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Manifestations of Horndeski theory in binary systems with pulsars.

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Abstract. Subclass of Horndeski gravity without Vainstein screening mechanism is tested in strong field regime of binary pulsars. Two specific cases of the Horndeski gravity are considered: the hybrid metric-Palatini f(R)-gravity and massive Brans-Dicke theory. The observational data of PSR J1738+0333, PSR J0737-3039, PSR J1012+5307 is used for obtaining constraints.

1 Introduction

There are two problems in modern phisics, which can not be explained by the generally recognized gravitational theory (General Relativity (GR)). The first of them is Dark Matter (DM), it's invisible matter, which fills up in galaxies and manifests itself only in the gravitational interaction [1]. The second problem is Dark Energy (DE), this phenomenon is responsible for accelerated expansion of the Universe [2]. One of the ways to solve these problems is to develope modified gravity. Adding the scalar field to GR is one of possible expansions of gravitational theory.

The Horndeski theory is the most general scalar-tensor theory with the second-order field equations which evades Ostrogradski instabilities [3]. Horndeski gravity suggests solutions for some GR's problems. For example, the scalar field can play the role of DE and explain the accelerating expansion of the Universe [4]. This theory has recently been studied extensively in the context of cosmology [5] and physics of black holes [6] and it has already been tested in cluster lensing [7], the cosmic microwave background (CMB) data [8] etc. Special attention should be paid to the recent work [9], where constraints for this theory were obtained by comparing LIGO data for event GW170817 [10] and the concomitant gamma-ray burst GRB 170817A [11]. In our work we also study the gravitational radiation in Horndeski gravity but use the binary pulsars data. In work [12] authors study the similar problem, but they restrict their consideration with massless case.

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In our work we test the subclass of Horndeski gravity without Vainstein screening mechanism in the strong field regime of binary pulsars. The discovery of binary pulsar PSR 1913 + 16 in 1974 [13] has opened a new posibility for testing gravitational theories [14]. In such systems gravitational field is stronger than in Solar System. Moreover, high stability of the pulse arrival allows to extract the dynamics of the orbital motion with such accuracy at which the effects of gravitational waves emission could appear. It provides an additional opportunity to verify the predictions of theories.

In this work, we test the Horndeski gravity in mixed binary systems and neutron star-neutron star binary (PSR J0737-3039) with quasi-circular orbits and impose restrictions on the parameters of Horndeski model.

2 Horndeski theory

2.1 Action

The action of Horndesky theory is given by [18]:

$$S = \frac{c^4}{16\pi G} \sum_{i=2}^{5} \int d^4x \sqrt{-g} L_i + S_m, \tag{1}$$

where c is the speed of light, g is the determinant of the metric, and S_m is the standard matter action. L_i are the gravitational Lagrangian densities:

$$\begin{aligned} L_2 = G_2(\phi, X), \;\; L_3 = -G_3(\phi, X)\Box\phi, \;\; L_4 = G_4(\phi, X)R + G_{4X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2], \\ L_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \tfrac{G_{5X}}{6}\Big[(\Box\phi)^3 + 2(\nabla_\mu\nabla_\nu\phi)^3 - 3(\nabla_\mu\nabla_\nu\phi)^2\Box\phi\Big], \end{aligned} \tag{2}$$

where $G_{\mu\nu}$ is the Einstein tensor, R is Ricci scalar, ϕ is the scalar field, $X = -1/2\nabla_\mu\phi\nabla^\mu\phi$, ∇_μ is the covariant derivative, $\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi$, $G_i(\phi, X)$ are arbitrary functions of the scalar field ϕ and its kinetic term X, $G_{iX} = \frac{\partial G_i}{\partial X}$. In subclass of Horndeski theory without Vinstein mechanism, where arbitrary functions take the following form:

$$G_2(\phi, X) = G_2(\phi, X), \;\; G_3(\phi, X) = G_3(\phi), \;\; G_4(\phi, X) = G_4(\phi), \;\; G_{4X} = 0, \;\; G_5(\phi, X) = 0 \tag{3}$$

where $G_2(\phi, X)$ contains no more than the first order in X.

