

Study of the double Gamow-Teller transitions using the shell model approach

Naftali Auerbach* and Minh-Loc Bui

¹School of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel

Abstract. The double Gamow-Teller strength distributions in the lightest double beta-decay candidate ⁴⁸Ca and its isotope ⁴⁶Ca were calculated using the nuclear shell model by applying the single Gamow-Teller operator two times sequentially on the ground state of parent nucleus. The nuclear matrix element of the double Gamow-Teller transition from the ground state to the ground state that goes into the double beta decay calculation was shown as a small fraction of the total transition.

1 Introduction

The double charge-exchange (DCX) processes are a promising tool to study nuclear structure in particular nucleon-nucleon correlations in nuclei. In the 1980s, the DCX reactions using pion beams that were produced in the three meson factories at LAMPF, TRIUMF, and SIN were performed [1, 2].

At present, there is a renewed interest in DCX reactions, to a large extent due to the extensive studies of double beta-decay, both the decay in which two neutrinos are emitted ($2\nu\beta\beta$) and neutrinoless double beta-decay ($0\nu\beta\beta$). The pion DCX reactions did not excite the states involving the spin, such as the double Gamow-Teller (DGT) state. The DGT strength is the essential part of the double beta decay transitions. The pion interacts weakly with states involving the spin. It was suggested in the past that one could probe such states using DCX reactions with light ions [3, 4]. The present day, DCX reactions are indeed performed using light ions [5]. One hopes that such studies might shed some light on the nature of the nuclear matrix element of the double beta-decay and serve as a “calibration” for the size of this matrix element. These DCX studies might also provide new interesting information about nuclear structure.

One of the outstanding resonances relevant to the double beta-decay is the DGT resonance. The notion of a DGT was introduced in Refs. [3, 4]. The DGT strength distributions in even-*A* Neon isotopes was discussed in Ref. [6] and recently the calculation for ⁴⁸Ca was performed in Ref. [7]. In both works, the Lanczos method [8] was used. In the present paper, the DGT transition strengths in even-*A* Calcium isotopes are calculated in the full *fp*-model space using the nuclear shell model code NuShellX@MSU [9, 10]. The properties of the DGT distribution are examined and limiting cases when the SU(4) holds or when the spin orbit-orbit coupling is put to zero are studied. DGT sum rules were derived in Refs. [6, 11–13]. The DGT sum rules in this paper were used as a tool

to asses whether in our numerical calculations most of the DGT strength is found.

2 Method of calculation

The nuclear shell-model wave functions of the initial ground state, intermediate states, and final states were obtained from the shell model code NuShellX@MSU [9, 10] using the FPD6 [14] interaction in the complete *fp*-model space. For $J_f = 0^+$ in ⁴⁶Ti, all 2343 possible states are taken into account. In the case of $J_f = 2^+$ in ⁴⁶Ti, the calculation was done for 5000 of 9884 states. We also calculated only 5000 of 14177 $J_f = 0^+$ states in ⁴⁸Ti. The number of intermediate states is 500 in our work. As one will see later, this is enough to exhaust almost the total DGT strength. The number of $J = 2^+$ in ⁴⁸Ti is too large (61953) to be calculated with the present computer codes.

After all wave functions were obtained, the single GT operator was applied two times sequentially. First, all transitions from the parent nucleus 0^+ to all 1^+ intermediate states are calculated and then all transitions from 1^+ intermediate states to each 0^+ or 2^+ in the final nucleus are computed. The single GT operator is defined as

$$Y_{\pm} = \sum_{i=1}^A \sigma t_{\pm}(i); \quad t_{\pm} = t_x \pm it_y, \quad (1)$$

with $t_{-n} = p$ and $t_{+p} = n$ where $2t_x$ and $2t_y$ are the Pauli isospin operators and σ is Pauli spin operator. Then the single GT transition amplitude $J_i^+ \rightarrow J_f^+$ is

$$M(GT_{\pm}) = \frac{\langle J_f^+ || Y_{\pm} || J_i^+ \rangle}{\sqrt{2J_i + 1}}, \quad (2)$$

and the GT transition strength given by

$$B(GT_{\pm}) = |M(GT_{\pm})|^2 \quad (3)$$

obeys the “ $3(N - Z)$ ” sum rule.

