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Theory of rare K decays

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Abstract. I review rare kaon decays. I introduce the flavor problem and possible solutions. Very rare kaon decays like $K \to \pi\nu\bar{\nu}$ are very important to this purpose: we study also $K \to \pi l^+ l^-$, $K \to \pi\pi ee$ where chiral dynamics is important to disentangle short distance effects. We discuss also the decays $K^0 \to \mu^+\mu^-$, which have received recently some attention due to the measurement by LHCB.

1 Introduction and $K \to \pi\nu\bar{\nu}$

Rare kaon decays furnish challenging MFV probes and will severely constrain additional flavor physics motivated by NP [1]. SM predicts the $V - A \otimes V - A$ effective hamiltonian

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} (\underbrace{V_{cs}^* V_{cd}\, X_{NL}}_{\lambda x_c} + \underbrace{V_{ts}^* V_{td} X(x_t)}_{A^2\lambda^5\,(1-\rho-i\eta)x_t})\, \bar{s}_L \gamma_\mu d_L\, \bar{\nu}_L \gamma^\mu \nu_L, \qquad (1)$$

$x_q = m_q^2/M_W^2$, θ_W the Weak angle and X's are the Inami-Lin functions with Wilson coefficients known at two-loop electroweak corrections and the main uncertainties is due to the strong corrections in the charm loop contribution. The structure in (1) leads to a pure CP violating contribution to $K_L \to \pi^0\nu\bar{\nu}$, induced only from the top loop contribution and thus proportional to $\Im m(\lambda_t)$ ($\lambda_t = V_{ts}^* V_{td}$) and free of hadronic uncertainties. This leads to the prediction

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\rm SM} = (2.9 \pm 0.2) \times 10^{-11} \quad \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} = (8.3 \pm 0.9) \times 10^{-11},$$

where the parametric uncertainty due to the error on $|V_{cb}|$, ρ and η is shown.

Typical BSM predict new flavor structures that might affect $K \to \pi\nu\bar{\nu}$ that now can be tested at NA62 and KOTO [2]; we describe two different BSM effects i) new flavor structures for ϵ' avoiding $\Delta S = 2$ constraints (Fig. 1) [1, 3] and ii) attempts to describe B-anomalies [4], typically induce large flavor effects at O(1) TeV [5]. i) the recent lattice results for $K \to 2\pi$ leave open the possibility of BSM for ϵ'; to isospin breaking terms in $\Im(A_2)$ have been studied [3] in Fig.1. We expect effects at most 10% in $K^+ \to \pi^+\nu\bar{\nu}$ while are more sizable for $K_L \to \pi^0\nu\bar{\nu}$. Theoretically addressing flavor in Randall Sundrum models is more challenging: we have studied the so called flavor anarchy scenario with 5D MFV and custodial symmetry; the only sources of flavor breaking are two 5D anarchic Yukawa matrices. These matrices also generate also the bulk masses, which are responsible for the

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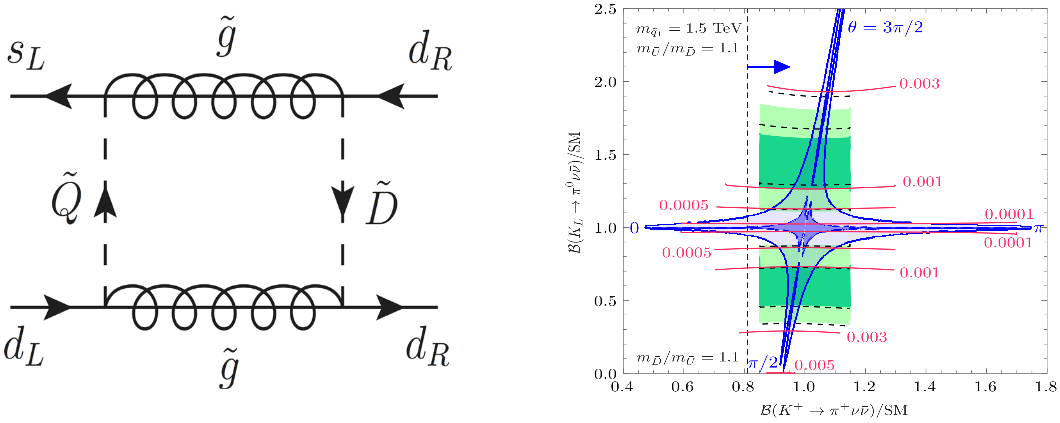

