

# Explanation of the puzzle of $^{164}\text{Dy}$ scissors

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**Abstract.** The solution of TDHFB equations by Wigner Function Moments method with the isovector-isoscalar coupling taken into account leads to the prediction of the new type of nuclear spin scissors mode. It turns out that the lower group of  $M1$  excitations in  $^{164}\text{Dy}$ , which is usually not included in the systematics of the scissors mode, can be quite naturally attributed to this new type of scissors.

## 1 Introduction

The nuclear scissors mode (rotational oscillations of protons with respect of neutrons in deformed nuclei) was predicted by R. Hilton in 1976 at the conference in Dubna [1] and discovered experimentally in a few years by the group of A. Richter [2]. In the rare earth nuclei it is represented by the group of  $1^+$  excitations in the energy region  $1 < E < 4$  MeV. However, the nucleus  $^{164}\text{Dy}$  is the exceptional case. The spectrum of its low lying  $1^+$  excitations consists, roughly speaking, of two well separated groups of levels. Experimentalists don't attribute to the scissors mode the lower group taking into account only the higher one, otherwise the result deviates very much from the rare earth systematics. The aim of this paper is to clarify this situation. It will be shown that experimentalists are only partly right, because these two groups of  $1^+$  states represent two different kinds of scissors.

We describe this phenomenon by the Wigner Function Moments (WFM) method.

## 2 A few words about the method

The basis of the method is the Time Dependent Hartree-Fock (TDHF) equation for the one-body density matrix  $\rho^\tau(\mathbf{r}_1, \mathbf{r}_2, t) = \langle \mathbf{r}_1 | \hat{\rho}^\tau(t) | \mathbf{r}_2 \rangle$ :

$$i\hbar \frac{\partial \hat{\rho}^\tau}{\partial t} = [\hat{h}^\tau, \hat{\rho}^\tau], \quad (1)$$

where  $\tau$  is an isotopic index.

With the help of Fourier (Wigner) transformation the density matrix  $\rho^\tau(\mathbf{r}_1, \mathbf{r}_2, t)$  is transformed into the Wigner function  $f^\tau(\mathbf{r}, \mathbf{p}, t)$  and equation (1) is transformed into TDHF equation for the Wigner function. Integrating this equation over the phase space with the weights

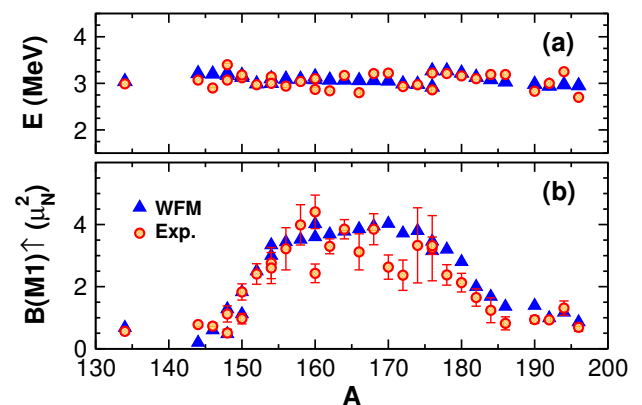
$$\{r \otimes r\}_{\lambda\mu}, \quad \{p \otimes p\}_{\lambda\mu}, \quad \{r \otimes p\}_{\lambda\mu},$$

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where  $\{r \otimes p\}_{\lambda\mu} = \sum_{\sigma,\nu} C_{1\sigma,1\nu}^{\lambda\mu} r_\sigma p_\nu$ , one derives dynamical equations for the following second order moments (collective variables):

$$\begin{aligned} R_{\lambda\mu}^\tau(t) &= 2(2\pi\hbar)^{-3} \int d\mathbf{p} \int d\mathbf{r} \{r \otimes r\}_{\lambda\mu} f^\tau(\mathbf{r}, \mathbf{p}, t), \\ P_{\lambda\mu}^\tau(t) &= 2(2\pi\hbar)^{-3} \int d\mathbf{p} \int d\mathbf{r} \{p \otimes p\}_{\lambda\mu} f^\tau(\mathbf{r}, \mathbf{p}, t), \\ L_{\lambda\mu}^\tau(t) &= 2(2\pi\hbar)^{-3} \int d\mathbf{p} \int d\mathbf{r} \{r \otimes p\}_{\lambda\mu} f^\tau(\mathbf{r}, \mathbf{p}, t). \end{aligned} \quad (2)$$

