Scalar leptoquarks: From GUT to B anomalies

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Abstract. Leptoquarks are often used to explain B meson anomalies. The minimalistic approach assumes only one light leptoquark explaining both puzzles. However, one scalar leptoquark cannot explain both anomalies. Two relatively light leptoquarks can be accommodated within $SU(5)$ GUT and might nicely explain both anomalies not being in a conflict with any low-energy flavour constraints or LHC results.

1 Introduction

Several experimental results points towards the possible violation of the lepton number universality in the B meson semileptonic decays. The charge current anomaly indicates the deviation from the measured and theoretically predicted ratios $R_{D^{(*)}} = \mathcal{B}(B \to D^{(*)}\tau\bar{\nu})/\mathcal{B}(B \to D^{(*)}\bar{\nu})$, $l = e,\mu$. It was found that the measured ratios are larger than the Standard Model predictions $R_{D^{(*)}}^{SM}$. The experimental results on $R_{D}$ [1–3] differ by $\sim 2\sigma$ with respect to the SM prediction [4] and by $\sim 3\sigma$ in the case of $R_{D^{*}}$ [5–7]. The similar deviations were found in transitions $b \to c\ell\nu$ ($\ell = \mu, \tau$), observed in $R_{J/\psi}$ ratio between $B_c \to J/\psi\ell\nu$ decay widths [8].

In the case of flavour changing neutral current transition (FCNC) $b \to s\mu^+\mu^-$ the LHCb experiment has measured ratios $R_{K^{(*)}} = \mathcal{B}(B \to K^{(*)}\mu\mu)/\mathcal{B}(B \to K^{(*)}ee)$ at low di-lepton invariant mass. Interestingly, these ratios were found to be systematically lower than expected in the SM. In the case of the $K$ meson in the final state, the ratio $R_{K}$ was measured in the kinematical region $q^2 \in [1.1, 6]$ GeV$^2$ [9], while $R_{K^{*}}$ was measured in the region $q^2 \in [0.045, 1.1]$ GeV$^2$ [10]. The three measured $R_{K^{(*)}}$ deviate from the SM predictions at $\sim 2.5\sigma$ level [11, 12].

2 $B$ meson anomalies and leptoquarks

Both $B$ anomalies are usually explained by contributions of New Physics (NP). The effective Lagrangian studies suggest that NP appears at TeV scale [12–15]. Most of these approaches favour the left-handed fermionic interactions. In addition, this new interaction prefers interactions of third fermion generation. As suggested in Ref. [16] the scale of such NP, accommodating the $R_{D^{(*)}}$ anomaly, can be introduced as

$$\mathcal{L}_{NP}^{D^{(*)}} = \frac{2}{\Lambda_{D^{(*)}}} \bar{c}_L Y_{\mu} b_L \tau_L \gamma^\mu \nu_L,$$

where \(\Lambda_{D^{(*)}}\) is the scale of such NP.
The experimental data can be explained for the scale $\Lambda_{D^{(\ast)}_S} \approx 3$ TeV. The $R_{K^{(\ast)}}$ anomaly can be explained by the Lagrangian

$$L_{NP}^{K^{(\ast)}} = \frac{1}{\Lambda_{K^{(\ast)}}} \bar{s}_L Y_a b_L \bar{\mu}_L Y'_a \mu_L,$$

(2)

from which one finds that the scale of NP is $\Lambda_{K^{(\ast)}} \approx 30$ TeV. Obviously, by requiring that the same model of NP explains both anomalies, that the NP in $R_{K^{(\ast)}}$ should be suppressed in comparison with $R_{D^{(\ast)}}$. The suppression can be realised in a number of different ways. For example $R_{D^{(\ast)}}$ receives contribution of NP at tree level, while in $R_{K^{(\ast)}}$ NP enters at loop level in particular model of leptoquarks [17, 18]. The approach of [12, 13] assumes that the second generation of quarks and leptons couples to a NP mediator weakly in comparison with the coupling to third generation. The third possibility is a “Cabibbo mechanism” [19, 20] in which $\cos \theta$ ($\sin \theta$) can determine size of NP contribution. Among many NP proposals, leptoquarks (LQs) explanations are very appealing due to their interactions with quarks and leptons [12, 13] can relatively good explain both anomalies. This state gives only V-A form of the interaction with quarks and leptons [12, 13].

