Confinement, Monopoles and Gauge Invariance

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Abstract. The status is reviewed of the lattice studies of confinement by dual superconductivity of *QCD* vacuum. Progress is made by proving that the existence and the creation of a monopole are gauge invariant concepts. This proof fully legitimates an existing order parameter, and with it the evidence it provides of monopole condensation in the confining vacuum.

1 Introduction

Quarks are strictly confined in nature. No free quark has ever been observed neither in ordinary matter nor in the final state of high energy reactions. This results in upper limits on the abundance n_q of quarks and on the inclusive cross section σ_q for production of quarks in, say, proton-proton collisions

$$R_n \equiv \frac{n_q}{n_p} \leq 10^{-27} \qquad\qquad R_\sigma \equiv \frac{\sigma_q}{\sigma_{TOT}} \leq 10^{-15} \tag{1}$$

Here n_q is the abundance of protons, σ_{TOT} the total $p-p$ cross section. The expectations for these numbers would be, if quarks could exist as free particles[1],

$$R_n \approx 10^{-12} \qquad\qquad R_\sigma \approx 1 \tag{2}$$

The natural explanation of an inhibition factor smaller than 10^{-15} is that R_n and σ_q are strictly zero: confinement is an absolute property protected by some symmetry. As a consequence the deconfinement transition is a change of symmetry.

The symmetry involved can not be a subgroup of the gauge group, since the order parameter has to be gauge invariant [2]. A hint for the nature of this symmetry is provided by the following argument. Let $U_G(\vec{x})$ be a time-independent transformation of the gauge group on a system of fields. In general

$$U_G(\vec{x}) = U(\vec{x})U_B(\vec{x}) \tag{3}$$

where $U(\vec{x})$ is the usual gauge transformation acting on the bulk of the system, which is trivial [$U(\vec{x}) = 1$] on the surface S_2 at spatial infinity. $U_B(\vec{x})$ is instead trivial [$U_B(\vec{x}) = 1$] everywhere except on S_2.

If all the fields vanish on S_2 $U_B(\vec{x})$ is irrelevant and can be put equal to 1 and U_G reduces to $U(\vec{x})$, the usual gauge transformation. It is instead relevant if there are degrees of freedom living on S_2. These degrees of freedom can produce a gauge invariant change of symmetry. Living on a sphere they can have non trivial topology.

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The natural topological excitations in 3 spatial dimensions are monopoles, defined by the mapping $S_2 \to SU(2)$. Vortices would be the natural excitations in 2 spatial dimensions with a mapping $S_1 \to U(1)$.

The above argument supports the idea that confinement be produced by dual superconductivity of the vacuum [3] [4]. Dual means that electric and magnetic are interchanged with respect to ordinary superconductors. Monopoles condense in the QCD vacuum in the same way as Cooper pairs do in ordinary superconductors, and quarks are confined by Abrikosov chromo-electric flux tubes in the same way as magnetic charges are confined by magnetic flux tubes in ordinary superconductors.

Lattice is a convenient non perturbative tool to explore large distance properties of QCD like confinement, for which the perturbative expansion fails. In particular monopoles have been extensively studied on lattice to verify the mechanism of confinement by dual superconductivity of the vacuum, proposed in [3] [4], and further developed in [5]. Two main strategies have been used to that purpose

1) Define an operator $\mu(\vec{x})$ which creates a monopole in the point $\vec{x}$ and use its vacuum expectation value $\langle\mu\rangle \equiv \langle 0|\mu|0\rangle$ as the order parameter for dual superconductivity of vacuum [6] [7][8][9][10][11].

2) Detect monopoles in lattice configurations and determine the constraint effective potential of their field. A minimum at non zero field signals dual superconductivity of the vacuum [12].

Both approaches look for a change of symmetry at the deconfinement transition. Both have to choose a $U(1)$ subgroup of the gauge group in which monopoles live, which is named Abelian Projection (AP) . A natural AP exists in the Higgs model in which monopoles were originally discovered [13][14], and is the unitary representation in which the Higgs field Φ is diagonal. Monopoles sit at the zeroes of Φ.[See Sect. 2 below]

In QCD there is no fundamental Higgs field. A possibility could be that any operator in the adjoint representation can act as effective Higgs field [5], and its zeroes as locations of the monopoles. Somehow physics should be independent on the specific choice of the operator [5].