The weak and strong equivalence principles are violated in some modified gravitational models. In scalar-tensor theories the inertial mass and internal structure depend on the scalar field. Eardley first showed that in the scalar-tensor gravity the influence of the scalar field on the internal structure of the body can be expressed through the assumption that the mass of the body is an arbitrary function of the scalar field [15]. So the matter action for a system of point-like masses can be written as

$$S_m = -c^2 \sum_a \int m_a(\phi) d\tau_a, \tag{4}$$

where $m_a(\phi)$ are inertial masses of particles labeled by a and τ_a is the proper time of the particle a measured along its world-line x_a^μ.

2.2 Field equations

Pulsars have the strong surface gravitational potential $\Phi_{NS} = (G_N M_{NS})/(c^2 R_{NS}) = 0.2$ (where G_N is Newtonian gravitational constant, M_{NS} is the mass of the neutron star, R_{NS} is its radius), but we intend to calculate the energy flux carried away by gravitational radiation from binary pulsars at large distances from the source, and surface gravitational potential does not have a significant influence on the metric $g_{\mu\nu}$ and the scalar field ϕ. Therefore, we can expand the scalar and tensor fields in the approximation of weak field limit and small velocities ($v/c \ll 1$) around cosmological background ϕ_0 and Minkowski background [19] $\eta_{\mu\nu}$ (post-Newtonian limit (PPN) [20]):

$$\phi = \phi_0 + \varphi, \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{5}$$

$h_{\mu\nu}$ and φ are the small perturbations of tensor and scalar fields of order $O(v^2/c^2)$, respectively. Also four arbitrary functions $G_2(\phi, X), G_3(\phi, X), G_4(\phi, X), G_5(\phi, X)$ can be expanded in Taylor's series around the scalar cosmological background:

$$G(\phi, X) = \sum_{m,n=0}^{\infty} G_{(m,n)} \varphi^m X^n, \quad G_{(m,n)} = \frac{1}{m!n!} \frac{\partial^{m+n}}{\partial^m \phi \partial^n X} G(\phi, X)\bigg|_{\phi=\phi_0, X=0}, \tag{6}$$

where $G_{(m,n)}$ are constants.

Also we can write stress-energy tensor, its trace, and $\frac{\partial T}{\partial \varphi}$ in the near zone in terms of Eardly method [19] for point mass approximation (binary system case):

$$\begin{aligned} T^{\mu\nu} &= \textstyle\sum_a m_a(\phi) u^\mu u^\nu \left(1 - \frac{h^k_k}{2} - \frac{v_a^2}{2c^2} + s_a \frac{\varphi}{\phi_0}\right) \delta^3(\vec{r} - \vec{r}_a(t)), \\ T &= -c^2 \textstyle\sum_a m_a(\phi) \left(1 - \frac{h^k_k}{2} - \frac{v_a^2}{2c^2} + s_a \frac{\varphi}{\phi_0}\right) \delta^3(\vec{r} - \vec{r}_a(t)), \\ \frac{\partial T}{\partial \varphi} &= -c^2 \textstyle\sum_a \frac{m_a(\phi)}{\phi_0} \left[s_a \left(1 - \frac{h^k_k}{2} - \frac{v_a^2}{2c^2}\right) - (s'_a - s_a^2 + s_a) \frac{\varphi}{\phi_0} \right] \delta^3(\vec{r} - \vec{r}_a(t)), \end{aligned} \tag{7}$$

where v_a is the velocity of the object labeled in a. The quantities s_a and s'_a are the "first and second sensitivities". These parameters firstly were introduced by Eardley [15]:

$$s_a \equiv \frac{\partial(\ln m_a)}{\partial(\ln \phi)}\bigg|_{\phi_0}, \quad s'_a \equiv \frac{\partial^2(\ln m_a)}{\partial(\ln \phi)^2}\bigg|_{\phi_0}. \tag{8}$$

