

# Exotic Tetraquark Mesons in Large- $N_c$ Limit: an Unexpected Great Surprise

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**Abstract.** Two-ordinary-meson scattering in large- $N_c$  QCD implies consistency criteria for intermediate-tetraquark contributions. Their fulfilment at  $N_c$ -leading order constrains the nature of the spectrum of genuinely exotic tetraquark states.

## 1 Introduction: $N_c$ -Leading First-Principles Approach to Tetraquarks

Qualitative information on systems controlled by quantum chromodynamics may be collected by considering large- $N_c$  QCD [1, 2], a quantum field theory generalizing QCD by enabling the number  $N_c$  of colour degrees of freedom to differ from  $N_c = 3$  and grow beyond bounds while, simultaneously, demanding that the product  $N_c \alpha_s$  of  $N_c$  and the strong fine-structure coupling

$$\alpha_s \equiv \frac{g_s^2}{4\pi}$$

approaches a finite value in the large- $N_c$  limit. For the  $N_c$  behaviour of  $\alpha_s$ , this demand implies

$$\alpha_s \propto \frac{1}{N_c} \quad \text{for } N_c \rightarrow \infty.$$

In an attempt to stay as close as possible to intuition, we adhere to the presumably very natural (but clearly not compulsory) assumption that the fermionic dynamical degrees of freedom, the quarks, continue to transform according to the  $N_c$ -dimensional, fundamental representation of the gauge group  $SU(N_c)$ . By utilizing QCD's large- $N_c$  limit ( $N_c \rightarrow \infty$ ) and  $1/N_c$  expansion (in powers of  $1/N_c$ ) about this limit, we extract constraints on crucial features (*e.g.*, decay widths) [3, 4] of tetraquarks, meson bound states of two quarks and two antiquarks predicted by QCD.

With respect to their *flavour* degrees of freedom, tetraquarks can be classified (Table 1) by specifying — for the two quarks and two antiquarks constituting the tetraquark bound state —

- the number of different quark flavours encountered in such bound state, in combination with
- the total number of *open* quark flavours, defined as the number of quark flavours that are not counterbalanced by an antiquark of same flavour and hence carried by the observed mesons.

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**Table 1.** Classification of tetraquark mesons by content of *different* and *open* quark flavour. The notion *open-flavour number* relates to the net sum of flavours not compensated by a corresponding antiflavour.

number of different quark flavours involved	tentative quark configuration $\bar{q}_\square q_\square \bar{q}_\square q_\square$	number of open quark flavours involved
4	$\bar{q}_1 q_2 \bar{q}_3 q_4$	4
3	$\bar{q}_1 q_2 \bar{q}_3 q_2$	4
	$\bar{q}_1 q_2 \bar{q}_1 q_3$	4
	$\bar{q}_1 q_2 \bar{q}_2 q_3$	2
	$\bar{q}_1 q_2 \bar{q}_3 q_3$	2
2	$\bar{q}_1 q_2 \bar{q}_1 q_2$	4
	$\bar{q}_1 q_1 \bar{q}_1 q_2$	2
	$\bar{q}_1 q_1 \bar{q}_2 q_1$	2
	$\bar{q}_1 q_2 \bar{q}_2 q_1$	0
	$\bar{q}_1 q_1 \bar{q}_2 q_2$	0
1	$\bar{q}_1 q_1 \bar{q}_1 q_1$	0

Whereas, by definition, for ordinary mesons a count of open flavour should yield zero or two, the case of four open flavours signals the tetraquark nature of the respective quark bound state.

We focus to two variants of tetraquarks with particularly interesting quark flavour content:

1. *flavour-exotic* tetraquarks  $T = (\bar{q}_1 q_2 \bar{q}_3 q_4)$  with all four (anti-) quark flavours different;
2. *flavour-cryptoexotic* tetraquarks  $T = (\bar{q}_1 q_2 \bar{q}_2 q_3)$  involving three different (anti-) quark flavours, by containing a quark–antiquark pair of same flavour differing from the others.