Following this suggestion the operator $\mu(\vec{x})$ of strategy 1) was defined and developed in Ref [8][10] using the simplest choice for the colour orientation of the effective Higgs field, the nominal 3-direction used in the numerical updating procedure.

As for strategy 2) what one needs is a reliable way to detect monopoles in lattice configurations in order to determine numerically from them the effective potential. The way this is done is by measuring the magnetic flux of the abelian field in the colour direction of the effective Higgs field: excess of magnetic flux through a plaquette is interpreted as Dirac string of a monopole[15]. If one repeats this game in different abelian projections and compares, the result is rather disturbing: the number and the location of monopoles strongly depend on the abelian projection. The existence of a monopole seems to be a gauge dependent concept.

In this paper we discuss these issues and in particular we show that the existence or the creation of a monopole is a gauge invariant, AP independent concept [Sect. 2]. This legitimates the construction of $\langle\mu\rangle$ used to implement strategy 1) and the idea that all choices for effective Higgs field are equivalent [5].

As for strategy 2) we discuss in some detail the operators in the theory which could act as effective Higgs fields in the sense of Ref [5]. The result is that a key role is played by the temporal component $\vec{A}_4 = i\vec{A}_0$ of the gauge field in the gauge $\partial_4 \vec{A}_4 = 0$, i.e. the AP in which the Polyakov line is diagonal.

An intrinsic difficulty in the method is to detect monopoles in the confined phase, in which the magnetic field is shielded, the magnetic photon being massive [Sect. 3].

Regarding the AP dependence of the monopoles observed in lattice configurations we argue that it is due to discretisation artefacts in the procedure used to detect monopoles on the lattice. This is discussed in Sect.3. Sect 4 contains some concluding remarks.

A copious literature exists on monopoles in QCD based on the concept of dominance, i.e. on the demonstration that removing somehow the monopoles from the lattice configurations removes confinement [16] [17]. In this optics some abelian projection (Maximal Abelian Gauge) looks better than others, with less lattice artefacts. We do not consider this approach relevant to understand the symmetry governing confinement.

2 Monopoles in Non-Abelian Gauge Theories

Monopoles were discovered as solitons in the broken phase of the $SU(2)$ Higgs model with the scalar field $\vec{\Phi}$ in the adjoint representation [13][14]. The Lagrangian is

$$L = -\frac{1}{4}\vec{G}_{\mu\nu}\vec{G}_{\mu\nu} + \frac{1}{2}D_\mu\vec{\Phi}D_\mu\vec{\Phi} - \frac{m^2}{2}\vec{\Phi}\vec{\Phi} - \frac{\lambda}{4}(\vec{\Phi}\vec{\Phi})^2 \tag{4}$$

The monopole solution has the form

$$\Phi^a = \rho\frac{x^a}{|\vec{x}|}H(\xi), \quad A_0^a = 0, \quad A_i^a = -\frac{1}{gx^2}\epsilon_{iab}x_b[1-K(\xi)] \tag{5}$$

Here $\xi = g\rho|\vec{x}|$, $\qquad \rho \equiv \sqrt{(-\frac{m^2}{\lambda})}$.

$H(\xi)$ and $K(\xi)$ can be determined numerically. In order to have a solution with finite energy (soliton) they must obey the boundary conditions

$$K(\infty) = 0, \quad K(0) = 1 \qquad H(\infty) = 1, \quad H(0) = 0 \tag{6}$$

$\vec{\Phi}$ has a zero at $\vec{x} = 0$, which is conventionally assumed to be the position of the monopole. At small $|\vec{x}|$

$$\Phi^a_{\vec{x}\to 0} \approx constant \times x^a \tag{7}$$

At large $|\vec{x}|$

$$\Phi^a{}_{|\vec{x}|\to\infty} = \rho\frac{x^a}{|\vec{x}|} \tag{8}$$

Eq(8) is a non trivial mapping of the sphere S_2 on $SU(2)/U(1)$: its winding number is the magnetic charge.