*e-mail: auerbach@post.tau.ac.il

Table 1. The total strength of DGT transition.

Nucleus	$^{46}\text{Ti } 0^+$	$^{46}\text{Ti } 2^+$	$^{48}\text{Ti } 0^+$
Calculation	223.7	752.6	385.0
Sum rule	≤ 252	≥ 720	≤ 432
$0_1^+ \rightarrow J_1^+$	0.201	0.017	0.109

The dimensionless DGT transition amplitude is defined as

$$M(\text{DGT}_\pm)(J_f) = \sum_n M(\text{GT}_\pm; i \rightarrow n)M(\text{GT}_\pm; n \rightarrow f), \quad (4)$$

where n are the intermediate states. Note that this is a coherent sum. The DGT strength is given by

$$B(\text{DGT}_\pm)(J_f) = |M(\text{DGT}_\pm)(J_f)|^2. \quad (5)$$

The DGT sum rules for $J_f = 0^+$ and $J_f = 2^+$ are given in Refs. [6, 11–13]:

$$\begin{aligned} S_{\text{DGT}}^{J_f=0} &= 6(N-Z)(N-Z+1) - 2\Delta, \\ S_{\text{DGT}}^{J_f=2} &= 30(N-Z)(N-Z-2) + 5\Delta, \end{aligned} \quad (6)$$

where $\Delta = \sqrt{2}\langle 0|[Y_+ \times Y_-]^{(1)} \cdot \Sigma - \Sigma \cdot [Y_- \times Y_+]^{(1)}|0\rangle$, with $\Sigma = \sum_i \sigma(i)$. There is factor of three difference between the equations in Refs. [6, 13] and our work because the spin operator is not projected.

3 Results and discussions

We present here only the results for the two heaviest Calcium isotopes, ^{46}Ca and ^{48}Ca . Note that the double beta-decay from the ground state of ^{48}Ca to the ground state of ^{48}Ti is energetically allowed and studied extensively. A review was given in Ref. [15].

To ensure we can exhaust all the DGT strength, the sum rules of the DGT operator are presented numerically and compared to the values given in Refs. [11–13]. Our results are given in Table 1 and they are in agreement with the results in Ref. [12] for $J_f^\pi = 0^+$ and the recent work of Ref. [13] for both $J_f^\pi = 0^+$ and 2^+ . After the entire distributions are obtained, the cumulative sums of the DGT transitions are shown in Figs. 1–3. We remind that the entire DGT distributions of even- A Ne isotopes were obtained in Ref. [6] but a different method of calculation from our work was used. Note that Refs. [12, 13] calculated the DGT sum rule indirectly and therefore gave only the value of the total sum, not the cumulative sum. In Figs. 1–3, the horizontal line represents the value of the DGT strength in the case when the SU(4) is a good symmetry. It is the upper limit for DGT sum rule of $J_f = 0^+$ and lower limit for $J_f = 2^+$. We also show in above figures the computed strength in the limiting case when the spin-orbit coupling is put to zero (the SU(4) symmetry is approximately restored) [16]. Because all possible $J_f = 0^+$ final states in ^{46}Ti were taken into account, Fig. 1 shows that the sum rule, in this case, was exhausted and when the spin-orbit coupling is put to zero its cumulative sum approaches the limit value (the horizontal line). Figures 2 and 3 show that

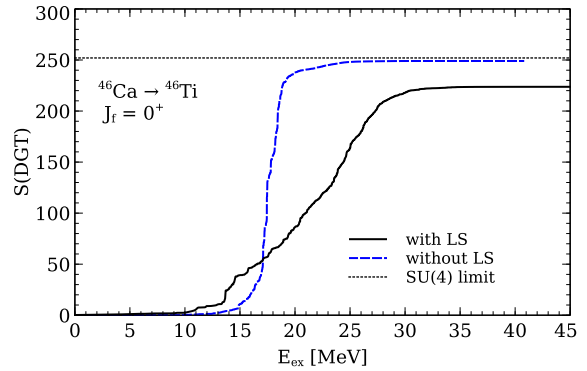


Figure 1. The cumulative sum of the DGT strength $B(\text{DGT}; 0^+ \rightarrow 0^+)$ in ^{46}Ca .