By definition of R. Hilton the scissors mode is the pure isovector mode. That is why dynamical equations are usually transformed into isovector and isoscalar ones. Neglecting by coupling terms one decouples isovector and isoscalar systems. In the case of the simple model (harmonic oscillator with q-q residual interaction) it is possible to find analytical solutions [3] which coincide with that of RPA [4]. Taking into account pair correlations [5, 6] and spin degrees of freedom [7], i.e. switching from TDHF to TDHF-Bogoliubov (TDHFB) equations, allows one to get the satisfactory agreement with experiment [8] in the description of energies and  $B(M1)$  values of the scissors mode in rare-earth nuclei (see figure 1).



**Figure 1.** Calculated (WFM) mean excitation energies (a) and summed  $M1$  strengths (b) of the scissors mode are compared with experimental data (Exp.).

The natural continuation of these investigations is to study the role of the coupling between isovector and isoscalar modes.

### 3 Isovector-isoscalar coupling

Let us look on the coupled dynamical equations in the simple model of the harmonic oscillator with q-q residual interaction:

$$H = \sum_{i=1}^A \left( \frac{\mathbf{p}_i^2}{2m} + \frac{1}{2} m \omega^2 \mathbf{r}_i^2 \right) + \sum_{\mu=-2}^2 (-1)^\mu \left\{ \bar{\kappa} \sum_i^Z \sum_j^N + \frac{\kappa}{2} \left[ \sum_{i \neq j}^Z + \sum_{i \neq j}^N \right] \right\} q_{2\mu}(\mathbf{r}_i) q_{2-\mu}(\mathbf{r}_j), \quad (3)$$

where  $q_{2\mu} = \sqrt{16\pi/5} r^2 Y_{2\mu}$  and  $N, Z$  are the number of neutrons and protons, respectively.

Isoscalar and isovector variables are defined as

$$X_{\lambda\mu}(t) = X_{\lambda\mu}^n(t) + X_{\lambda\mu}^p(t), \quad \bar{X}_{\lambda\mu}(t) = X_{\lambda\mu}^n(t) - X_{\lambda\mu}^p(t),$$

where  $X \equiv \{R, L, P\}$ . Coupled nonlinear isovector and isoscalar dynamical equations read:

$$\begin{aligned} \dot{\bar{R}}_{\lambda\mu} &= \frac{2}{m} \bar{L}_{\lambda\mu}, \\ \dot{\bar{L}}_{\lambda\mu} &= \frac{1}{m} \bar{P}_{\lambda\mu} - m \omega^2 \bar{R}_{\lambda\mu} \\ &+ 12 \sqrt{5} \sum_{j=0}^2 \sqrt{2j+1} \{1\}_{2,\lambda 1}^{11j} \left[ \kappa_0 \{\bar{R}_2 \otimes \bar{R}_j\}_{\lambda\mu} + \kappa_1 \{\bar{R}_2 \otimes R_j\}_{\lambda\mu} \right], \\ \dot{\bar{P}}_{\lambda\mu} &= -2m \omega^2 \bar{L}_{\lambda\mu} \\ &+ 24 \sqrt{5} \sum_{j=0}^2 \sqrt{2j+1} \{1\}_{2,\lambda 1}^{11j} \left[ \kappa_0 \{\bar{R}_2 \otimes \bar{L}_j\}_{\lambda\mu} + \kappa_1 \{\bar{R}_2 \otimes L_j\}_{\lambda\mu} \right], \\ \dot{R}_{\lambda\mu} &= \frac{2}{m} L_{\lambda\mu}, \\ \dot{L}_{\lambda\mu} &= \frac{1}{m} P_{\lambda\mu} - m \omega^2 R_{\lambda\mu} \\ &+ 12 \sqrt{5} \sum_{j=0}^2 \sqrt{2j+1} \{1\}_{2,\lambda 1}^{11j} \left[ \kappa_0 \{R_2 \otimes R_j\}_{\lambda\mu} + \kappa_1 \{\bar{R}_2 \otimes \bar{R}_j\}_{\lambda\mu} \right], \\ \dot{P}_{\lambda\mu} &= -2m \omega^2 L_{\lambda\mu} + 24 \sqrt{5} \sum_{j=0}^2 \sqrt{2j+1} \{1\}_{2,\lambda 1}^{11j} \left[ \kappa_0 \{R_2 \otimes L_j\}_{\lambda\mu} \right. \\ &\left. + \kappa_1 \{\bar{R}_2 \otimes \bar{L}_j\}_{\lambda\mu} \right], \end{aligned} \quad (4)$$