Leptoquarks are listed by quantum numbers of $SU(3)_{C}$ colour, electroweak quantum numbers of $SU(2)_{L}$ and hypercharge $U(1)_{Y}$ [21]. The hypercharge is defined as $Y = Q - I_{3}$. According to their SM quantum numbers, scalar leptoquarks are: one weak triplet $S_3 = (\bar{3},3,1/3)$, two weak doublets $R_2 = (3,2,7/6), \tilde{R}_2 = (3,2,1/6)$ and three singlets $\bar{S}_1 = (\bar{3},1,4/3), S_1 = (\bar{3},1,1/3)$ and $\tilde{S}_1 = (\bar{3},1,-2/3)$. Among all these states only weak doublet $R_2$ and $\tilde{R}_2$ do not cause proton decay at tree level [21, 22]. The rest of leptoquarks, might mediate proton decay at tree level if there is no additional symmetry forbidding it. For example it was noticed in [22, 23] that within particular $SU(5)$ GUT model $S_3$ does not cause proton decay. Spin-1 leptoquarks are following: $U_3 = (3,3,2/3), V_2 = (\bar{3},2,5/6), \bar{V}_2 = (\bar{3},2,-1/6), U_1 = (3,1,5/3), U_1 = (3,1,2/3)$ and $\tilde{U}_1 = (3,1,-1/3)$. Among all these states it was found that $U_1$ leptoquarks coupling to the left handed quarks and leptons [12, 13] can relatively good explain both anomalies. This state gives only V-A form of the interaction with quarks and leptons.

The scalar leptoquark $S_3$, being a weak triplet, can couple only to the left-handed quarks and leptons. It can alone well explain $R_{K^{(\ast)}}$ puzzle. In the model presented in [24] we succeeded to accommodate $b \rightarrow s t^{\pm} l^{\mp}$ sector. Because of its weak triplet nature it couples also to up-type quarks and neutrinos which is a couplings needed to address $R_{D^{(\ast)}}$. If one explains $R_{D^{(\ast)}}$ puzzle, $S_3$ then inevitably contributes to other well constrained flavour observables. For example, after detailed study we have demonstrated [24] that the most constraining bounds from $R_{D^{(\ast)}}^{\nu \nu} = B(B \rightarrow K^{(\ast)} \bar{\nu} \nu)/B(B \rightarrow K^{(\ast)} \bar{\nu} \nu)_{SM}$ and $\Delta m_{s}$, allow only for minor improvement of $R_{D^{(\ast)}}$ puzzle.

The $R_2 = (3,2,1/6)$ leptoquark can accommodate $R_{D^{(\ast)}}$ by the coupling to a right-handed neutrino [26]. Such mechanism induces contributions which is not interfering with SM. In this year the idea of right-handed neutrino contributing to $R_{D^{(\ast)}}$ was further explored by [27–29]. The $\tilde{R}_2 = (3,2,1/6)$ leptoquark can explain $R_K$ while it cannot accommodate $R_{K^{(\ast)}}$ [26]. Namely $R_K$, due to the matrix element of two pseudoscalar is blind on the axial part of the current and therefore the measurement of $R_K^{(\ast)}$ helps to distinguish between the chiralities of NP operators [30].

The $S_1 = (\bar{3},1,1/3)$ leptoquark was extensively explored by [17]. Being a weak singlet this leptoquark has left-handed and right-handed couplings. Unfortunately, this state is severely constrained by the low-energy flavour dynamics [18]. Among scalar leptoquarks remaining states are $\bar{S}_1 = (\bar{3},1,4/3)$ and $\tilde{S}_1 = (\bar{3},1,-2/3)$. The chirality of the charged leptons of the $S_1$ is not adequate for the $R_{K^{(\ast)}}$ explanation, while $S_1$ cannot couple to charge leptons (see e.g. [21]).
As noticed in the studies of [13] and [25] one scalar leptoquark cannot satisfy satisfactorily described both anomalies without being in a conflict with all low-energy constraints. Two light leptoquarks can also generate neutrino masses [23]. Even more, two relatively light leptoquarks can be accommodated within GUT theories [21]. These are additional arguments why scenarios with two leptoquarks are appealing.