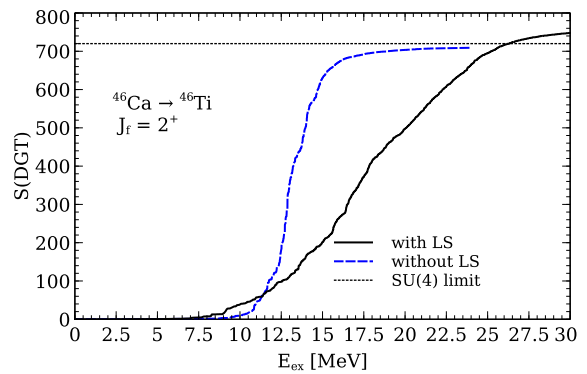


Figure 2. The cumulative sum of the DGT strength $B(\text{DGT}; 0^+ \rightarrow 2^+)$ in ^{46}Ca .

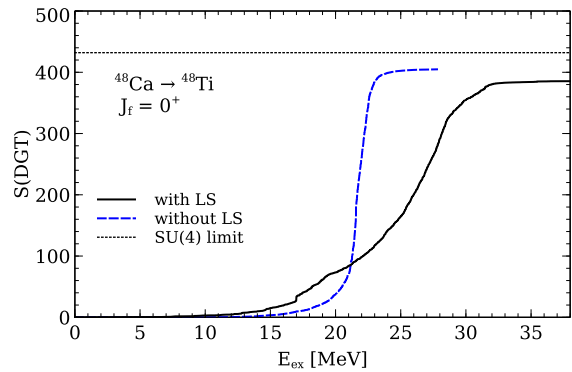


Figure 3. The cumulative sum of the double Gamow-Teller strength $B(\text{DGT}; 0^+ \rightarrow 0^+)$ in ^{48}Ca .

the cumulative sums are still increasing because the calculations were limited up to 5000 final states.

The detailed DGT strength distributions are shown in Figs. 4 and 5 for ^{46}Ca , and in Fig. 6 for ^{48}Ca . Figures 4–6 contain inserts which show the DGT strength in the low-lying states of $^{46,48}\text{Ti}$. The transition strength is a very tiny fraction of the total strength. For example, the strength in the ground state of ^{48}Ti is only 3×10^{-4} of the total strength (see Table 1). This strength enters in the calculation of the double beta-decay.

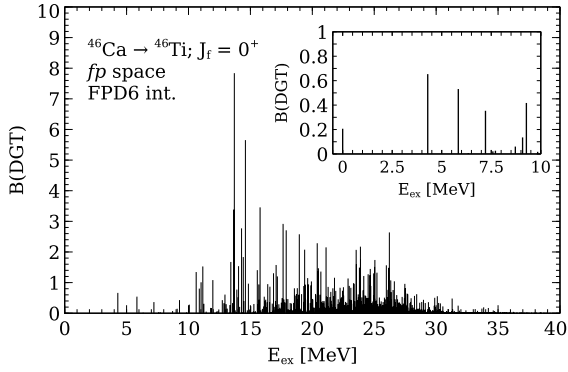


Figure 4. $B(\text{DGT}; 0^+ \rightarrow 0^+)$ for ^{46}Ca . The insert shows the DGT strength in the low-lying states.

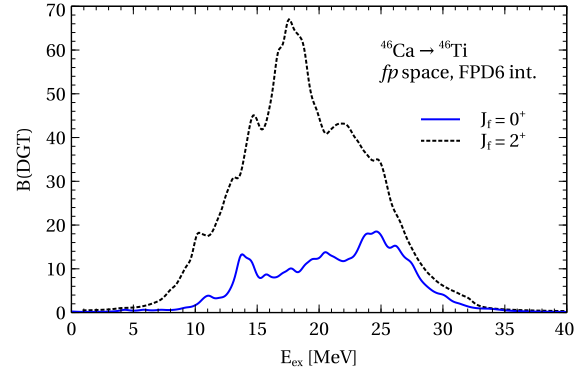


Figure 7. $B(\text{DGT}; 0^+ \rightarrow 0^+; 2^+)$ for ^{46}Ca . The strengths are smoothed by using Lorentzian averaging with the width of 1 MeV.