where  $\kappa_0 = (\kappa + \bar{\kappa})/2$ ,  $\kappa_1 = (\kappa - \bar{\kappa})/2$ ,  $\kappa_1 = \alpha \kappa_0$ . They are solved in a small amplitude approximation. To this end all collective variables are written as  $X_{\lambda\mu}(t) = X_{\lambda\mu}^{\text{eq}} + \mathcal{X}_{\lambda\mu}(t)$ , where  $\mathcal{X}_{\lambda\mu}(t)$  is an infinitesimally small deviation, and

equations are linearized by neglecting  $\mathcal{X}_{\lambda\mu}(t)^2$  terms

$$\begin{aligned} \dot{\bar{R}}_{\lambda\mu} &= \frac{2}{m} \bar{L}_{\lambda\mu}, \\ \dot{\bar{L}}_{\lambda\mu} &= \frac{1}{m} \bar{P}_{\lambda\mu} - m \omega^2 \bar{R}_{\lambda\mu} \\ &+ 12 \sqrt{5} \sum_{j=0}^2 \sqrt{2j+1} \{1\}_{2,\lambda 1}^{11j} \left[ \kappa_0 \{R_2^{\text{eq}} \otimes \bar{R}_j\}_{\lambda\mu} + \kappa_1 \{\bar{R}_2 \otimes R_j^{\text{eq}}\}_{\lambda\mu} \right. \\ &\left. + \underbrace{\kappa_1 \{\bar{R}_2^{\text{eq}} \otimes R_j\}_{\lambda\mu} + \kappa_0 \{R_2 \otimes \bar{R}_j^{\text{eq}}\}_{\lambda\mu}}_{\text{coupling terms}} \right], \\ \dot{\bar{P}}_{\lambda\mu} &= -2m \omega^2 \bar{L}_{\lambda\mu} \\ &+ 24 \sqrt{5} \sum_{j=0}^2 \sqrt{2j+1} \{1\}_{2,\lambda 1}^{11j} \left[ \kappa_0 \{R_2^{\text{eq}} \otimes \bar{L}_j\}_{\lambda\mu} + \kappa_1 \{\bar{R}_2 \otimes L_j^{\text{eq}}\}_{\lambda\mu} \right. \\ &\left. + \underbrace{\kappa_1 \{\bar{R}_2^{\text{eq}} \otimes L_j\}_{\lambda\mu} + \kappa_0 \{R_2 \otimes \bar{L}_j^{\text{eq}}\}_{\lambda\mu}}_{\text{coupling terms}} \right], \end{aligned} \quad (5)$$

$$\begin{aligned} \dot{R}_{\lambda\mu} &= \frac{2}{m} L_{\lambda\mu}, \\ \dot{L}_{\lambda\mu} &= \frac{1}{m} P_{\lambda\mu} - m \omega^2 R_{\lambda\mu} \\ &+ 12 \sqrt{5} \sum_{j=0}^2 \sqrt{2j+1} \{1\}_{2,\lambda 1}^{11j} \left[ \kappa_0 \left( \{R_2^{\text{eq}} \otimes R_j\}_{\lambda\mu} + \{R_2 \otimes R_j^{\text{eq}}\}_{\lambda\mu} \right) \right. \\ &\left. + \underbrace{\kappa_1 \left( \{\bar{R}_2^{\text{eq}} \otimes \bar{R}_j\}_{\lambda\mu} + \{\bar{R}_2 \otimes \bar{R}_j^{\text{eq}}\}_{\lambda\mu} \right)}_{\text{coupling terms}} \right], \\ \dot{P}_{\lambda\mu} &= -2m \omega^2 L_{\lambda\mu} \\ &+ 24 \sqrt{5} \sum_{j=0}^2 \sqrt{2j+1} \{1\}_{2,\lambda 1}^{11j} \left[ \kappa_0 \left( \{R_2^{\text{eq}} \otimes L_j\}_{\lambda\mu} + \{R_2 \otimes L_j^{\text{eq}}\}_{\lambda\mu} \right) \right. \\ &\left. + \underbrace{\kappa_1 \left( \{\bar{R}_2^{\text{eq}} \otimes \bar{L}_j\}_{\lambda\mu} + \{\bar{R}_2 \otimes \bar{L}_j^{\text{eq}}\}_{\lambda\mu} \right)}_{\text{coupling terms}} \right]. \end{aligned} \quad (6)$$