### 3 SU(5) GUT scenario with $R_2$ and $S_3$

In order to build an UV complete model the simplest proposal is to construct SU(5) GUT models [19]. In the simplest SU(5) scenario one can accommodate both $R_2$ and $S_3$ to be relativley light, masses being in the TeV regime. In this scenario scalar sector needs to contain 45 and 50 representations, whereas the SM fermions are set in $\bar{5}$ and 10 (for details see [19]). The interactions of $R_2$ and $S_3$ with the SM fermions are

$$
\mathcal{L} \supset Y_L^{ij} \bar{Q}_L^i \ell_R^j R_2 + Y_L^{ij} \bar{u}_R^i R_2^3 L_j^I + Y_R^{ij} \bar{Q}_R^i \ell_L^j L_j^I + Y_R^{ij} \bar{c}_R^i \ell_L^j S_3^I, \quad (3)
$$

where $Y_L$, $Y_R$, and $Y$ are Yukawa matrices, $\tau_i$ denote the Pauli matrices. For the Yukawa matrices we use $Y_R = Y_L^T$, $Y = -Y_L$ and assume that the matrix $(Y_RE_R^{\dagger})_{33} = y_R^{br}$, and $(U_R Y_L)_{22} = y_L^{c\mu}$ and $(U_R Y_L)_{23} = y_L^{c\tau}$, with the rest being negligible small.

Following [19], we allow the two $R_2$ scalars be members of representations 45 in 50. They can mix (for details see [19]) allowing us to end up with the two light scalars, i.e., $R_2$ and $S_3$, and one heavy $R_2$. The heavy $R_2$ state is completely decoupled from the low-energy spectrum. The relevant lagrangian, in the mass eigenstate basis of the two light LQs, is we write Lagrangian

$$
\mathcal{L} \supset + s_\phi (VbE_R^{\dagger})^i j \bar{u}_L^i \ell_R^j R_2^5 + s_\phi (bE_R^{\dagger})^i j \bar{d}_L^i \ell_R^j R_2^5
$$

$$
+ c_\phi (U_R aU)^i j \bar{u}_R^i \ell_L^j S_3^5 - c_\phi (U_R aU)^i j \bar{d}_R^i \ell_L^j S_3^5
$$

$$
2^{-\frac{1}{2}} (aU)^i j \bar{d}_L^i \ell_L^j S_3^5 - (V^* aU)^i j \bar{u}_L^i \ell_L^j S_3^5
$$

$$
+ a^i j \bar{d}_L^i \ell_L^j S_3^5 + 2^{-\frac{i}{2}} (V^* aU)^i j \bar{u}_L^i \ell_L^j S_3^5, \quad (4)
$$

where we define the mixing angle between the two $R_2$’s to be $\phi$. It is now easy to derive $Y = -a/\sqrt{2}$, $Y_L = c_\phi a$, and $Y_R = Y_L^T = s_\phi b$. For the maximal mixing $s_\phi = c_\phi = 1/\sqrt{2}$ for both $R_2$ states we can recover relation $Y_R = Y_L^*$. $Y = -Y_L$. Obviously, the two SU(5) operators proportional to $a$ and $b$ might to generate three Yukawa matrices $Y$, $Y_L$, and $Y_R$. After such set-up, we consider the NP parameters $m_{R_2}$, $m_{S_3}$, $y_R^{br}$, $y_L^{c\mu}$, $y_L^{c\tau}$, and $\theta$. With this set-up we find out that the low-energy Yukawa couplings can originate from SU(5). Even more, we checked weather the Yukawa couplings have perturbative nature, meaning that $y_R^{br}$, $y_L^{c\mu}$, and $y_L^{c\tau}$ remain below $\sqrt{4\pi}$. Therefore, we have checked the renormalization group running from the electroweak to the GUT scale $5 \times 10^{15}$ GeV. For example, as we mentioned in [19] the running of the Yukawa coupling $y_R^{br}$, crucial in explaining the $R_{D^{0}}$ puzzle is given by

$$
16\pi^2 \frac{d \ln y_R^{br}}{d \ln \mu} = |y_L^{c\mu}|^2 + |y_L^{c\tau}|^2 + \frac{9}{2} |y_R^{br}|^2 + \frac{1}{2} y_L^{c\tau}^2 + \ldots, \quad (5)
$$