In order to work out — at least, at a qualitative level — some basic features of such kind of tetraquarks, we investigate, for two ordinary mesons of appropriate flavour quantum numbers, their possible scattering reactions into two ordinary mesons with regard to potential *s*-channel contributions of intermediate poles interpretable as a manifestation of tetraquarks with narrow decay width. For both sets of tetraquark in our focus of interest, as well as for the one with two different flavours but no open flavour, we have to analyze two variants of scattering processes:

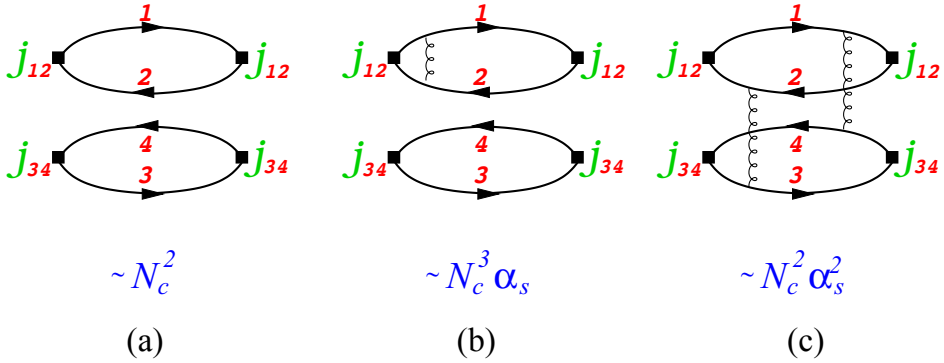
- flavour-preserving ones, with identical flavour content of the initial- and final-state mesons;
- flavour-rearranging ones, with unequal flavour content of the initial- and final-state mesons.

We identify all the contributions to four-point correlation functions of quark bilinear operators  $j_{ij} \equiv \bar{q}_i q_j$  interpolating ordinary mesons  $M_{ij}$  (notationally exploiting the actual irrelevance of parity and spin therein) that are capable of supporting a pole related to a tetraquark built up by four (anti-) quarks of masses  $m_i$ ,  $i = 1, \dots, 4$ , by imposing an unambiguous selection criterion [3, 4]: for the scattering of two mesons of momenta  $p_1$  and  $p_2$ , an allowable Feynman diagram not only has to depend on the Mandelstam variable  $s \equiv (p_1 + p_2)^2$  in a non-polynomial way but has to admit a four-quark intermediate state with related branch cut starting at the branch point

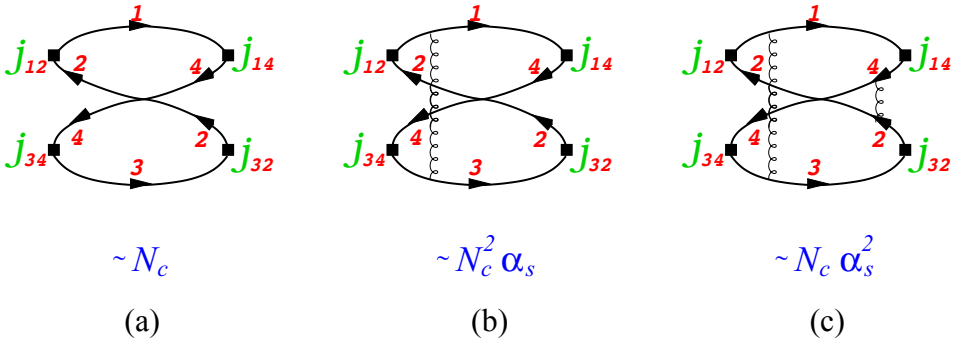
$$s = (m_1 + m_2 + m_3 + m_4)^2 .$$

## 2 Four different quark flavours $\iff$ flavour-exotic tetraquark meson

Surprisingly or not, for truly flavour-exotic tetraquark states,  $T = (\bar{q}_1 q_2 \bar{q}_3 q_4)$ , the leading- $N_c$  dependence of all contributions to the correlation functions of four quark bilinear operators  $j_{ij}$



**Figure 1.** Flavour-preserving four-point correlation function of quark bilinear operators  $j_{ij}$ : examples of Feynman diagrams contributing at low order to this correlator’s  $1/N_c$  expansion, viz., at order  $N_c^2$  (a,b) or  $N_c^0$  (c) [3, Fig. 1]. Tetraquark-friendly contributions of lowest order in  $1/N_c$  turn out to be of order  $N_c^0$  (c).



**Figure 2.** Flavour-rearranging four-point correlation function of quark bilinear currents  $j_{ij}$ : examples of Feynman diagrams contributing at low order to this correlator’s  $1/N_c$  expansion, that is, at order  $N_c$  (a,b) or  $N_c^{-1}$  (c) [3, Fig. 2]. Tetraquark-ophile contributions of lowest order in  $1/N_c$  prove to be of order  $N_c^{-1}$  (c).