Figure 1. Impact of $K \to 2\pi$ isospin breaking terms ($\Im(A_2)$) (left) on $K \to \pi\nu\bar{\nu}$, typical effects, (right)

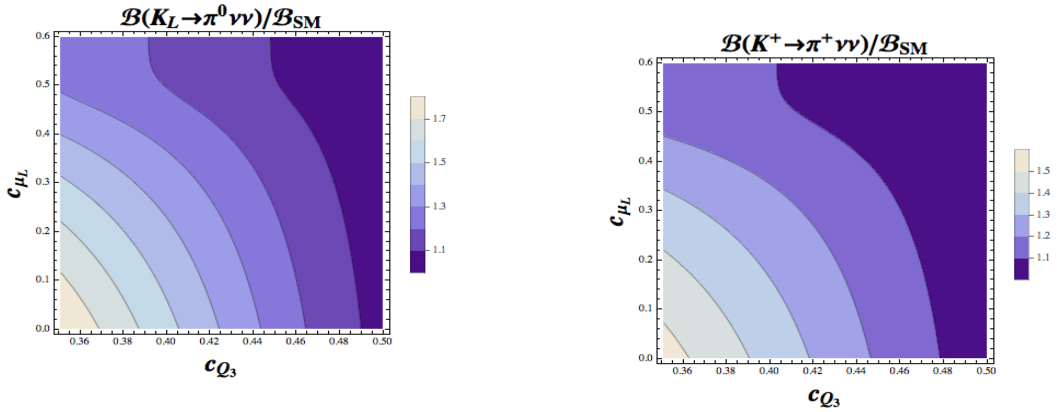

Figure 2. RS scenario to explain B-anomalies: $B(K \to \pi\nu\bar{\nu})$ ranges as a function of fermion profiles (c_i's)

resulting flavor hierarchy. The theory flows to a next to minimal flavor violation model where flavor violation is dominantly coming from the 3rd generation. We show that it is possible to find a range of parameters for bulk masses satisfying experimental flavor constraints, but also we explain the neutral B-anomalies, requiring NP flavor scale at O(1) TeV. Then we address $K \to \pi\nu\bar{\nu}$−decays: we show the TH predictions as a function f the bulk fermion masses in Fig.2 [5]. A natural issue is to test O(1) TeV physics at LHC; we are trying to apply the technique of Ref. [6] to this purpose.

2 $K_{L,S} \to \mu^+\mu^-$

Recent $K_S \to \mu\bar{\mu}$ LHCB measurement is very interesting and unexpected

$$B(K_S \to \mu\bar{\mu})_{LHCB} < 9 \times 10^{-9} \text{ at } 90\ \%\ \text{CL} \qquad B(K_S \to \mu\bar{\mu})_{SM} = (5.0 \pm 1.5) \times 10^{-12}. \qquad (2)$$

It represents an important milestone since it has improved the previous limit, $< 3.2 \times 10^{-7}$ at 90 % CL, lasted 40 years. It is based on a production of 10^{13} K_S per fb^{-1} inside the LHCB acceptance and it is obtained using 1.0 fb^{-1} of pp collisions at $\sqrt{s} = 7$ TeV collected in 2011.

Two photon exchange generates the dominant contribution for both K_L and K_S decays to two muons [7]. The structure of weak and electromagnetic interactions entails a vanishing CP conserving short distance contribution to $K_S \to \mu^+\mu^-$. Indeed the SM short diagrams (similar to $K \to \pi\nu\bar{\nu}$ in Fig. 2) lead to the SM effective hamiltonian similar to eq. (1).