where $y_t$ is the top Yukawa coupling.
4 B anomalies in GUT scenario

4.1 Charged-current decays

The relevant effective Lagrangian for (semi-)leptonic decays is

\[ \mathcal{L}_{\text{eff}}^{d \to u l \bar{u} \ell} = -\frac{4 G_F}{\sqrt{2}} V_{ud} [(1 + g_{V_L})(\bar{u}_L \gamma_\mu d_L)(\bar{\ell}_L \gamma^\mu \nu_L) + g_{S_L}(\mu)(\bar{u}_R \gamma_\mu d_L)(\bar{\ell}_R \gamma^\mu \nu_L)], \]

where neutrinos are in the flavor basis. The effective Wilson coefficients of \( d \to u \ell \bar{\nu}_\ell \) are related to the LQ couplings at the matching scale \( \Lambda \approx 1 \text{ TeV} \) via the expression

\[ g_{S_L}(\Lambda) = 4 g_T(\Lambda) = \frac{4 G_F}{\sqrt{2}} \frac{m^2_{\bar{R}}}{G_F V_{ud}}, \]

\[ g_{V_L} = \frac{-4 G_F}{\sqrt{2}} \frac{m^2_{\bar{S}} G_F V_{ud}}{G_F V_{ud}}. \]

From the above equations we learn that the only transitions affected by \( R_2 \) in our scenario are \( b \to c \tau \bar{\nu}_\tau \). \( S_3 \), on the other hand, contributes to processes involving \( u, c, s, b, \) and \( \ell, \ell' = \mu, \tau \), but gives a negligibly small contribution to \( R_{D^{\mu, \tau}} \).

To compute \( R_D \) we employ the \( B \to D \) form factors calculated using the lattice QCD [4, 31], resulting in prediction \( R_{D}^{\text{SM}} = 0.29(1) \) which is \( \approx 2 \sigma \) below the experimental average \( R_{D}^{\text{exp}} = 0.41(5) \) [32–34]. On the other hand, the \( B \to D^* \) form factors have never been computed on the lattice at nonzero recoil. Thus, for \( R_{D^{\mu, \tau}} \) we consider the leading form factors extracted from the \( B \to D^* \bar{\nu} \) (\( \ell = e, \mu \)) spectra [35], which are combined with the ratios \( A_0(q^2)/A_1(q^2) \) and \( T_{1-3}(q^2)/A_1(q^2) \) computed in Ref. [6]. We obtain the value \( R_{D^{\mu, \tau}}^{\text{SM}} = 0.26(1) \) which is \( \approx 3 \sigma \) below the experimental average \( R_{D^{\mu, \tau}}^{\text{exp}} = 0.30(2) \) [35]. Moreover, the scalar (tensor) effective coefficients in Eq. 7 have to account for the SM running from the matching scale \( \mu = \Lambda \) down to \( \mu = m_y \). In the case of the vector coefficient there is no QCD running effects.

Following analysis of Ref. [24] our fit includes constraints from several (semi-)leptonic decays. Interestingly, the ratios for the light leptons, the analog of the one with \( \tau \) and \( \mu \), \( R_{D^{\mu, \tau}}^{\text{eff}} = \mathcal{B}(B \to D^{(\mu, \tau)} \bar{\nu}_L) / \mathcal{B}(B \to D^{(\mu, \tau)} \bar{\nu}_\ell) \) gives very important constraint. [18]. We also consider \( \mathcal{B}(B \to \tau \bar{\nu}) \) as well as the kaon ratio \( R_{K^{e/\mu}} \) which both agrees with their SM predictions (see for details [24]).