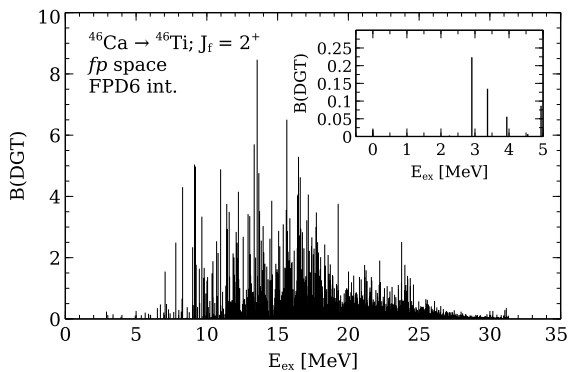


Figure 5. The same as Fig. 4 but now for $B(\text{DGT}; 0^+ \rightarrow 2^+)$.

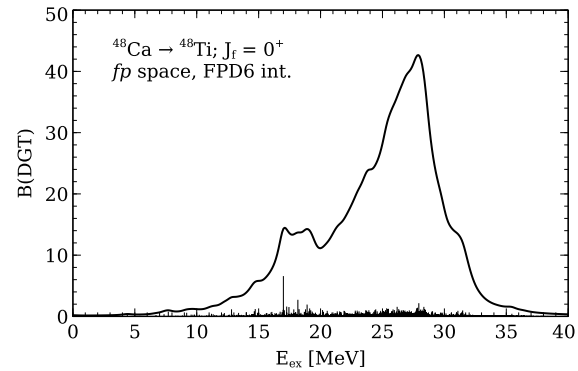


Figure 8. $B(\text{DGT}; 0^+ \rightarrow 0^+)$ for ^{48}Ca . The strengths are smoothed by using Lorentzian averaging with the width of 1 MeV.

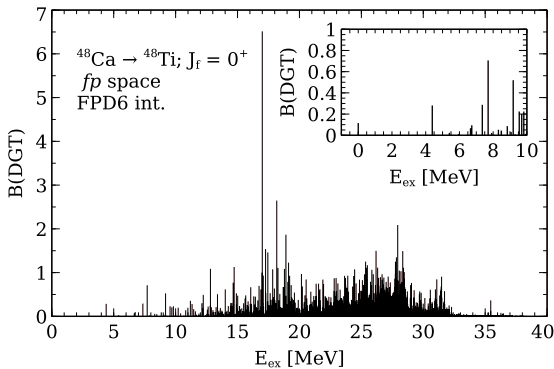


Figure 6. The same as Fig. 4 but now for ^{48}Ca .

After that, all the strengths are spread by using Lorentzian averaging with the width of 1 MeV. Figure 7 shows that the DGT transition to the $J_f = 2^+$ is stronger than the transition to $J_f = 0^+$. Figure 8 shows the distribution in ^{48}Ca before and after the Lorentzian averaging. We observe that the distributions are not single-peaked. There are at least two peaks and in some nuclei as many as four major peaks. We should remind that the single GT resonances have at least two peaks [17].

The average energy of the DGT strength \bar{E} is defined as:

$$\bar{E} = \frac{\sum_f E_f B_f(\text{DGT}_-)}{\sum_f B_f(\text{DGT}_-)}, \quad (7)$$

where $B_f(\text{DGT}_-)$ is the DGT transition at the energy E_f . In ^{46}Ti , this energy for the $J = 0^+$ is $\bar{E} = 21.2$ MeV and for the $J = 2^+$ it is lower $\bar{E} = 18.0$ MeV. In ^{48}Ti we calculated only the $J = 0^+$ DGT distribution. Its average energy is $\bar{E} = 24.6$ MeV. In a recent paper [18], the experimental results for the DCX reaction $^{56}\text{Fe}(^{11}\text{B}, ^{11}\text{Li})$ are presented. In this reaction several resonances were excited. There is a peak at 25 MeV excitation, that the authors indicate that it could be the DGT resonance.

4 Conclusion

The DCX interaction involving ions is much more complicated than the DGT operator, and the reaction mechanism is more evolved than the simple sequential process. However the DCX reaction will excite the DGT strength, and when the energy of the projectile is high enough it will excite the DGT resonance, as well as low-energy states containing DGT strength. A comparison between theory and the experimental cross-sections will provide useful information about the DGT strength and thus help to learn more about the double beta-decay nuclear matrix element. More work is needed on the DCX reaction theory before this goal is achieved.

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