(exemplified in Figs. 1 and 2) potentially capable of developing this tetraquark pole *differs*, for the flavour-preserving (Fig. 1(c)) and flavour-rearranging (Fig. 2(c)) cases, by one order of  $N_c$ :

$$\langle j_{12}^\dagger j_{34}^\dagger j_{12} j_{34} \rangle_T = O(N_c^0), \quad \langle j_{14}^\dagger j_{32}^\dagger j_{14} j_{32} \rangle_T = O(N_c^0), \quad \langle j_{14}^\dagger j_{32}^\dagger j_{12} j_{34} \rangle_T = O(N_c^{-1}).$$

The resulting  $N_c$ -leading tetraquark–two-ordinary-meson amplitudes  $A$  imply the presence of, at least, two<sup>1</sup> tetraquarks  $T_{A,B}$  with, however, decay rates  $\Gamma(T_{A,B})$  of similar large- $N_c$  decrease:

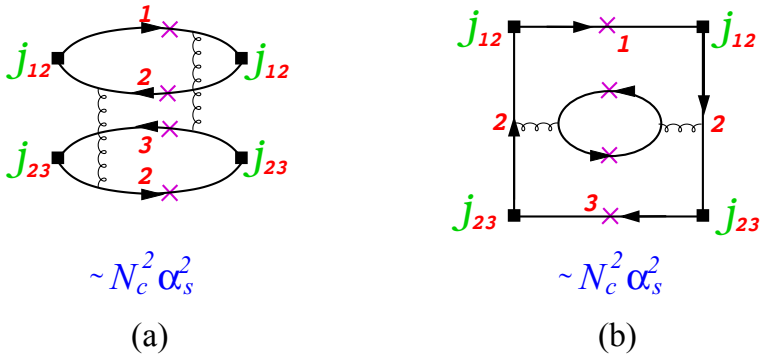
$$\begin{aligned} \underbrace{A(T_A \longleftrightarrow M_{12} M_{34}) = O(N_c^{-1})}_{\implies \Gamma(T_A) = O(N_c^{-2})} & \stackrel{N_c}{>} A(T_A \longleftrightarrow M_{14} M_{32}) = O(N_c^{-2}), \\ A(T_B \longleftrightarrow M_{12} M_{34}) = O(N_c^{-2}) & \stackrel{N_c}{<} \underbrace{A(T_B \longleftrightarrow M_{14} M_{32}) = O(N_c^{-1})}_{\implies \Gamma(T_B) = O(N_c^{-2})}. \end{aligned}$$

<sup>1</sup>In such a situation, their conclusions some people in the form may phrase “always two there are, . . . , no less” [5].

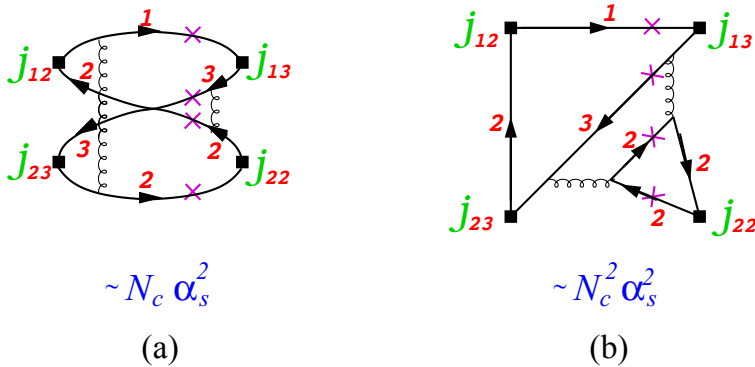
### 3 Two open quark flavours $\equiv$ flavour-cryptoexotic tetraquark meson

In the case of flavour-cryptoexotic tetraquark mesons built from three different quark flavours,  $T = (\bar{q}_1 q_2 \bar{q}_2 q_3)$ , the large- $N_c$  behaviour of flavour-preserving (Fig. 3) and flavour-reshuffling (Fig. 4) subcategories of those contributions to the correlation functions of four quark bilinear currents  $j_{ij}$  which might support the development of a tetraquark pole turns out to be identical:

$$\langle J_{12}^\dagger J_{23}^\dagger j_{12} j_{23} \rangle_T = O(N_c^0), \quad \langle J_{13}^\dagger J_{22}^\dagger j_{13} j_{22} \rangle_T = O(N_c^0), \quad \langle J_{13}^\dagger J_{22}^\dagger j_{12} j_{23} \rangle_T = O(N_c^0).$$



**Figure 3.** Flavour-preserving four-point correlation function of quark bilinear operators  $j_{ij}$ : examples of tetraquark-philic Feynman diagrams contributing at the  $N_c$ -leading order  $N_c^0$  to this correlation function's  $1/N_c$  expansion [3, Fig. 3]. (Purple crosses indicate quarks potentially contributing to a tetraquark pole.)



