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Heavy-light mesons in a contact interaction

Marco A. Bedolla[1,2,*] and *Elena* Santopinto[1,**]

[1]INFN, Sezione di Genova, via Dodecaneso 33, 16146 Genova, Italy.
[2]Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, Edificio C-3, Ciudad Universitaria, Morelia, Michoacán 58040, México.

Abstract. We present a unified formalism for the analysis of mesons. It is provided by a symmetry preserving Schwinger-Dyson-Bethe-Salpeter-Equation (SDBSE) treatment of a vector-vector contact interaction. The contact interaction (CI) model offers a simple-to-implement alternative to perform exploratory studies of QCD, within the SDBSE framework. Within the limitation of this model, we calculate the spectrum of D and B mesons. Our results are in agreement within 2% when compare with experimental data, lattice QCD and other SDBSE calculations involving sophisticated interaction kernels.

1 Introduction

The Schwinger-Dyson-Bethe-Salpeter Equation (SDBSE) approach has proven to be a reliable instrument to explain a wide range of QCD phenomena [1–3]. However, brute force numerical evaluations depict problems when evaluating at large momentum transfer regions [4, 5], and the community endeavor consists in solving these problems from other perspectives [6–9].

The CI model appeared as an alternative to full QCD studies. In this model, quarks interact not via mass-less vector-boson exchanges, but instead through a symmetry preserving vector-vector contact interaction [10–14]. This interaction is embedded within the SDBSE approach in the rainbow-ladder approximation, implement confinement through a proper time regularization scheme. A fully consistent treatment of the CI model is simple to execute, and it produces useful results that can be compared and contrasted with full QCD calculations and experimental data.

In this work, we present a study of heavy-light systems with the contact interaction. Our results are a direct application of this model, that we developed in our previous papers [15–17]. We present our heavy-light mesons results and compare them experimental data and other covariant model setups [18–21]. For a complete model revision refer to Ref. [15].

2 SDBSE approach and CI model

Since this work is a direct application of the unified model presented in Ref. [15], we only sketch the basic formulae. The complete description of the CI model is found in Refs. [14, 17].

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[*] e-mail: bedolla@ifm.umich.mx
[**] e-mail: santopinto@ge.infn.it

2.1 Contact Interaction Model

The DSBSE formalism solves the bound-state problem in terms of their building blocks (quarks) and their interactions with gluons. In order to solve the meson bound state equation, we need to know the quark propagator, the gluon propagator and the quark-gluon interaction. In the contact interaction model, we assume that the quark-gluon interaction is led by symmetry-preserving vector×vector contact interaction; here, quarks are attached through the interaction defined as

$$g^2 D_{\mu\nu}(k) = \frac{4\pi\alpha_{\rm IR}}{m_g^2}\delta_{\mu\nu} \equiv \frac{1}{m_G^2}\delta_{\mu\nu}, \tag{1}$$

$$\Gamma^a_\mu(p,q) = \frac{\lambda^a}{2}\gamma_\mu, \tag{2}$$

where $m_g = 800\,\text{MeV}$ is a gluon mass scale which is in fact generated dynamically in QCD [22], and $\alpha_{\rm IR}$ is the CI model parameter, which can be interpreted as the interaction strength in the infrared [23, 24].

With this interaction, we obtain a constant mass function. Because the integrals we need to solve are divergent, we must adopt a regularization procedure. We employ the proper time regularization scheme [25] an get the mass function expression

$$M_f = m_f + \frac{M_f^3}{3\pi^2 m_G^2}\Gamma(-1, \tau_{\rm UV} M_f^2, \tau_{\rm IR} M_f^2)\,, \tag{3}$$

where $\Gamma(a, z_1, z_2)$ is the generalized incomplete Gamma function:

$$\Gamma(a, z_1, z_2) = \Gamma(a, z_1) - \Gamma(a, z_2)\,. \tag{4}$$

The parameters $\tau_{\rm IR}$ and $\tau_{\rm UV}$ are infrared and ultraviolet regulators, respectively. A nonzero value for $\tau_{\rm IR} \equiv 1/\Lambda_{\rm IR}$ implements confinement [26]. Since the CI is nonrenormalizable theory, $\tau_{\rm UV} \equiv 1/\Lambda_{\rm UV}$ becomes part of the model and therefore sets the scale for all dimensional quantities.