4.2 Neutral-current decays

The standard effective Hamiltonian for the \( b \to s \) (semi-)leptonic transition can be written as

\[ \mathcal{H}_{\text{eff}}^{b \to s \ell \bar{\nu}_\ell} = -\frac{4 G_F \lambda_t}{\sqrt{2}} \sum_{i=7,9,10} C_i(\mu) O_i(\mu), \]

where \( \lambda_t = V_{tb} V_{ts}^* \). The relevant operators for our discussion are

\[ O_{8(10)} = \frac{e^2}{(4 \pi)^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}_R \gamma^\mu (\gamma^5) l). \]

In our scenario, only \( S_3 \) contributes at tree-level [24]

\[ \delta C_9^{\mu} = -\delta C_{10}^{\mu} = \frac{\pi}{\lambda_t a_{em}} \frac{\nu^2}{m^2_{S_3}} y^{\mu} (y'^{\mu})^*. \]
Relying on the results of [36, 37] the 1σ interval $C^{ij}_{9} = -C^{ij}_{10} \in (-0.85, -0.50)$ is obtained by performing a fit to the clean $b \rightarrow sl l$ observables, namely, $R_{K}$, $R_{K^{*}}$, and $\mathcal{B}(B_{s} \rightarrow \mu\mu)$ [36, 37]. Contributions to the left-handed current operators in $b \rightarrow sl l$ transition immediately contribute to $B \rightarrow K^{(*)\nu\nu}$ decay amplitudes which are well constrained by experiments. These decays are governed by

$$\mathcal{L}_{\text{eff}}^{b \rightarrow sl l} = \frac{\sqrt{2}G_F\alpha_{\text{em}}\lambda_{i}}{\pi} c^{ij}_{L} (\bar{s}_{L} i \gamma_{\mu} b_{L})(\bar{\nu}_{l_{L}} \gamma_{\mu} \nu_{l_{L}}),$$

(12)

with $C^{ij}_{L} = \delta_{ij} C_{L}^{SM} + \delta C^{ij}_{L}$ being the Wilson coefficient with the SM contributions $C_{L}^{SM} = -6.38(6)$ [38] and the contribution $\delta C^{ij}_{L}$ from NP. Similarly as in the $b \rightarrow sl l$ transition, the only tree-level contribution to $b \rightarrow sv\bar{v}$ comes from the $S_{3}$ state and reads [24]

$$\delta C^{ij}_{L} = \frac{\pi}{2\alpha_{\text{em}}\lambda_{i}} \frac{e^{2}}{m_{S_{3}}^{2}} y^{b_{j}}(x_{t})^{*}, \quad i, j = \mu, \tau. \quad (13)$$

These effective coefficients modify the ratios $R_{\nu\nu}^{(s)} = \mathcal{B}(B \rightarrow K^{(*)\nu\nu})/\mathcal{B}(B \rightarrow K^{(*)\nu\nu})^{SM}$ in the following way

$$R_{\nu\nu}^{(s)} = \frac{\sum_{ij} \delta_{ij} C_{L}^{SM} + \delta C^{ij}_{L}/2}{|C_{L}^{SM}|^{2}}. \quad (14)$$

In our fit we use experimental bounds $R_{\nu\nu} < 3.9$ and $R_{\nu\nu}^{(s)} < 2.7$ [39] and data on the $B_{s} - \bar{B}_{s}$ mixing amplitude as in [19], modified by the box-diagrams containing two $S_{3}$, leading to the proportionality to $[(y^{t_{R}})^{2}]^{2}/m_{S_{3}}^{2}$. Then we explore the experimental limits on LFV $\tau$ decays, $\mathcal{B}(\tau \rightarrow \mu\phi)^{\text{exp}} < 8.4 \times 10^{-8}$ and $\mathcal{B}(\tau \rightarrow \mu\gamma)^{\text{exp}} < 4.4 \times 10^{-8}$ [40], muon anomalous magnetic moment with the $\approx 3.6 \sigma$ discrepancy with respect to the SM [41]. In out approach, however, $(g - 2)_{\mu}$, receives very small corrections. Then we explored bounds on the $Z \rightarrow l^{+}l^{-}$ couplings measured at LEP [42]. Namely, both $R_{2}$ and $S_{3}$ contribute at on loop level to these decays. The strong constraints come from the measured $D - \bar{D}$ mixing parameters and we have checked that our model is not in a conflict with experimental results obtained for the charm meson mixing.