The unitary gauge, in which $\vec{\Phi}\vec{\sigma}$ is diagonal ($\vec{\Phi}\vec{\sigma} = |\vec{\Phi}|\sigma_3$) is by definition the AP identified by Φ. The abelian magnetic field in this gauge $\quad B_k^3 \equiv \frac{1}{2}\epsilon_{kij}F_{ij}^3$, ($F_{ij}^3 \equiv \partial_i A_j^3 - \partial_j A_i^3$), can be given a manifestly gauge invariant form in terms of the 'tHooft tensor F_{ij} [13] . If we denote by $\hat{\Phi} = \frac{\vec{\Phi}}{|\vec{\Phi}|}$ is the orientation of $\vec{\Phi}$ in colour space

$$F_{ij} \equiv \hat{\Phi}[\vec{G}_{ij} - \frac{1}{g}D_i\hat{\Phi}\wedge D_j\hat{\Phi}] \tag{9}$$

In F_{ij} the terms quadratic in the gauge field cancel by construction, linear terms are proportional to a spatial derivative $\partial_k\hat{\Phi}$ and vanish in the unitary gauge in which $\hat{\Phi}$ is constant, so that only the abelian part of $\vec{G}_{ij}$ is left , $F_{ij} = F^3_{ij}$ and for the magnetic field $B^3_k = B_k = \frac{1}{2}\epsilon_{kij}F_{ij}$. At distances $r > \frac{1}{\rho}$ the field $\vec{B}$ is that of an abelian magnetic monopole

$$\vec{B} \approx_{r\to\infty} \frac{1}{g}\frac{\vec{r}}{r^3} + Dirac - string \tag{10}$$

We shall choose to put the Dirac string along the positive z-axis $z+$. This can be done since the magnetic charge obeys the Dirac quantisation condition and any string crossing space from side to side is invisible. The magnetic charge M is equal to the magnetic flux through the Dirac string and is given by

$$M = \oint_c \hat{\Phi}\vec{A}_i dx_i \tag{11}$$

with c a closed path on S_2 encircling $z+$. Indeed, by use of Stokes theorem, in the unitary gauge where $\vec{\nabla}\hat{\Phi}^a = 0$, $\quad M = \int_S d\vec{\sigma}\hat{\Phi}^a\vec{\nabla} \wedge (\vec{A}^a) = \int_S d\vec{\sigma}\vec{B}$. S is a surface which has c as border, and is thus pierced by the Dirac string.

M is gauge invariant in the sense that $\quad M' = \oint_c \hat{\Phi}'\vec{A}'_i dx_i = M \quad$. Indeed if $\hat{\Phi}' = R\hat{\Phi}$ and $\vec{A}'_i = \vec{A}_i + \frac{i}{g}R\partial_i R^\dagger$

$$\oint_c \hat{\Phi}'\vec{A}'_i dx_i = \oint_c (\hat{\Phi}\vec{A}_i + \frac{i}{g}\hat{\Phi}\partial_i R^\dagger) dx_i \tag{12}$$

The second term in Eq(12) vanishes if R is a global transformation, but also if $\hat{\Phi}$ is constant i.e. in the unitary representation.

Suppose that two (or more) different fields of the theory both in the adjoint representation, $\vec{\Phi}$ and $\vec{\Phi}'$ can act as effective Higgs fields and define monopole configurations and let $\hat{\Phi}$ and $\hat{\Phi}'$ be the corresponding directions in colour space. A transformation R of the gauge group exists , with non trivial U_B Eq.(3), such that on S_2 $\hat{\Phi}' = R\hat{\Phi}$, and hence also a transformation R' which transforms the AP of Φ into the AP of Φ', which is a global transformation. As a consequence $M = M'$: the magnetic charge is AP invariant. Creating a monopole in one specific AP means creating it in all AP's.