The LD contributions to $K_S \to \mu^+\mu^-$ Fig. (4) have been computed reliably in CHPT ($B = (5.0 \pm 1.5) \times 10^{-12}$). The relevant short distance contributions are

$$B(K_S \to \mu\bar{\mu})^{SD}_{SM} = 1 \times 10^{-5} |\Im(V^*_{ts} V_{td})|^2 \sim 10^{-13} \qquad \text{vs} \qquad B(K_S \to \mu\bar{\mu})_{NP} \leq 10^{-11} \tag{3}$$

We have shown that in some appealing susy scenario in Fig. (3) [8] large allowed new physics contributions (NP) can be substantially larger that SM SD contributions.

The short distance hamiltonian will contribute also to $K_L \to \mu\bar{\mu}$, through a CP conserving amplitude, $\Re(A_{\mathrm{short}})$, that has to be disentangled from the large LD two-photon exchange contributions, $A_{\gamma\gamma}$: the absorptive LD contribution is much larger than SD, in the rate respectively 25 times larger than dispersive; total ($B_{\mathrm{expt}} = (6.84 \pm 0.11) \times 10^{-9}$) To extract SD info the situation would be better if we would know the sign of $A_{\gamma\gamma}$, theoretically and experimentally unknown. While K_L-decays outside the LHCB fiducial volume the interference $A(K_L \to \mu\bar{\mu})^* A(K_S \to \mu\bar{\mu})$ may affect the LHCB K_S-rates: we can study the time interference $K_{S,L} \to \mu\mu$; this can be done by flavor tagging $K^0 \bar{K}^0$, specifically by detecting the associated $\pi^{\pm}$ and (or) $K^{\mp}$, determining the impurity parameter $D = \frac{K^0 - \bar{K}^0}{K^0 + \bar{K}^0}$. Then interference term will affect the measured branching [7]:

$$\begin{aligned}\mathcal{B}(K^0_S \to \mu^+\mu^-)_{\mathrm{eff}} = \tau_S \left(\int_{t_{\min}}^{t_{\max}} dt e^{-\Gamma_S t} \varepsilon(t)\right)^{-1} &\left[\int_{t_{\min}}^{t_{\max}} dt \left\{\Gamma(K^0_S \to \mu^+\mu^-) e^{-\Gamma_S t}\right.\right. \\ &\left.\left.+ \tfrac{D f_K^2 M_K^3 \beta_\mu}{8\pi} \mathrm{Re}\left[i\left(A_S A_L - \beta_\mu^2 B_S^* B_L\right) e^{-i\Delta M_K t}\right] e^{-\frac{\Gamma_S + \Gamma_L}{2} t}\right\} \varepsilon(t)\right],\end{aligned} \tag{4}$$

Then we are i) increasing the sensitivity to short distance and ii) possibly determining the sign $A_{L\gamma\gamma}$

$$\sum_{\mathrm{spin}} \mathcal{A}(K_1 \to \mu^+\mu^-)^* \mathcal{A}(K_2 \to \mu^+\mu^-) \sim \underbrace{\mathrm{Im}[\lambda_t] y'_{7A}}_{SD} \left\{ \underbrace{A^{\mu}_{L\gamma\gamma}}_{LD} - 2\pi \sin^2\theta_W \left(\mathrm{Re}[\lambda_t] y'_{7A} + \mathrm{Re}[\lambda_c] y_c\right) \right\} \tag{5}$$

Experimentally, one can also access an *effective* branching ratio of $K^0_S \to \mu^+\mu^-$ [7] which includes an interference contribution with $K^0_L \to \mu^+\mu^-$ in the neutral kaon sample.

