Two-phonon structures for beta-decay theory

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Abstract. The β-decay rates of ^{60}Ca have been studied within a microscopic model, which is based on the Skyrme interaction T45 to construct single-particle and phonon spaces. We observe a redistribution of the Gamow–Teller strength due to the phonon-phonon coupling, considered in the model. For ^{60}Sc, the spin-parity of the ground state is found to be 1^+. We predict that the half-life of ^{60}Ca is 0.3 ms, while the total probability of the βxn emission is 6.1%. Additionally, the random matrix theory has been applied to analyze the statistical properties of the 1^+ spectrum populated in the β-decay to elucidate the obtained results.

The multi-neutron emission is basically a multi-step process consisting of (a) the β-decay of the parent nucleus (N, Z) which results in feeding the excited states of the daughter nucleus $(N-1, Z+1)$ followed by the (b) γ-deexcitation to the ground state or (c) multi-neutron emissions to the ground state of the final nucleus $(N-1-X, Z+1)$ (see, e.g., Ref. [1]). There is a strong need in a consistent estimate of the multi-neutron emission for the analysis of radioactive beam experiments and for modeling of astrophysical r-process. Recent experiments indicate on evidence of strong shell effects in exotic calcium isotopes [2, 3]. Therefore, the β-decay properties of neutron-rich isotope ^{52}Ca provide valuable information [4], with important tests of theoretical calculations (see, e.g., Ref. [5]). In the report we demonstrate the importance of the phonon-phonon coupling (PPC) on the β-delayed multi-neutron emission of the neutron-rich nucleus ^{60}Ca discovered in [6].

One of the successful tools for studying charge-exchange nuclear modes is the quasiparticle random phase approximation (QRPA) with the self-consistent mean-field derived from a Skyrme energy-density functional (EDF). Indeed, such QRPA calculations enable one to describe the properties of the parent ground state and Gamow–Teller (GT) transitions using the same EDF. In the case of the β-decay of ^{60}Ca, we use the EDF T45 which takes into account the tensor force added with refitting the parameters of the central interaction [7]. The set T45 gives positive value of the spin-isospin Landau parameter calculated at the saturation density (G'_0=0.1), and a reasonable description of the Q_β and S_{xn} values for the βxn-emission of the even neutron-rich Ca isotopes. The pairing correlations are generated by a zero-range volume force with a strength of -315 MeVfm^3, and a smooth cutoff at 10 MeV above the Fermi energies [8]. This value of the pairing strength has been fitted to reproduce the experimental neutron pairing energy of ^{50,52,54}Ca [5, 9]. The calculated Q_β-window of the β-decay of ^{60}Ca and S_{xn} values of ^{60}Sc are shown in Fig. 1. There is the possibility of the nonzero probability of one-, two-, three-, four- and fifth-neutron emission. As expected, the largest contribution to the calculated β-decay half-life comes from the 1^+_1 state, which structure is dominated by one unperturbed configuration. The lowest two-quasiparticle (2QP) state has the structure $\{\pi 1f\frac{7}{2}, \nu 1f\frac{5}{2}\}$.

We employ the standard procedure [10] to construct the QRPA equations on the basis of HF-BCS quasiparticle states of the parent nucleus. The residual interactions in the particle-hole channel and the particle-particle channel are derived consistently from the Skyrme EDF. The eigenvalues of the QRPA equations within the finite rank separable approximation are found numerically as the roots of the secular equation for the cases of electric excitations [8, 11] and charge-exchange excitations [12, 13]. It enables us to perform the QRPA calculations in a large 2QP spaces. In particular, the cutoff of the discretized continuous part of the single-particle spectra is performed at the energy of 100 MeV. This is sufficient for exhausting the Ikeda sum rule $S_- - S_+ = 3(N-Z)$.