The argument can be generalized to larger gauge groups G [21][22], where monopoles live in $SU(2)$ subgroups.

In QCD there is no fundamental Higgs field, but effective Higgs fields can exist allowing the existence of monopoles. One example is the temporal component $\vec{A}_4 = i\vec{A}_0$ in the Gauge $\partial_0\vec{A}_0 = 0$ [18] [19] [20]. We shall discuss this issue in Sect.3. Anyhow the statement that the magnetic charge is AP invariant is the same no matter if the Higgs field is fundamental or effective.

This rigorously justifies the simplification used in the strategy 1) by creating the monopole in the AP defined by the 3-axis used in numerical updates of the configurations [9][10]. However it makes hard to explain the empirical observation made on lattice in strategy 2) that monopoles defined by different abelian projections look different both in their number and in their locations.

3 Monopoles in QCD

Monopoles are needed in QCD to produce by condensation the dual superconductivity of the vacuum, but no fundamental Higgs field exists in the system to produce a non trivial mapping of S_2 on $SU(2)$ subgroups of the gauge group. A solution to this difficulty proposed in Ref[5] is that any operator $\vec{O}$ in the adjoint representation can act as an effective Higgs field. The corresponding AP is the representation in which $\vec{O}\vec{\sigma}$ is diagonal, and the zeroes of $\vec{O}$ are the locations of the monopoles. The argument is [5] that in that gauge near the zero

$$\langle \vec{O}\vec{\sigma} \rangle \approx costant \times |\vec{x}|\sigma_3 \tag{13}$$

at least if there is invariance under rotations and O is a scalar under space rotations. For a classical configuration $\vec{O} = \langle \vec{O} \rangle$ and the behaviour Eq(13) is the same as that of the Higgs field in the monopole solution Eq(7). Of course the behaviour Eq(7) is a necessary, but not generally sufficient condition for a monopole solution. The condition Eq(8) must also be satisfied which expresses the topological nature of monopoles. The effective Higgs field is a function of the classical gauge fields, transforming in the adjoint representation

$$\Phi_{eff} = \Phi(\vec{A}_\mu) \tag{14}$$

at least for pure gauge theory (no quarks).

If we insist in putting $\vec{A}_0 = 0$ as in the solution Eq(5) Φ_{eff} only depends on $\vec{A}_i$ which vanishe on S_2, and must have at the same time a constant length on S_2 to satisfy Eq.(8). This looks impossible.

If instead we allow non zero $\vec{A}_0$ the requirement that the solution be static implies

$$\partial_0 \vec{A}_0 = 0 \tag{15}$$

An obvious possibility for the effective Higgs field is the choice $\vec{\Phi}_{eff} = \vec{A}_4 = i\vec{A}_0$. To have a monopole the boundary conditions Eq(7) and Eq(8) must be satisfied by $\vec{A}_4$. This is the solution of Ref. [18] [19][20].

Of course $\vec{A}_4$ on S_2 is the same for all monopoles: this fixes the monopoles which can coexist. A detailed discussion is presented in Ref.[22], where the construction is also extended to $SU(N)$ gauge groups with $N > 2$.

$\partial_0 \vec{A}_0 = 0$ is a gauge condition. In this gauge the Polyakov line $\Pi(\vec{x})$ has a simple expression

$$\Pi(\vec{x}) \equiv P \exp(ig \int_0^T \vec{A}_0(\vec{x}, t)\frac{\vec{\sigma}}{2}) = \exp(i\frac{g}{T}\vec{A}_0(\vec{x})\frac{\vec{\sigma}}{2}) \tag{16}$$

T is the temperature, P indicates time ordered product, which simplifies to ordinary product if $\vec{A}_0$ is time independent.

The gauge in which $\vec{A}_0(\vec{x})\vec{\sigma}$ is diagonal coincides with that in which the Polyakov line is diagonal, and has been extensively studied on lattice in connection with confinement.