2.2 CI model running coupling

quark	$\hat{\alpha}_{IR}$ [GeV^{-2}]	Λ_{UV} [GeV]	α	Ratio
u, d, s	4.565	0.905	3.739	1
c	0.228	2.400	1.547	0.414
b	0.035	6.400	1.496	0.400

Table 1. Dimensionless coupling constant $\alpha = \hat{\alpha}_{IR}\Lambda^2_{\rm UV}$, where $\hat{\alpha}_{IR} = \alpha_{\rm IR}/m_g^2$, for the contact interaction, extracted from a best-fit to data, as explained in Ref. [15]. Fixed parameters are $m_g = 0.8\,\text{GeV}$ and $\Lambda_{\rm IR} = 0.24\,\text{GeV}$.

In a previous paper [15], we explained how the CI can be used to study light and heavy mesons. When studying the heavy sector, a change in the model parameters has to be done: an increase in the ultraviolet regulator, and a reduction in the coupling strength. Subsequently, we figured out that different set of parameters are needed in order to study each sector: light, charm and bottom, as displayed in Table 1. With these parameters, we defined a dimensionless coupling α guided by [27, 28]

$$\alpha = \frac{\alpha_{IR}}{m_g^2}\Lambda_{\rm UV}^2 . \tag{5}$$

The drop in α, in relation to its value in the light-quarks sector, can be read off from the last column of Table 1. Indeed, α is reduced by a factor of $2.1 - 2.3$ on going from the light to the heavy sector, instead of the apparent large factors listed in Refs. [15, 17].

Moreover, as a reminiscent of the running coupling QCD with the momentum scale at which it is measured, an inverse logarithmic curve can fit reasonably well the functional dependence of $\alpha(\Lambda_{UV})$. The fit reads

$$\alpha(\Lambda_{\rm UV}) = a \ln^{-1}\left(\Lambda_{UV}/\Lambda_0\right) , \tag{6}$$

where $a = 0.923$ and $\Lambda_0 = 0.357$ [15]. With this fit, we can recover the value of the strength coupling α once given a value of Λ_{UV}.

3 Results

In order to calculate mass spectra and decay constants, we follow the expressions found in Ref. [17]. Additionally, we include two distinct spin-orbit(SO) parameters,

$$g_{SO}^{1^+} = 0.312/3, \quad g_{SO}^{0^+} = 0.4/3 . \tag{7}$$

to calculate the spectrum of scalar and axial-vector mesons, respectively. With similar values, it is possible to match the mass splitting $m_{a_1} - m_\rho = 0.45$GeV and $m_\sigma - \rho = 0.29$GeV [29].

In our approach, to solve these unequal mass systems, we find $\Lambda_{\rm IR}$ and α through Eq. (6) to obtain the pseudoscalar meson experimental mass value. Then, with these parameters, we calculate the other mesons masses, the pseudoscalar and vector decay constants.

We present plots and tables comparing our results on D and B mesons, We compare with experimental data and other models results.

3.1 *D* Mesons

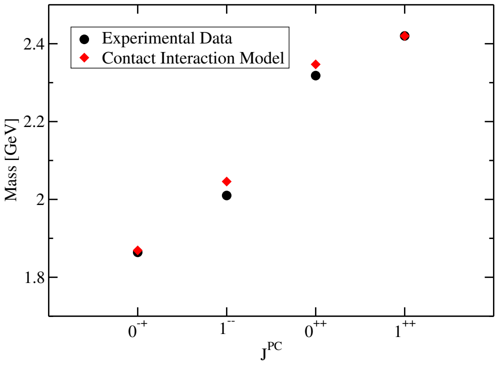

Figure 1. Contact interaction results for $c\bar{u}$ ($\bar{c}u$) mass spectrum using model parameters found in Table 2. PDG data are from Ref. [31].

Table 2. Ground-state of D $u\bar{c}(\bar{u}c)$ mass spectrum. The CI results were obtained with the best-fit parameter set: $m_g = 0.8\,\mathrm{GeV}$, $\alpha_{IR} = 0.93\pi/8.047$, $\Lambda_{\mathrm{IR}} = 0.24\,\mathrm{GeV}$, and $\Lambda_{\mathrm{UV}} = 1.905\,\mathrm{GeV}$. The current-quark mass are $m_n = 0.007\,\mathrm{GeV}$ and $m_c = 1.09\,\mathrm{GeV}$ and the dynamically generated constituent-like mass are $M_n = 0.049\,\mathrm{GeV}$ and $M_c = 1.451\,\mathrm{GeV}$. *The decay constants are from a Lattice calculation [32].