If we want to consider gravitational radiation in far zone at the point of detector, we must solve a field equation in near zone, where gravitational radiation is generated. The field equations within the post-Newtonian (PN) approximation in the 1st PN order $O(v/c)^2$ take the form [21]:

$$\Box \theta_{\mu\nu} = -\frac{16\pi G}{c^4 G_{4(0,0)}} T_{\mu\nu}, \tag{9}$$

$$\Box \varphi - m_\varphi^2 \varphi = \frac{16\pi G}{c^4} c_\varphi S, \tag{10}$$

where

$$\begin{gathered} m_\varphi^2 = \frac{G_{2(2,0)}}{2G_{3(1,0)} - G_{2(0,1)} - 3\frac{G_{4(1,0)}^2}{G_{4(0,0)}}}, \quad c_\varphi = -\frac{G_{4(1,0)}}{2G_{4(0,0)}\left(2G_{3(1,0)} - G_{2(0,1)} - 3\frac{G_{4(1,0)}^2}{G_{4(0,0)}}\right)}, \\ S = T - \frac{2G_{4(0,0)}}{G_{4(1,0)}} \frac{\partial T}{\partial \varphi}. \end{gathered} \tag{11}$$

In these equations we also use the following notations:

$$\theta_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h - \frac{G_{4(1,0)}}{G_{4(0,0)}}\eta_{\mu\nu}\varphi, \quad \theta = -h - 4\frac{G_{4(1,0)}}{G_{4(0,0)}}\varphi, \tag{12}$$

and transverse gauge $\partial_\mu \theta^{\mu\nu} = 0$. The Eq.(10) is analogue of inhomogeneous Klein-Gordon-Fock equation, where the parameter m_φ is the inverse Compton wavelength of the scalar field. In this paper we work in the CGS system thus here and further the scalar field mass m_φ has the dimension of inverse centimeter (cm^{-1}).

Let's obtain the solution of these equations at leading order. For the scalar field, it takes the following form [21]:

$$\varphi = \frac{4c_\varphi}{c^2}\sum_a \frac{Gm_a}{r_a}\left(1 - \frac{2s_a}{\phi_0}\frac{G_{4(0,0)}}{G_{4(1,0)}}\right)e^{-m_\varphi r_a}, \tag{13}$$

where $r_a = |\vec{r} - \vec{r}_a(t)|$. Then we solve Eq.(9) within the 1PN approximation in the near zone [21]:

$$\theta_{00} = \tfrac{4}{c^2 G_{4(0,0)}}\sum_a \tfrac{Gm_a}{r_a} + O\left(\tfrac{v}{c}\right)^4, \quad \theta_{ij} = \tfrac{4v_i v_j}{c^4 G_{4(0,0)}}\sum_a \tfrac{Gm_a}{r_a} + O\left(\tfrac{v}{c}\right)^6,$$
$$\theta = -\tfrac{4}{c^2 G_{4(0,0)}}\sum_a \tfrac{Gm_a}{r_a} + O\left(\tfrac{v}{c}\right)^4. \tag{14}$$

It is important to emphasize that in scalar-tensor theories weak equivalence principle (WEP) can be violated [22]. So that is why we must modify equations of motion. In Horndeski theory the changes are following: gravitational constant becomes a function of the scalar field, parameters of the theory, sensitivities and distance between objects. We obtain the explicit form for Kepler's third law using the A. Einstein, L. Infeld, and B. Hoffmann (EIH) method [23]:

$$a^3(2\pi/P_b)^2 = \mathcal{G}_{12}m \tag{15}$$

and the orbital binding energy of such system is

$$E = -\frac{\mathcal{G}_{12}m\mu}{2a}, \tag{16}$$

here a is the semi-major axis, $m = m_1 + m_2$, and $\mu = m_1 m_2/m$, P_b is the orbital period of a binary system, $\mathcal{G}_{12}$ is the effective gravitational coupling constant between two compact objects:

$$\mathcal{G}_{ab} = \frac{G}{G_{4(0,0)}}\left\{1+(1+m_\varphi r_{ab})e^{-m_\varphi r_{ab}}\left[2c_\varphi G_{4(1,0)}\left(1-\frac{2s_b}{\phi_0}\frac{G_{4(0,0)}}{G_{4(1,0)}}\right)-\frac{4s_a c_\varphi G_{4(0,0)}}{\phi_0}\left(1-\frac{2s_b}{\phi_0}\frac{G_{4(0,0)}}{G_{4(1,0)}}\right)\right]\right\}, \tag{17}$$