LHCB has also a beautiful kaon physics program [10–12]

	PDG	Prospects
$K_S \to \mu\mu$	$< 9 \times 10^{-9}$ at 90% CL	(LD)$(5.0 \pm 1.5) \cdot 10^{-12}$ NP $< 10^{-11}$
$K_L \to \mu\mu$	$(6.84 \pm 0.11) \times 10^{-9}$	difficult : SD << LD
$K_S \to \mu\mu\mu\mu$	–	SM LD $\sim 2 \times 10^{-14}$
$K_S \to ee\mu\mu$	–	$\sim 10^{-11}$
$K_S \to eeee$	–	$\sim 10^{-10}$
$K_S \to \pi^+\pi^-\mu^+\mu^-$	–	SM LD $\sim 10^{-14}$

(6)

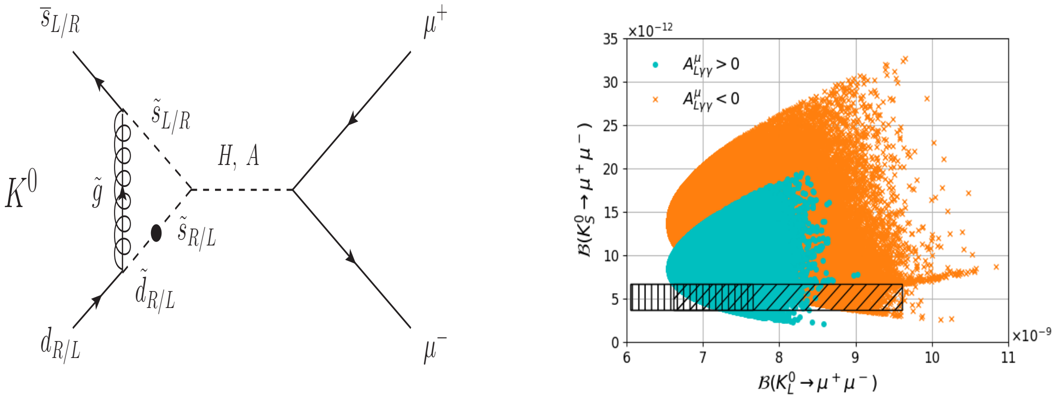

Figure 3. Susy scenario: $K_S \to \mu\mu$ diagram (left), theory predictions: in dashed area no interference effects are considered (right)

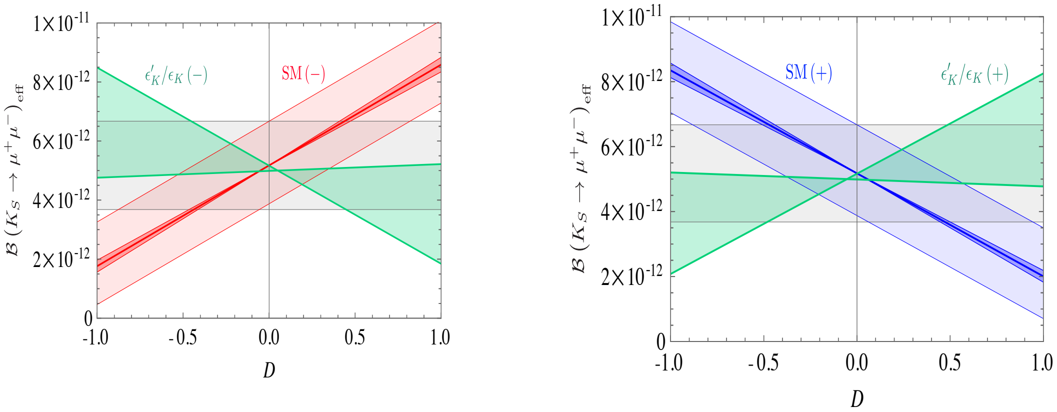

Figure 4. K_S LD diagram (left), interference effect from eq. 5 on $\mathcal{B}(K_S \to \mu^+\mu^-)$ depending on the $A_{L\gamma\gamma}$ sign: negative (center and in red SM while in green NP contributions) and positive (right and in blue SM while in green NP contributions).

