}$$

Solving the 00-part of equation (7) in the second post-Newtonian order and substituting the solution (20), we obtain:

$$\begin{aligned}
h_{00}^{(4)} = \quad & \frac{G^2\phi_0}{9(1+\phi_0)c^4}\mathscr{U}^2 e^{-2m_\varphi\vec{r}} - \frac{2G^2}{(1+\phi_0)^2c^4}\mathscr{U}^2\left[1-\frac{\phi_0}{3}e^{-m_\varphi\vec{r}}\right]^2 \\
- \quad & \frac{2G^2\phi_0^2}{3(1+\phi_0)^2c^4}\int d^3r' U'\rho' e^{-m_\varphi\vec{r}'}\frac{e^{-m_\varphi|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} + \frac{4G^2}{(1+\phi_0)^2c^4}\int d^3r'\frac{U'\rho'}{|\vec{r}-\vec{r}'|} \\
+ \quad & \frac{4G^2\phi_0}{3(1+\phi_0)^2c^4}\int d^3r' U'\rho'\frac{e^{-m_\varphi\vec{r}'}}{|\vec{r}-\vec{r}'|} + \frac{2G^2\phi_0}{9(1+\phi_0)^2c^4}\int d^3r' U'\rho' e^{-m_\varphi\vec{r}'}\frac{e^{-m_\varphi|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \\
+ \quad & \frac{2G}{(1+\phi_0)c^4}\int d^3r'\Pi'\rho'\left[1-\frac{\phi_0}{3}e^{-m_\varphi|\vec{r}-\vec{r}'|}\right]\frac{1}{|\vec{r}-\vec{r}'|} \\
+ \quad & \frac{6G}{(1+\phi_0)c^4}\int d^3r' p'\left[1+\frac{\phi_0}{3}e^{-m_\varphi|\vec{r}-\vec{r}'|}\right]\frac{1}{|\vec{r}-\vec{r}'|} + \frac{4G}{(1+\phi_0)c^4}\int d^3r'\rho' v'^2\frac{1}{|\vec{r}-\vec{r}'|} \\
- \quad & \frac{1}{4\pi(1+\phi_0)}\int d^3r'\Bigg(-\frac{(7\phi_0+1)G^2\phi_0}{9(1+\phi_0)c^4}m_\varphi^2 U'^2 e^{-2m_\varphi\vec{r}'} - \frac{4G^2\phi_0}{3(1+\phi_0)c^4}m_\varphi U'^2 e^{-m_\varphi\vec{r}'} \\
+ \quad & \frac{4G^2\phi_0^2}{27c^4}\left[V' - (1+\phi_0)V'' - \frac{(1+\phi_0)\phi_0}{2}V'''\right]U'^2 e^{-2m_\varphi\vec{r}'}\Bigg)\frac{e^{-m_\varphi|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \\
- \quad & \frac{2G^2\phi_0^2}{36\pi(1+\phi_0)c^4}\int d^3r' U'^2 V''\frac{e^{-2m_\varphi\vec{r}'}}{|\vec{r}-\vec{r}'|} + \frac{G}{c^2(1+\phi_0)}\partial_t\partial_t\int d^3r'\rho'|\vec{r}-\vec{r}'| \\
+ \quad & \frac{G\phi_0}{3c^2(1+\phi_0)}\partial_t\partial_t\int d^3r'\rho'\frac{e^{-m_\varphi|\vec{r}-\vec{r}'|}}{m_\varphi} \qquad (21)
\end{aligned}$$

Comparing obtained PPN-metric for hybrid gravity with the general one (4) it is possible to identify PPN parameters $\alpha_3 = \zeta_1 = \zeta_3 = \zeta_4 = \xi = 0$. Also we can find the analytical expression for β:

$$\beta = 1 - \frac{\phi_0(\phi_0+1)e^{-2m_\varphi r}}{18(1-\frac{\phi_0}{3}e^{-m_\varphi r})^2}. \qquad (22)$$