In this work we use approximation where scalar field is responsible for DE effect: $m_\varphi r_{ab} \ll 1$ and $e^{-m_\varphi r_{ab}} \to 1$.

For obtaining constraint on parameters of Horndeski theory, we selected the binary systems where the most significant dissipative effect is caused by gravitational radiation. Therefore, the energy losses can be expressed via the first derivative of the orbital period using Eq.(15) and Eq.(16):

$$\frac{\dot{E}}{E} = -\frac{2}{3}\frac{\dot{P}_b}{P_b}. \tag{18}$$

Further, we'll use this expression for obtaining constraints.

3 Graviational radiation from binary pulsars

Energy flux of binary system is defined as [19]

$$\left\langle \dot{E} \right\rangle = -cr^2 \int d\Omega \left\langle t_{0i} \right\rangle , \tag{19}$$

where t_{0i} is component of stress-energy pseudotensor. For calculation pseudotensor, we use Noether current method [24]:

$$t^{\alpha}_{\gamma} = \frac{c^4}{16\pi G}\left[\frac{G_{4(0,0)}}{2}\,\partial_\gamma\theta^{\mu\nu}\partial^\alpha\theta_{\mu\nu} + \frac{G_{4(1,0)}}{2G_{4(0,0)}c_\varphi}\partial_\gamma\varphi\partial^\alpha\varphi + \delta^\alpha_\gamma\left(G_{2(2,0)}\varphi^2 - \frac{G_{4(0,0)}}{4}\partial_\alpha\theta^{\mu\nu}\partial^\alpha\theta_{\mu\nu} - \right.\right.$$
$$\left.\left. - \frac{G_{4(1,0)}}{4G_{4(0,0)}c_\varphi}\partial_\mu\varphi\partial^\mu\varphi\right)\right]. \tag{20}$$

The energy flux consist of two part: tensor and scalar. The tensor part is

$$\left\langle \dot{E}_g \right\rangle = -\frac{32\mu^2 G(\mathcal{G}_{12}m)^3}{5c^5 G_{4(0,0)}R^5} = -\frac{32\mu^2 m^3}{5c^5 G^4_{4(0,0)}R^5}\left\{1 + \left[2c_\varphi G_{4(1,0)}\left(1 - \frac{2s_b}{\phi_0}\frac{G_{4(0,0)}}{G_{4(1,0)}}\right) - \right.\right.$$
$$\left.\left. - \frac{4s_a c_\varphi G_{4(0,0)}}{\phi_0}\left(1 - \frac{2s_b}{\phi_0}\frac{G_{4(0,0)}}{G_{4(1,0)}}\right)\right]\right\}^3, \tag{21}$$

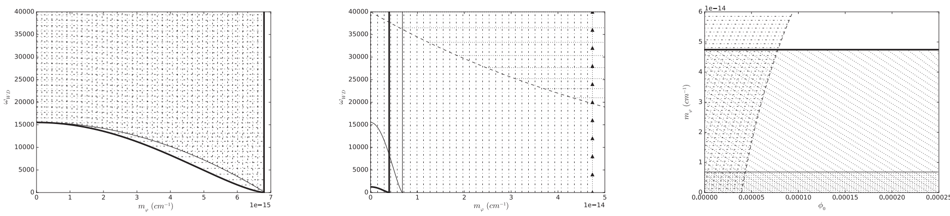
where R is radius of the reduced mass μ orbit and in the case of quasi-circular orbit in Eq.(15) $R = a$. The quantity $\mathcal{G}_{12}$ is the effective gravitational constant between components of binary system (17). The scalar part is