4 The order parameter

An operator $\mu(\vec{x}, t)$ which creates a monopole in the location $\vec{x}$ at time t is [8]

$$\mu(\vec{x}, t) = \exp\left(i \int d^3y \vec{E}^3_\perp(\vec{y}, t)\vec{A}^0_\perp(\vec{y} - \vec{x})\right) \tag{17}$$

$\vec{A}^0_\perp(\vec{y} - \vec{x})$ is the classical vector potential produced in the point $\vec{y}$ by a monopole sitting at the point $\vec{x}$, in the transverse gauge $\vec{\nabla}\vec{A}^0_\perp = 0$. It can either be a point magnetic charge with a Dirac string carrying the flux at infinity , or a Wu-Yang type configuration in which the flux is carried to infinity by a 2-dimensional surface of singularity. In the convolution in Eq(17) only the transverse components of the electric field $\vec{E}^3$ contribute. $\vec{E}^3$ is the (canonical) electric field if the theory has gauge group $U(1)$, or its component in the direction 3 of the effective Higgs field in a AP of non abelian theories.

In any case, no matter how the theory is quantized, $\vec{E}^3_\perp(\vec{y},t)$ is the canonically conjugate field to $\vec{A}^3_\perp(\vec{y},t)$. Therefore, in the same way as for a motion in one dimension

$$\exp(ipa)|x\rangle = |x+a\rangle \tag{18}$$

in the Schroedinger representation of the field theory

$$\mu(\vec{x},t)|\vec{A}^3_\perp(\vec{z},t)\rangle = |\vec{A}^3_\perp(\vec{z},t) + \vec{A}^0_\perp(\vec{z}-\vec{x})\rangle \tag{19}$$

The operator $\mu(\vec{x},t)$ adds a monopole to field configurations.

The peculiar dependence on the coupling constant allows to compute $\langle\mu\rangle$ in a convenient way [7] [9]. If we normalise the fields as done in lattice formulation the action depends on the coupling constant as βS , with $\beta \equiv \frac{2N}{g^2}$ for gauge group $SU(N)$ and S independent on β

In Eq(17) the electric field is the canonical field, i.e. $\frac{1}{g}$ times the field in the lattice normalisation. An extra factor $\frac{1}{g}$ comes from the magnetic charge in the monopole field $\vec{A}_0$, so that finally μ has the form

$$\mu(\vec{x}) = \exp(-\beta\Delta S)$$

with $\Delta S = \frac{g^2}{2N}(i\int d^3y \vec{E}^3_\perp(\vec{y},t)\vec{A}^0_\perp(\vec{y}-\vec{x}))$ independent on β.

In the language of Feynman integrals

$$\mu(\beta) \equiv \langle\mu(\beta)\rangle = \frac{\int dA \exp\left(-\beta(S+\Delta S)\right)}{\int dA \exp\left(-\beta S\right)} = \frac{Z(S+\Delta S)}{Z(S)} \tag{20}$$

the ratio of two partition functions. The quantity $\rho(\beta) = \frac{d}{d\beta}\ln\left(\langle\mu(\beta)\rangle\right)$ is easily computed from the expression (20) giving

$$\rho(\beta) = \langle S\rangle_S - \langle(S+\Delta S)\rangle_{(S+\Delta S)} \tag{21}$$

The subscript indicates the action used to perform the average. Since $\mu(0) = 1$

$$\mu(\beta) = \exp\left(\int_0^\beta \rho(\beta')d\beta'\right) \tag{22}$$

Computing numerically ρ on using the expression (21) presents no difficulty. The expectation is, if confinement is produced by dual superconductivity, that in the confined phase ($\beta < \beta_c$) ρ stays finite in the infinite volume limit so that $\mu(\beta)$ is different from zero; instead ρ goes large and negative in the deconfined phase, $\beta > \beta_c$ so that $\mu(\beta)$ vanishes in the infinite volume limit. In pure gauge theory there is an independent order parameter for confinement, the string tension measured from correlators of Polyakov lines, which permits the check.