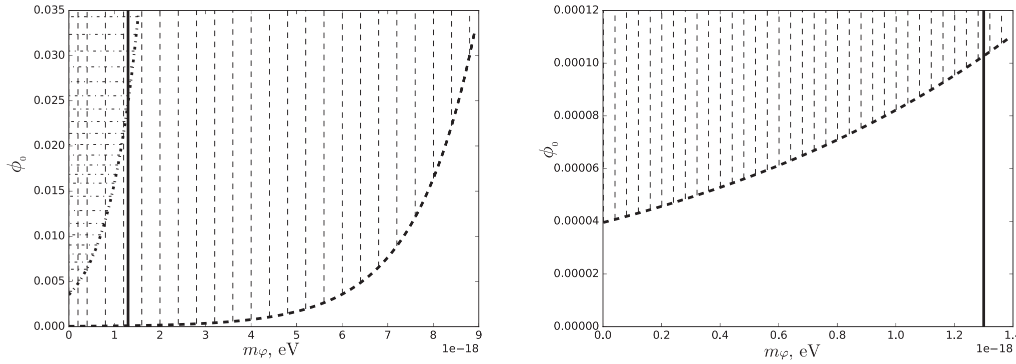

Figure 1. Dependence of scalar field background value upon the scalar field mass. Two figures are shown allowable regions at different scales. The vertical dotted region corresponds to excluded values obtained from parameter γ, the horizontal dotted region corresponds to excluded values obtained from parameter β, the vertical solid line is the critical value of scalar mass $m_\varphi = \frac{1}{r=1AU}$.

After all these procedures, only terms which are multiplied by m_φ, V', V'', V''' remain. All these terms contribute in the parameter ζ_2. But we should neglect terms which include m_φ because we consider case $m_\varphi << 1/r$. According to theorem of Lee [14] in every Lagrangian-based metric theory of gravity PPN parameters $\alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 \equiv 0$, which means that $V' - (1+\phi_0)V'' - \frac{(1+\phi_0)\phi_0}{2}V''' << 1/r$ and $\phi_0^2 V'' << 1/r$. In hybrid f(R)-gravity $\zeta_2 = 0$.

To obtain the values of remaining parameters α_1 and α_2 it is necessary to consider the equation (8). The solution of this equation is

$$h_{0j}^{(3)} = -\frac{4G}{(1+\phi_0)c^3}\int \rho' v_j \frac{1}{|\vec{r}-\vec{r}'|}. \tag{23}$$

According to [11] it is possible to transfer the terms with two temporal derivatives from equation (21) to (8) changing one temporal derivative to the spatial one. As the result we have

$$\begin{aligned} h_{0j}^{(3)} = & - \frac{4G}{(1+\phi_0)c^3}\int \rho' v_j \frac{1}{|\vec{r}-\vec{r}'|} + \frac{G}{2c^2(1+\phi_0)}\partial_t\partial_j \int \rho' |\vec{r}-\vec{r}'| d^3\vec{r}' \\ & + \frac{G\phi_0}{6c^2(1+\phi_0)}\partial_t\partial_j \int \rho' \frac{e^{-m_\varphi|\vec{r}-\vec{r}'|}}{m_\varphi} d^3\vec{r}' = -\frac{7G}{2(1+\phi_0)c^3}\int \frac{\rho' v'_j}{|\vec{r}-\vec{r}'|} d^3 r' \\ & - \frac{G}{2(1+\phi_0)c^3}\int \frac{\rho' r'_j(\vec{v}'\cdot\vec{r}')}{|\vec{r}-\vec{r}'|^3}\left[1 - \frac{\phi_0}{3}e^{-m_\varphi|\vec{r}-\vec{r}'|}\right] d^3 r' \\ & - \frac{G\phi_0}{6(1+\phi_0)c^3}\int \frac{\rho' v'_j}{|\vec{r}-\vec{r}'|} e^{-m_\varphi|\vec{r}-\vec{r}'|} d^3 r' \end{aligned} \tag{24}$$

From above equation it follows that $\alpha_1 = \alpha_2 = 0$.

5 Imposing of restrictions on m_φ and ϕ_0

Now we have all instruments for obtaining constraints on the parameters of hybrid f(R)-gravity. In this theory only two PPN parameters (γ and β) differ from GR quantities. Using the analytical expressions of these parameters (19,22) and experimental data from Tab.1 we restrict the asymptotic value of the field far away from the local system ϕ_0 and mass of the scalar field m_φ on the Fig.1.

The constraints obtained from the γ are much more stringent than those obtained from the β. The considering limit of very light scalar field implies a small value of ϕ_0. This fact could allow for the existence of a long-range scalar field able to modify the cosmological dynamics, but leaving unaffected the Solar System.

6 Conclusion

In this work we developed the post-Newtonian limit for hybrid metric-Palatini f(R)-gravity. On the example of this theory it was shown that in the limiting case $m_\varphi << 1/r$ it is possible to apply post-Newtonian formalism to models with massive scalar field. We obtained values of 10 PPN parameters in this theory in the considered limit and proved that 8 PPN parameters are equal to zero $\alpha_1 = \alpha_2 = \alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = \xi = 0$. Using the experimental data from the Solar System (values of γ and β) the parameters of hybrid f(R)-gravity (ϕ_0 and m_φ) were constrained and the most stringent restrictions are imposed by the parameter γ.

Thus, we showed that the hybrid f(R)-gravity satisfies the experiments in the Solar System in the case of very light scalar field. In the future, it is planned to study hybrid f(R)-gravity in the strong-field limit and compare it with observational data from binary pulsars.

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