Taking into account the basic ideas of the quasi-particle-phonon model (QPM) [14, 15], the Hamiltonian is then diagonalized in a space spanned by states composed of one and two QRPA phonons [16, 17]

$$\Psi_\nu(JM)=\Bigg(\sum_i R_i(J\nu)Q^+_{JMi}+ \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\Big[Q^+_{\lambda_1\mu_1 i_1}\bar{Q}^+_{\lambda_2\mu_2 i_2}\Big]_{\lambda\mu}\Bigg)|0\rangle, \quad (1)$$

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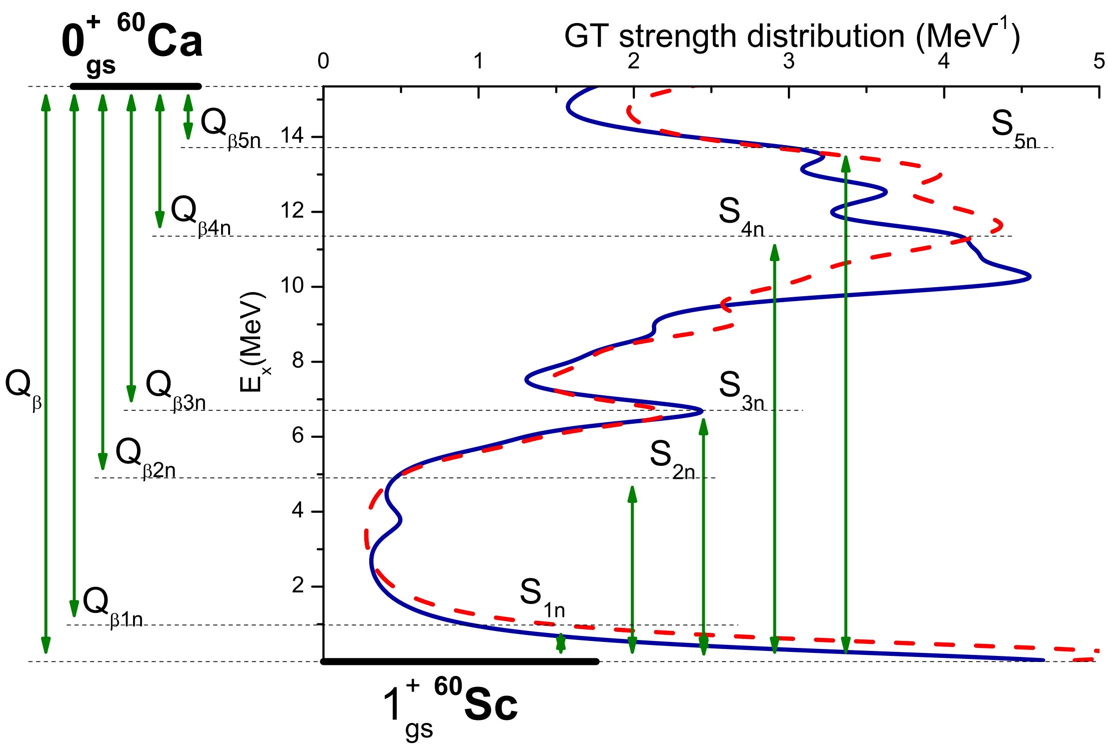

Figure 1. (Color online) GT strength distributions of ^{60}Ca as functions of the excitation energy of the daughter nuclei. The calculations, taking into account the microscopic and random coupling matrix elements between the one- and two-phonon configurations, are shown as solid lines and dashed lines, respectively. The smoothing parameter 1 MeV is used for the strength distribution described by the Lorentzian function.