	masses and decay constants [MeV]			
	$(m, f)_{D(1S)}$	$(m, f)_{D^*(1S)}$	$m_{D_0(1P)}$	$m_{D_1(1P)}$
Experiment [31]*	(1864, 149)	(2010, 196)	2318	2420
CI-model	(1869, 425)	(2046, 182)	2347	2422
CI-subtr [30]	(1869, 146)	(2011, 169)	···	···
NST1 [18]	(1850, 108)	(2040, 113)	···	···
NST2 [18]	(1880, 183)	···	···	···
HGKL1 [21]	(1868, 228)	···	···	···
HGKL2 [21]	(1869, 678)	···	···	···
	amplitudes			
E_H	4.292	0.592	0.073	0.039
F_H	0.064	···	···	···

The spectrum of D mesons is displayed in Fig. 1. In Table 2, we compare the spectrum with experimental data and other models predictions. Our mass spectrum is in excellent agreement with experimental results and other models predictions. However, the decay constants predictions with different SDBSE methods still need more improvement.

3.2 *B* mesons

Table 3. Ground-state of B $n\bar{b}(\bar{n}b)$ mass spectrum. The CI results were obtained with the best-fit parameter set: $m_g = 0.8\,\mathrm{GeV}$, $\alpha_{IR} = 0.93\pi/168.569$, $\Lambda_{\mathrm{IR}} = 0.24\,\mathrm{GeV}$, and $\Lambda_{\mathrm{UV}} = 6.605\,\mathrm{GeV}$. The current-quark mass are $m_n = 0.007\,\mathrm{GeV}$ and $m_b = 3.8\,\mathrm{GeV}$ and the dynamically generated constituent-like mass are $M_n = 0.0140\,\mathrm{GeV}$ and $M_b = 4.575\,\mathrm{GeV}$. *The decay constants are from a recent Lattice calculation [33].

	masses and decay constants [GeV]			
	$(m, f)_{B(1S)}$	$(m, f)_{B^*(1S)}$	$m_{B_0(1P)}$	$m_{B_1(1P)}$
Experiment [31]*	(5279, 186)	(5325, 175)	···	5725
CI-model	(5273, 129)	(5314, 371)	5660	5704
NST1 [18]	(5270, 74)	(5150, 187)	···	···
NST2 [18]	(5150, 187)	···	···	···
	amplitudes			
E_H	0.143	0.149	0.002	0.001
F_H	0.000	···	···	···

Table 3 and Fig. 2 provide the CI model predicted B-mesons mass spectrum. Similar to our previous results, the mass spectrum agrees perfectly with experimental data and other models results. Additionally, thanks to the model simplicity, we present a B_0 mass value. We expect that our value will be in good agreement once it is an experimental value.

It is important to mention that the light quark gets a really small dressing when it is bounded to the bottom quark. This means that in a heavy-light systems, the light quark plays

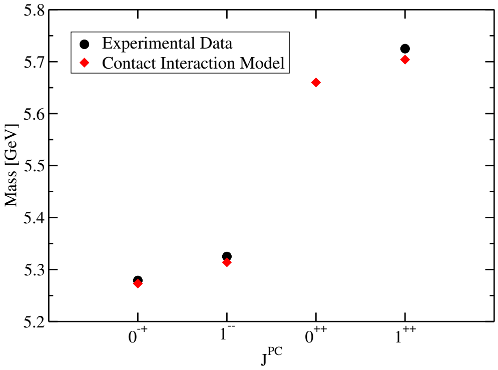

Figure 2. Contact interaction results for $b\bar{n}$ ($\bar{b}n$) mass spectrum using model parameters found in Table 3. PDG data are from Ref. [31].

a secondary role while the heavy quark has the most significant effects. Speaking about the decay constants, due to the mass difference between the bottom and light quarks, we aim to improve the results in future works.

4 Conclusions

We calculated the spectrum of D and B mesons. Our mass spectrum is in a good agreement with experimental data and other model results. However, the decay constants still need more improvement in these unequal mass systems. On the other hand, in D mesons we reduce the light coupling by a factor of ≈ 8, while in B mesons this factor is ≈ 168. This indicates that our model coupling with high sensitivity in the heavy sector

Thus, we found that the reduction in the coupling model is dependent on the quark masses involved in the bound-state. We fitted the contact interaction coupling with an inverse logarithmic curve. In future works, we will exploit this feature to study baryons and exotics with a variety of quarks components.

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