At the beginning this research program, which was already rigorously checked in the simple case of gauge group $U(1)$ [7], worked perfectly within the statistical errors for gauge groups $SU(2)$ and $SU(3)$ [9]. In trying to extend the analysis to G_2 gauge group, to test the general validity of the mechanism a divergence of ρ in the large volume limit showed up, which would give $\mu(\beta) = 0$ both below ad above the critical temperature β_c[23]. A reanalysis of the previous work showed that the same disease would appear at large volumes or with larger statistics ad smaller errors also for the case of $SU(2)$ and $SU(3)$ gauge groups.

The origin of the problem was understood and removed in Ref [10] by improving the order parameter

$$\langle 0|\mu|0\rangle \to \mu' = \frac{\langle 0|\mu|0\rangle}{\sqrt{(\langle 0|0\rangle)}\sqrt{(\langle\mu 0|\mu 0\rangle)}} \tag{23}$$

which acts as an order parameter even when $\langle\mu 0|\mu 0\rangle$ and $\langle 0|\mu|0\rangle$ vanish in the infinite volume limit. In the case of gauge group $U(1)$ the problem does not exist and μ and μ' differ by a finite multiplicative constant.

For gauge group $SU(2)$ the improvement was numerically checked in all details in Ref [10], including the scaling behaviour at the deconfining phase transition. For $SU(3)$ and possibly G_2 we are still collecting data.

The last open question about the order parameter, the choice of the abelian projection along the 3-axis used in numerical updating, is given a positive answer by the arguments presented above. Any AP is legitimate, in particular the one used in [9] and [10]. A rigorous test thus exists of the confinement mechanism of Ref. [3] [4].

Of course strategy 2) should provide the same answer. However the last existing works on the subject, by the ITEP group and the Kanazawa Group in which the effective potential of the monopole field is computed by a sophisticated procedure from the monopoles observed in lattice configurations, do not provide stable and clear evidence for a minimum at non zero field [See Ref. [24] for a detailed discussion and an accurate bibliography]. One reason for that can be that a fraction of the monopoles used in that analysis are not AP invariant, they are discretisation artefacts, and can pollute the procedure employed to determine the potential. Maybe repeating the procedure at smaller lattice spacings thus reducing the probability of large fluctuations of the abelian magnetic flux through plaquettes would improve the result.

5 Discussion

The stringent experimental limits on the existence of free quarks indicate that there is a symmetry behind confinement, and that deconfinement is a change of symmetry. The only possible gauge invariant symmetry breaking in a gauge theory involves degrees of freedom living on the 2-dimensional surface at spatial infinity, and monopoles come out as natural candidates, and with them dual superconductivity of the vacuum [3] [4].

Monopoles are $U(1)$ configurations, and there exist infinitely many ways of choosing that $U(1)$ in a non abelian system. Each choice is called an Abelian Projection. For long time it has been believed that monopoles living in different AP 's can be different from each other. The number and the positions of the monopoles observed in lattice configurations look indeed generically different.

Monopoles, however, are defined as classical configurations with non trivial topology [13] [14], and topology can not be changed by continuous gauge transformations. What we show is that the magnetic charge of a configuration is gauge and AP invariant. Creating a monopole in any abelian projection is equivalent to create it in all AP's. This allows to define in a simple way the operator which creates a monopole and with it the order parameter, which is its vacuum expectation value [10]. Both in $U(1)$ [7] and in $SU(2)$ pure gauge [10] theories it has been demonstrated that confinement is due to condensation of monopoles in the vacuum. Work is in progress on $SU(3)$ and G_2 pure gauge systems.

An alternative technique to compute the order parameter consists in computing the effective potential of the monopole field and in showing that it has a minimum at non zero field. The effective potential is computed from the positions of the monopoles in lattice configurations. The fact that no convincing results are achieved [24] can depend on the fact that the input monopoles are strongly AP dependent, which means that many of them are probably lattice artefacts. The situation, however, can be improved e.g. by going at larger values of

$\beta = \frac{2N}{g^2}$ thus reducing fluctuation on the magnetic flux across plaquettes in terms of which monopole are usually defined on Lattice. Finally the two strategies must give the same result.

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