3 The weak chiral lagrangian

In Ref. [9] we have studied how to determine the weak O(p^4) chiral countertems in

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K\to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+\to\pi^+\gamma\gamma,\,K\to\pi l^+ l^-} + \cdots .$$

Due to the accurate NA48/2 study of the decays $K^\pm \to \pi^\pm\pi^0\gamma$ and $K^\pm \to \pi^\pm\pi^0 e^+e^-$ the subset of CT's in the table can be determined

$$\begin{array}{llll} K^\pm \to \pi^\pm\gamma^* & N^r_{14} - N^r_{15} & a_+ = -0.578 \pm 0.016 & \mathrm{NA48/2} \\ K_S \to \pi^0\gamma^* & 2N^r_{14} + N^r_{15} & a_S = (1.06^{+0.26}_{-0.21} \pm 0.07) & \mathrm{NA48/1} \\ K^\pm \to \pi^\pm\pi^0\gamma & N^r_{14} - N^r_{15} - N^r_{16} - N_{17} & X_E = (-24 \pm 4 \pm 4)\ \mathrm{GeV}^{-4} & \mathrm{NA48/2} \\ K^+ \to \pi^+\gamma\gamma & N^r_{14} - N^r_{15} - 2N^r_{18} & \hat{c} = 1.56 \pm 0.23 \pm 0.11 & \mathrm{NA48/2} \end{array} \tag{7}$$

4 $K_{S,L} \to l^+l^-$, $K_{S,L} \to l^+l^-l^+l^-$ and $K_S \to \pi^+\pi^-l^+l^-$

Recent LHCB limit on $K_S \to \mu\overline{\mu}$ in the table is close to test interesting New Physics (NP) models. A high precision measurement can test the short distance (SD) SM but it requires to improve the long distance (LD) prediction with auxiliary channels [10]. $K_L \to \mu\mu$: the small ratio SD/LD $\sim \frac{1}{30}$ may obscure an experimental improvement on the rate. The situation would be a bit ameliorated if the sign for $A(K_L \to \gamma\gamma)$ would be known. Help to this ambiguity could come from the experimental study of $K_{S.L} \to l^+l^-l^+l^-$ [10] As shown in table these channels are at reach in a high intensity machine and they may also give LD distance info needed for a better control of $K_L \to \mu\mu$. These four body decays have also a peculiar feature, similarly to $K_{S,L} \to \pi^+\pi^-e^+e^-$, the two different helicity amplitudes interfere; then one can measure the sign $K_L \to\to \gamma^*\gamma^* \to l^+l^-l^+l^-$ by studying the time interference K_S K_L which it has a decay length $2\Gamma_S$ [10].

The interplay between LHCB and NA62 program is nicely shown in ref [12].

$K_S \to \pi^+\pi^-l^+l^-$ Following the study of $K^+ \to \pi^+\pi^0l^+l^-$in Ref. [13] we have studied the decay $K_S \to \pi^+\pi^-l^+l^-$ [9], that it has been studied by NA48/2 and it is a target of LHCB. One finds that the long-distance contributions to $K_S \to \pi^+\pi^-\gamma^*$ can be determined with remarkable accuracy, namely

$$BR(K_S \to \pi^+\pi^-e^+e^-) = \underbrace{4.74 \cdot 10^{-5}}_{\text{Brems.}} + \underbrace{4.39 \cdot 10^{-8}}_{\text{Int.}} + \underbrace{1.33 \cdot 10^{-10}}_{\text{DE}} . \tag{8}$$

This number is in excellent agreement with the PDG average [14]:

$$BR(K_S \to \pi^+\pi^-e^+e^-)_{exp} = (4.79 \pm 0.15) \times 10^{-5} . \tag{9}$$

Similarly, one can predict that

$$BR(K_S \to \pi^+\pi^-\mu^+\mu^-) = \underbrace{4.17 \cdot 10^{-14}}_{\text{Brems.}} + \underbrace{4.98 \cdot 10^{-15}}_{\text{Int.}} + \underbrace{2.17 \cdot 10^{-16}}_{\text{DE}} . \tag{10}$$

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References

[1] A. Crivellin, G. D'Ambrosio, T. Kitahara and U. Nierste, Phys. Rev. D **96**, no. 1, 015023 (2017) doi:10.1103/PhysRevD.96.015023 [arXiv:1703.05786 [hep-ph]].