where $Q^+_{\lambda\mu i}|0\rangle$ are the wave functions of the neutron-proton one-phonon states having energy $\omega_{\lambda i}$ of the daughter nucleus $(N-1, Z+1)$; $\bar{Q}^+_{\lambda\mu i}|0\rangle$ is the one-phonon excitation having energy $\bar{\omega}_{\lambda i}$ of the parent nucleus (N, Z). For the unknown amplitudes $R_i(J\nu)$ and $P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)$ the variational principle leads to the set of linear equations with the rank equal to the number of one- and two-phonon configurations,

$$(\omega_{\lambda i}-\Omega_\nu)R_i(J\nu)+\sum_{\lambda_1 i_1 \lambda_2 i_2} U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)=0, \quad (2)$$

$$(\omega_{\lambda_1 i_1}+\bar{\omega}_{\lambda_2 i_2}-\Omega_\nu)P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)+\sum_i U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)R_i(J\nu)=0. \quad (3)$$

To solve these equations it is required to compute the Hamiltonian matrix elements coupling one- and two-phonon configurations [16, 17],

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)=\langle 0|Q_{Ji}H\left[Q^+_{\lambda_1 i_1}\bar{Q}^+_{\lambda_2 i_2}\right]_J|0\rangle. \quad (4)$$

In order to construct the wave functions Eq. (1) of the low-energy 1^+ states, in the present study we assume that the two-phonon configurations are constructed from the 1^+ and 3^+ charge-exchange phonons, and the 2^+ and 4^+ phonons. The redistribution of the GT strength due to the PPC is mostly sensitive to the multi-neutron emission probability [18].

In the allowed GT approximation, the β^--decay rate is expressed by summing up the probabilities (in units of $G_A^2/4\pi$) of the energetically allowed transitions ($E_k^{\rm GT}\leq Q_\beta$) weighted with the integrated Fermi function

$$T^{-1}_{1/2}=D^{-1}\left(\frac{G_A}{G_V}\right)^2\sum_k f_0(Z+1,A,E_k^{\rm GT})B(GT)_k, \quad (5)$$

$$E_k^{\rm GT}=Q_\beta-E_{1^+_k}, \quad (6)$$

where G_A/G_V=1.25 is the ratio of the weak axial-vector and vector coupling constants and D=6147 s (see Ref. [19]). $E_{1^+_k}$ denotes the excitation energy of the daughter nucleus. As proposed in Ref. [20], this energy can be estimated by the following expression:

$$E_{1^+_k}\approx\Omega_k-E_{\rm 2QP,lowest}, \quad (7)$$

where Ω_k are the 1^+_k eigenvalues of the Eqs. (2) and (3); $E_{\rm 2QP,lowest}$ corresponds the lowest 2QP energy, i.e., the energy $\{\pi 1f\frac{7}{2},\nu 1f\frac{5}{2}\}$ in the case of ^{60}Ca. Moreover, the ground state of ^{60}Sc is found as 1^+. The wave functions allow us to determine GT transitions whose operator is $\hat{O}_-=\sum_{i,m}t_-(i)\sigma_{m(i)}$:

$$B(GT)_k=\left|\langle N-1,Z+1;1^+_k|\hat{O}^-|N,Z;0^+_{\rm gs}\rangle\right|^2. \quad (8)$$

Since the tensor correlation effects are taken into account within the 1p−1h and 2p−2h configurational spaces, any quenching factors are redundant [21].

The difference in the characteristic time scales of the β decay and subsequent neutron emission processes justifies an assumption of their statistical independence. As proposed in Ref. [22], the P_{xn} probability of the βxn emission accompanying the β decay to the excited states in the daughter nucleus can be expressed as

$$P_{xn}=T_{1/2}D^{-1}\left(\frac{G_A}{G_V}\right)^2\sum_{k'}f_0(Z+1,A,E_{k'}^{\rm GT})B(GT)_{k'}, \quad (9)$$

where the GT transition energy ($E_{k'}^{\rm GT}$) is located within the neutron emission window $Q_{\beta xn}\equiv Q_\beta - S_{xn}$. For P_{1n} we have $Q_{\beta 2n}\leq E_{k'}^{\rm GT}\leq Q_{\beta n}$, while for P_{xn} this becomes $Q_{\beta x+1n}\leq E_{k'}^{\rm GT}\leq Q_{\beta xn}$. Since we neglect the γ-deexcitation of the daughter nucleus, some overestimation of the resulting P_{xn} values can be obtained [18].