5 Constraints and predictions

The observables we described above were used in the fit described in [19] by varying the parameters $y^{b_{R}}$, $y^{c_{L}}$, $y^{t_{R}}$ and $\theta$. First we learn that our fit requires $S_{3}$ to have larger mass than $R_{2}$. The masses $m_{R_{2}}$ and $m_{S_{3}}$, we consider to be the lowest values allowed by projected LHC constraints with $m_{R_{2}} = 800$ GeV and $m_{S_{3}} = 2$ TeV. Very interesting feature of our model is that $y^{t_{R}}$ should be a complex number [19]. The complex phase does not induce any observable CP violation. In our study the LHC constraints are taken into account [19]. Our best fit points are

$$\text{Re}[g_{S_{L}}] = 0, \quad \text{Im}[g_{S_{L}}] = 0.59 \left(\frac{+0.13}{-0.14}\right)_{1\sigma} \left(\frac{+0.20}{-0.29}\right)_{2\sigma}. \quad (15)$$

We find very interesting prediction for the lepton flavour violating process $\mathcal{B}(B \rightarrow K\mu\tau)$ which is found to be bounded, as presented in Fig. 2. At $1 \sigma$ level we find

$$1.1 \times 10^{-7} \leq \mathcal{B}(B \rightarrow K\mu^{\pm}\tau^{\mp}) \leq 6.5 \times 10^{-7}. \quad (16)$$

This value is bellow the current bound $\mathcal{B}(B \rightarrow K\mu\tau)^{\text{exp}} < 4.8 \times 10^{-5}$ [32].
Figure 1. The flavor fit constraints on the \( g_{S_L} \) plane for the transition \( b \to c\tau\bar{\nu}_\tau \). The allowed 1\( \sigma \) (2\( \sigma \)) regions are rendered in red (orange). Separate constraints from \( R_D \) and \( R_{D'} \) to 2\( \sigma \) accuracy are shown by the blue and purple regions, respectively. The LHC exclusion area are presented by the grey colour.

It is easy to obtain the bound \( \mathcal{B}(B \to K^*\mu\tau) \approx 1.9 \times \mathcal{B}(B \to K\mu\tau) \) and \( \mathcal{B}(B_s \to \mu\tau) \approx 0.9 \times \mathcal{B}(B \to K\mu\tau) \) [43–45]. We also predict about \( \sim 50\% \) enhancement of the ratio \( \mathcal{B}(B \to K^{(*)}\nu\nu) \), which can be tested in the near future at Belle-II. Remarkably, these two observables are highly correlated as depicted in Fig. 2. Within our scenario we make prediction for \( \mathcal{B}(\tau \to \mu\gamma) \), which lies just below the current experimental limit

\[
\mathcal{B}(\tau \to \mu\gamma) \sim 1.5 \times 10^{-8}.
\]  

Finally, our description of the \( B \)-physics anomalies, and most particularly \( R_{D^0} \), strongly depends on the assumption that the LQ states are not too far from the TeV scale. Thus, these particles are necessarily accessible at the LHC, yielding also predictions for the direct searches [19].

Figure 2. \( \mathcal{B}(B \to K\mu\tau) \) is presented. \( R_{\nu\nu} = \mathcal{B}(B \to K^{(*)}\nu\nu)/\mathcal{B}(B \to K^{(*)}\nu\nu)^{SM} \) for the 1\( \sigma \) (red) and 2\( \sigma \) (orange) regions of Fig. 1. The black line denotes the current experimental limit, \( R_{\nu\nu} < 2.7 \) [39].
Figure 3. The flavor fit constraints on the $g_{SL}$ plane for the transition $b \rightarrow c \tau \bar{\nu}_\tau$. The allowed 1 $\sigma$ (2 $\sigma$) regions are rendered in red (orange). Separate constraints from $R_D$ and $R_{D^*}$ to 2 $\sigma$ accuracy are shown by the blue and purple regions, respectively. The LHC exclusion area are presented by the gray colour.

6 Conclusions

We find out that the simple $SU(5)$ GUT model might accommodate two light scalar leptoquarks that can explain all measured $B$ meson anomalies. Therefore, our scenario gives the explanation of the possible lepton flavour universality violation. The model is not in a conflict with low-energy phenomenology or with direct search constraints at the LHC. It is significant that all Yukawa couplings remain perturbative up to the unification scale. The model predicts the branching ratio for the lepton flavour violating decays $B \rightarrow K\mu\tau$, $B \rightarrow K^{(*)}\nu\nu$ and $\tau \rightarrow \mu\gamma$ as well as indicates their correlation.

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