[2] Riccardo Fantechi, these Proceedings

[3] T. Kitahara, U. Nierste and P. Tremper, Phys. Rev. Lett. **117**, no. 9, 091802 (2016) doi:10.1103/PhysRevLett.117.091802 [arXiv:1604.07400 [hep-ph]].

[4] T. Hurth, C. Langenbruch and F. Mahmoudi, JHEP **1711**, 176 (2017) doi:10.1007/JHEP11(2017)176 [arXiv:1708.04474 [hep-ph]].

[5] G. D'Ambrosio and A. M. Iyer, arXiv:1712.08122 [hep-ph].

[6] A. Chakraborty, A. M. Iyer and T. S. Roy, arXiv:1707.07084 [hep-ph].

[7] G. D'Ambrosio and T. Kitahara, Phys. Rev. Lett. **119**, no. 20, 201802 (2017) doi:10.1103/PhysRevLett.119.201802 [arXiv:1707.06999 [hep-ph]].

[8] V. Chobanova, G. D'Ambrosio, T. Kitahara, M. Lucio Martinez, D. Martinez Santos, I. S. Fernandez and K. Yamamoto, JHEP **1805**, 024 (2018) doi.org/10.1007/JHEP05(2018)024 [arXiv:1708.04474 [hep-ph]] arXiv:1711.11030 [hep-ph].

[9] L. Cappiello, O. Catà and G. D'Ambrosio, Eur. Phys. J. C **78**, no. 3, 265 (2018) doi:10.1140/epjc/s10052-018-5748-6 [arXiv:1712.10270 [hep-ph]].

[10] G. D'Ambrosio, D. Greynat and G. Vulvert, Eur. Phys. J. C **73**, 2678 (2013) [arXiv:1309.5736 [hep-ph]].

[11] V. Cirigliano, G. Ecker, H. Neufeld, A. Pich and J. Portoles, Rev. Mod. Phys. **84**, 399 (2012) doi:10.1103/RevModPhys.84.399 [arXiv:1107.6001 [hep-ph]].

[12] A. A. Alves Junior *et al.*, arXiv:1808.03477 [hep-ex].

[13] L. Cappiello, O. Cata, G. D'Ambrosio and D. N. Gao, Eur. Phys. J. C **72**, 1872 (2012) Erratum: [Eur. Phys. J. C **72**, 2208 (2012)] doi:10.1140/epjc/s10052-012-2208-6, 10.1140/epjc/s10052-012-1872-x [arXiv:1112.5184 [hep-ph]].

[14] M. Tanabashi *et al.* [ParticleDataGroup], Phys. Rev. D **98** (2018) no.3, 030001. doi:10.1103/PhysRevD.98.030001

[15] F. Feruglio, P. Paradisi and A. Pattori, Phys. Rev. Lett. **118** (2017) no.1, 011801 doi:10.1103/PhysRevLett.118.011801 [arXiv:1606.00524 [hep-ph]].

[16] A. Crivellin, G. D'Ambrosio, M. Hoferichter and L. C. Tunstall, Phys. Rev. D **93**, no. 7, 074038 (2016) doi:10.1103/PhysRevD.93.074038 [arXiv:1601.00970 [hep-ph]].

[17] E. Coluccio Leskow, G. D'Ambrosio, D. Greynat and A. Nath, Phys. Rev. D **93**, no. 9, 094031 (2016) doi:10.1103/PhysRevD.93.094031 [arXiv:1603.09721 [hep-ph]].