The largest contribution (93%) in half-life comes from the 1_1^+ state calculated with the PPC. The dominant contribution (94%) in the wave function of the first 1^+ state comes from the $[1_1^+]_{\rm QRPA}$ configuration which is mainly built on the configuration $\{\pi 1f\frac{7}{2}, \nu 1f\frac{5}{2}\}$. The inclusion of the PPC leads to a redistribution of the GT strength and the fragmentation is shown in Fig. 1. The excitation energies refer to the ground state of the daughter nucleus ^{60}Sc. The half-life $T_{1/2}$=0.3 ms and the total probability of the βxn emission of $P_{\rm tot}$=4.8% are calculated within the QRPA. The inclusion of the two-phonon terms results in the same half-life and $P_{\rm tot}$=6.1%. Using the large β-decay window, we obtain the unexpectedly small value of $P_{\rm tot}$, the effect which was predicted within the one-phonon approximation before. Similar $P_{\rm tot}$-value of 7.7% was predicted within the pn-RQRPA [23], however with nearly 18 times longer half-life of 5.3 ms. The DF3+cQRPA calculation predicted a 6 times longer half-life of $T_{1/2}$=1.8 ms but also a low $P_{\rm tot}$-value of 11.3% [24].

A natural question arises: what the complexity of the configurational space should be enough in order to obtain the half-life and the β-delayed neutron emission at extreme N/Z ratio? This restriction can be justified by the rough estimate from the random matrix theory [25]. We generalize the approach based on the ideas of the random matrix distribution of the coupling between one- and two-phonon states generated in the QRPA [26]. We find that the distribution of the matrix elements Eq. (4) is well reproduced by a truncated Cauchy distribution. The similar tendency is observed for the description of a gross structure of the spreading widths of monopole, dipole, and quadrupole giant resonances [26, 27]. Considering truncated Cauchy distributions, according to the central limit theorem, the resulting shape (the average of the sum) is driving the Gaussian distribution. Since the distribution of the matrix elements Eq. (4) is symmetric with a finite root-mean square (rms) value, we may generate the random matrix elements from the truncated Cauchy distribution or from a Gaussian distribution [27].

With the motivation above, we assume that the matrix elements Eq. (4) can be replaced by a random interaction where the matrix elements are Gaussian distributed random numbers,

$$P(U)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(\frac{-U^2}{2\sigma^2}\right), \quad (10)$$

with the rms value σ calculated with the microscopic PPC. Solutions are ensemble averaged over the random interaction and give the GT strength distribution, see Fig. 1. The value $P_{\rm tot}$=5.3% and the GT strength distribution calculated with the random coupling matrix elements are close to those that were calculated within the microscopic PPC. We conclude that the present approach makes it possible to perform the new analysis of the rates of the β-delayed multi-neutron emission. The vitality of the obtained results enables to us to extend the validity of our approach to the next level of simplifications. Namely, considering the microscopic one-phonon states coupled randomly to the two-phonons energies generated from the Gaussian Orthogonal Ensembles distribution. The computational developments that would allow us to conclude on this point are under way.

In summary, by means of the Skyrme mean-field calculations and considering the coupling between the phonons, we have studied the β-decay properties of ^{60}Ca. Using the Skyrme interaction T45 in conjunction with the volume pairing interaction, the unexpectedly low probability of the βxn emission is obtained. To check this, the statistical properties of the 1^+ spectrum populated in the β-decay are analyzed. The restriction of the two-phonon configurations can be justified by the rough estimate from the random matrix theory, which demonstrate the unimportance of other two-phonon composition on the half-life and the neutron-emission probability.

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