$$\left\langle \dot{E}_\varphi \right\rangle = -\frac{32\mu^2(\mathcal{G}_{12}m)^3}{5c^5 G_{4(0,0)}R^5}\left[\frac{5c^2 G_{4(1,0)}c_\varphi R}{48\mathcal{G}_{12}m}\left(A_d^2 + A_d\bar{A}_d\frac{2\mu}{c^2R}\right)\left[1 - \left(\frac{P_b c m_\varphi}{2\pi}\right)^2\right]^{\frac{3}{2}} + \right.$$
$$\left. + \frac{G_{4(1,0)}c_\varphi}{3}A_q^2\left[1 - \left(\frac{P_b c m_\varphi}{2\pi}\right)^2\right]^{\frac{5}{2}} - \frac{G_{4(1,0)}c_\varphi}{96}A_d A_o\left[1 - \left(\frac{P_b c m_\varphi}{2\pi}\right)^2\right]^{\frac{5}{2}}\right], \tag{22}$$

where $A_d, \bar{A}_d, A_q, A_o$ are functions of theory parameters, sensitivities and cosmological background. As seen from (22), the scalar part includes dipole term (term with A_d), PN correction to dipole term (term with $\bar{A}_d$), quadrupole term (term with A_q), dipole-octupole term (term with A_o). In contrast to the tensor part, which includes only the quadrupole term.

We can obtain the expression for first derivative of orbital period:

$$\dot{P}^{th}_b = -\frac{192\pi\mu}{5m}\left(\frac{2\pi Gm}{c^3 G_{4(0,0)}P_b}\right)^{\frac{5}{3}}\left\{1+\left[2c_\varphi G_{4(1,0)}\left(1-\frac{2s_2}{\phi_0}\frac{G_{4(0,0)}}{G_{4(1,0)}}\right)-\frac{4s_1 c_\varphi G_{4(0,0)}}{\phi_0}\left(1-\frac{2s_2}{\phi_0}\frac{G_{4(0,0)}}{G_{4(1,0)}}\right)\right]\right\}^{\frac{2}{3}}$$
$$\times\left\{1+\frac{5c^2 c_\varphi G_{4(0,0)}(s_2 - s_1)}{24\phi_0}\left\{\left(\frac{2G_{4(0,0)}(s_2 - s_1)}{G_{4(1,0)}\phi_0}\right)\left(\frac{P_b G_{4(0,0)}}{2\pi m}\right)^{2/3}\left(1+\left[2c_\varphi G_{4(1,0)}\left(1-\frac{2s_2}{\phi_0}\frac{G_{4(0,0)}}{G_{4(1,0)}}\right)-\right.\right.\right.\right.$$
$$\left.\left.-\frac{4s_1 c_\varphi G_{4(0,0)}}{\phi_0}\left(1-\frac{2s_2}{\phi_0}\frac{G_{4(0,0)}}{G_{4(1,0)}}\right)\right]\right)^{-2/3}+\frac{2\mu G_{4(0,0)}}{c^2 m}\left[-\frac{7}{2G_{4(0,0)}}\left(\frac{m_2}{m_1}-\frac{m_1}{m_2}\right)+\frac{1}{G_{4(1,0)}\phi_0}\left(\frac{7m_2 s_1}{m_1}-\frac{7m_1 s_2}{m_2}+\right.\right.$$
$$\left.+6s_1-6s_2\right)+\frac{23}{4}c_\varphi\frac{G_{4(1,0)}}{G_{4(0,0)}}\left(\frac{m_2}{m_1}-\frac{m_1}{m_2}\right)+\frac{c_\varphi}{\phi_0}\left(\frac{7m_1 s_2}{m_2}-\frac{7m_2 s_1}{m_1}+\frac{23m_1 s_1}{2m_2}-\frac{23m_2 s_2}{2m_1}+4s_1-4s_2\right)+\frac{G_{4(0,0)}c_\varphi}{G_{4(1,0)}\phi_0^2}$$
$$\times\left(\frac{14s_1 s_2 m_2}{m_1}-\frac{14s_1 s_2 m_1}{m_2}+8s_1-8s_2+\frac{8m_2 s_1}{m_1}-\frac{8m_1 s_2}{m_2}+\frac{9s_2^2 m_1}{m_2}-\frac{9s_1^2 m_2}{m_1}+\frac{8m_2 s_1'}{m_1}-\frac{8m_1 s_2'}{m_2}+8s_1'-8s_2'+\right.$$
$$\left.+8s_2^2-8s_1^2\right)+\frac{G^2_{4(0,0)}c_\varphi}{G^2_{4(1,0)}\phi_0^3}\left(\frac{18s_1^2 s_2 m_2}{m_1}-\frac{18s_2^2 s_1 m_1}{m_2}+16s_1^2 s_2-16s_2^2 s_1+\frac{16m_1}{m_2}-\frac{16m_2}{m_1}+\frac{16s_1 s_2' m_1}{m_2}-\right.$$