**Figure 4.** Flavour-reshuffling four-point correlation function of quark bilinear currents  $j_{ij}$ : examples of tetraquark-philic Feynman diagrams contributing at low order to such correlator's  $1/N_c$  expansion, *i.e.*, at order  $N_c^{-1}$  (a) or  $N_c^0$  (b) [3, Fig. 4]; among these, the  $N_c$ -leading contributions prove to be of order  $N_c^0$  (b).

So, a single tetraquark  $T_C$  (which, due to its cryptoexotic nature, can mix with the meson  $M_{13}$ ) satisfies all constraints induced by the  $N_c$ -leading tetraquark–two-ordinary-meson amplitudes

$$\underbrace{A(T_C \longleftrightarrow M_{12} M_{23}) = O(N_c^{-1}) \quad \stackrel{N_c}{\equiv} \quad A(T_C \longleftrightarrow M_{13} M_{22}) = O(N_c^{-1})}_{\implies \Gamma(T_C) = O(N_c^{-2})}.$$

## 4 Insights: $N_c$ -Leading Conclusions for (Crypto-) Exotic Tetraquarks

In summary, we find that self-consistency conditions arising from the inspection of tetraquark contributions to the scattering amplitudes of two ordinary mesons into two ordinary mesons in the  $1/N_c$  expansion of large- $N_c$  QCD provide rigorous constraints on the features of tetraquark states [3, 4]. Demanding these constraints to be satisfied at  $N_c$ -leading order implies [3, 4] that

- genuinely exotic tetraquarks need to appear in pairs  $(T_A, T_B)$  the members of which differ in the large- $N_c$  behaviour of their dominant or preferred decay modes to two ordinary mesons;
- both genuinely exotic  $(T_{A,B})$  and cryptoexotic  $(T_C)$  tetraquarks exhibit narrow decay widths

$$\Gamma(T) \propto 1/N_c^2 \xrightarrow{N_c \rightarrow \infty} 0 \quad \text{for } T = T_A, T_B, T_C .$$

Table 2 confronts these findings for the rates of the large- $N_c$  decrease of the total decay widths of exotic and cryptoexotic tetraquarks with corresponding outcomes of earlier analyses [6–8]. Imposition of additional requirements clearly may strengthen the predicted large- $N_c$  decrease. Differences to our results arise from misidentifying the actually  $N_c$ -leading contribution to the tetraquark pole or from consideration of merely a single (say, the flavour-reshuffling) channel.

**Table 2.** Comparison: predictions of *upper bounds* on the large- $N_c$  behaviour of tetraquark decay rates.

Author Collective	Decay Width $\Gamma$		Reference
	Exotic Tetraquarks	Cryptoexotic Tetraquarks	
Lucha <i>et al.</i>	$O(1/N_c^2)$	$O(1/N_c^2)$	[3, 4]
Knecht and Peris	$O(1/N_c^2)$	$O(1/N_c)$	[6]
Cohen and Lebed	$O(1/N_c^2)$	—	[7]
Maiani <i>et al.</i>	$O(1/N_c^3)$	$O(1/N_c^3)$	[8]

## Acknowledgements

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## References

- [1] G. 't Hooft, Nucl. Phys. B **72** (1974) 461.
- [2] E. Witten, Nucl. Phys. B **160** (1979) 57.
- [3] W. Lucha, D. Melikhov, and H. Sazdjian, Phys. Rev. D **96** (2017) 014022, arXiv:1706.06003 [hep-ph].
- [4] W. Lucha, D. Melikhov, and H. Sazdjian, Eur. Phys. J. C **77** (2017) 866, arXiv:1710.08316 [hep-ph].
- [5] Yoda, JM, in *Star Wars. Episode I: The Phantom Menace* (1999, in a galaxy far, far away), sc. 197.
- [6] M. Knecht and S. Peris, Phys. Rev. D **88** (2013) 036016, arXiv:1307.1273 [hep-ph].
- [7] T. D. Cohen and R. F. Lebed, Phys. Rev. D **90** (2014) 016001, arXiv:1403.8090 [hep-ph].
- [8] L. Maiani, A. D. Polosa, and V. Riquer, J. High Energy Phys. **06** (2016) 160, arXiv:1605.04839 [hep-ph].