a) b) c)

Figure 1. Brans-Dicke theory. a) Dependence of the ω_{BD} upon the scalar field mass in the case of the PSR J1738+0333. The solid line and region of vertical dot-dash lines correspond to the results of the article [16]. The bold line and region of horizontal dot-dash lines describe the results of our work. The vertical bold line is the critical value of scalar mass ($m_\varphi = \omega/c$). b) Dependence of the ω_{BD} upon the scalar field mass. The solid line corresponds to the PSR J1738+0333. The bold line describes the results for the PSR J1012+5307. The line with triangles represents the PSR J0737-3039. The region with vertical dot-dash lines contains the allowable values obtained from binary pulsars. The region with horizontal dotted lines contains the allowable values obtained from Cassini data for γ_{PPN} [28]. Dependence of the scalar field mass upon scalar field background value. Figure c) represent allowable regions at different scales. The region with oblique dotted lines corresponds to allowable values in the case of the PSR J0737-3039, the horizontal solid lines are the critical value of scalar mass $m_\varphi = 2\omega/c$, the vertical dotted lines describe allowable values in the case of the PSR J1738+0333, the horizontal dot-dash lines correspond to allowable values from the γ_{PPN}.

$$-\frac{16 s_2 s_1' m_2}{m_1}+16 s_1 s_2'-16 s_2 s_1'\Big)\Big]\Big(1+\Big[2c_\varphi G_{4(1,0)}\Big(1-\frac{2s_2}{\phi_0}\frac{G_{4(0,0)}}{G_{4(1,0)}}\Big)-\frac{4 s_1 c_\varphi G_{4(0,0)}}{\phi_0}\Big(1-\frac{2s_2}{\phi_0}\frac{G_{4(0,0)}}{G_{4(1,0)}}\Big)\Big]\Big)^{-1}\Big\}$$

$$\times\Big[1-\Big(\frac{P_b c m_\varphi}{2\pi}\Big)^2\Big]^{\frac{3}{2}}+\frac{G_{4(1,0)}c_\varphi}{3}\Big(1-\frac{2G_{4(0,0)}(s_2 m_1+s_1 m_2)}{G_{4(1,0)}m\phi_0}\Big)^2\Big[1-\Big(\frac{P_b c m_\varphi}{4\pi}\Big)^2\Big]^{\frac{5}{2}}-\frac{c_\varphi}{96}\Big(\frac{2G_{4(0,0)}(s_2-s_1)}{\phi_0}\Big)$$

$$\times\Big(\frac{m_1-m_2}{m}-\frac{2G_{4(0,0)}}{G_{4(1,0)}\phi_0}\frac{s_2 m_1^2-s_1 m_2^2}{m^2}\Big)\Big[1-\Big(\frac{P_b c m_\varphi}{2\pi}\Big)^2\Big]^{\frac{5}{2}}\Big\}. \tag{23}$$

4 Observational constrains on Horndeski gravity and special cases from binary pulsars

For obtaining constraints on the theories we use the following method:

$$\left|\frac{\dot{P}_b^{th}}{\dot{P}_b^{GR}}-\frac{\dot{P}_b^{obs}}{\dot{P}_b^{GR}}\right|\leq 2\sigma. \tag{24}$$

Here we compare the predicted quantity $\dot{P}_b^{th}/\dot{P}_b^{GR}$ and the observational quantity $\dot{P}_b^{obs}/\dot{P}_b^{GR}$ at 95% confidence level, where σ is the observational uncertainty.

For case with sensitivities we obtain the constraints from PSR J1738+0333 [26]:

$$\left|\frac{\mathcal{G}_{12}^{\frac{2}{3}}}{G_{4(0,0)}}\left[1+\frac{5c_\varphi}{12}\Big(\frac{P_b c^3}{2\pi m\mathcal{G}_{12}}\Big)^{\frac{2}{3}}\Big(1-\frac{m_\varphi^2 c^2 P_b^2}{4\pi^2}\Big)^{\frac{3}{2}}\Big(\frac{G_{4(0,0)}^2(s_{NS}-s_{WD})^2}{G_{4(1,0)}\phi_0^2}\Big)\right]-0.93\right|\leq 0.26. \tag{25}$$

where s_{NS} is sensitivity of neutron star and s_{WD} is sensitivity of white dwarf. Here we keep only leading orders in scalar and tensor parts. For case without sensitivities from PSR J0737-3039 [27]:

$$\left|1.003 - \frac{\mathcal{G}_{12}^{\frac{2}{3}}}{G_{4(0,0)}}\left[1 + \frac{G_{4(1,0)}c_\varphi}{3}(1 - 4\times 10^{26} m_\varphi^2)^{\frac{5}{2}}\right]\right| \leq 0.028. \tag{26}$$

Let's take a closer look to some important special cases of Hordeski theory. One of them is Brans-Dicke theory. We can pass over to consideration of this case via following transformations:

$$G_2 = \frac{2\omega_{BD}}{\phi}X + V(\phi),\ G_3 = 0,\ G_4 = \phi,\ G_5 = 0, \tag{27}$$

The dependence of the quantity ϕ_0 upon ω_{BD} for massive and massless Brans-Dicke cases [16] is

$$\phi_0 = \frac{4 + 2\omega_{BD}}{3 + 2\omega_{BD}}. \tag{28}$$

Manifestations of Brans Dicke theory in binary pulsars has already account in this article [16], but in this work we take into consideration PN corrections to the scalar dipole term and scalar dipole-octupole term. This difference present on Fig.1a. Constraints for this theory from binary pulsars and Solar System are presented on Fig.1b. The best result is provided by combining PSR J1012+5307[25] and Solar System test [28]:

$$\omega_{BD} \geq 36000, \quad m_\varphi \leq 4\times 10^{-15}(cm^{-1}). \tag{29}$$

Now, let's consider hybrid f(R)-gravity [29]. Transfer from Horndeski theory to this theory is given by:

$$G_2 \quad = -\tfrac{3X}{\phi} - V(\phi),\ G_3 = 0,\ G_4 = 1 + \phi,\ G_5 = 0 \tag{30}$$

Constraints on parameters of this theory from binary pulsars and Solar System are presented on Fig.1c and Fig.1d. The bests results is provided by combining PSR J1738+0333 and Solar System tests [28]:

$$\phi_0 \leq 0.00004, \quad m_\varphi \leq 1.4\times 10^{-14}(cm^{-1}). \tag{31}$$

5 Conclusion

In this paper we obtained constraints in the strong-field regime on the subclass of Horndeski theories without the Vainstein mechanism for case with (25) and without sensitivities(26). We showed that more accurate than in [16] consideration significant.

As we can see on Fig.1b and Fig.1c the constraints in the Solar system are better than the restrictions from pulsars in the case of massless theories, but in the case of massive theories, both kinds of constraints make their unique contribution.

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