In our previous papers we used two ways of decoupling of isovector and isoscalar equations:

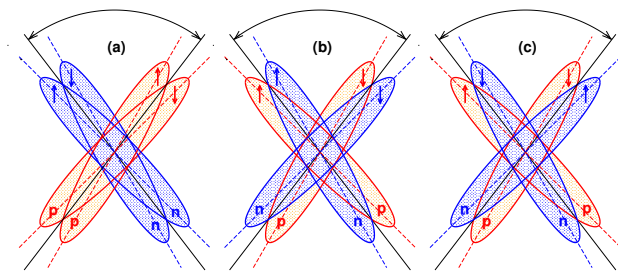
1. Just neglect by the coupling terms, that is equivalent to the assumption that all equilibrium characteristics of protons are equal to that of neutrons (that is true for  $N = Z$ ).
2. One assumes that all amplitudes (deviations) of protons are proportional to that of neutrons:  $\mathcal{X}_{\lambda\mu}^p/Z = \pm \mathcal{X}_{\lambda\mu}^n/N$  (+ for isoscalar motion, - for isovector one).

As a matter of fact, we don't need in the artificial decoupling, because there are no problems to solve the set of coupled equations without any approximations. So, let us consider the full problem: Nilsson potential with pair correlations, quadrupole-quadrupole and spin-spin residual interactions, taking into account the coupling terms. The result turned out rather unexpected (see table 1). Both methods of decoupling produce similar qualitative results. The high-lying levels are less sensitive to the decoupling. The most remarkable and unexpected change happens with third low-lying level - it acquires rather big magnetic strength. So, in the decoupled case there were

**Table 1.** Energies and excitation probabilities of  $^{164}\text{Dy}$  obtained by the exact and approximate solutions of the set of equations for second order moments (2) of TDHFB equation. The low-lying isovector modes in the decoupled cases are shown by bold face. The values of all model parameters are taken from our paper [8].

Coupled equations			Decoupling 2			Decoupling 1		
$E$ , MeV	$B(M1)$ , $\mu_N^2$	$B(E2)$ , W.u.	$E$ , MeV	$B(M1)$ , $\mu_N^2$	$B(E2)$ , W.u.	$E$ , MeV	$B(M1)$ , $\mu_N^2$	$B(E2)$ , W.u.
1.47	0.17	25.44	1.38	0.02	34.00	1.29	0.01	53.25
2.20	1.76	3.30	<b>2.36</b>	<b>1.52</b>	<b>0.28</b>	<b>2.44</b>	<b>2.03</b>	<b>0.34</b>
2.87	2.24	0.34	2.54	0.07	2.41	2.62	0.09	2.91
3.59	1.56	4.37	<b>3.26</b>	<b>1.04</b>	<b>1.38</b>	<b>3.35</b>	<b>1.36</b>	<b>1.62</b>
10.92	0.04	50.37	11.35	0.00	42.94	10.94	0.00	55.12
13.10	0.00	2.85	14.31	0.00	2.39	14.04	0.00	2.78
15.42	0.07	0.57	14.81	0.05	0.43	14.60	0.06	0.48
15.55	0.00	1.12	16.01	0.00	0.50	15.88	0.00	0.55
16.78	0.06	0.53	16.56	0.06	0.30	16.46	0.07	0.36
17.69	0.01	0.68	17.70	0.00	0.43	17.69	0.00	0.45
17.91	0.00	0.53	17.95	0.00	0.43	17.90	0.00	0.51
18.22	0.13	0.89	18.20	0.14	1.48	18.22	0.18	1.85
19.32	0.08	0.61	19.31	0.08	0.78	19.32	0.10	0.97
21.26	2.03	21.60	21.26	1.90	25.26	21.29	2.47	31.38

2 isovector magnetic and 2 isoscalar electric levels, and in the coupled case there are 3 magnetic and 1 electric levels of the mixed isovector-isoscalar nature. It turns out that all 3 magnetic states correspond to 3 physically possible scissors modes: figure 2 (a) – spin-scalar isovector (conventional, orbital scissors), (b) – spin-vector isoscalar (spin scissors), (c) – spin-vector isovector (spin scissors).



**Figure 2.** Schematic representation of three physically possible scissors modes: (a) – spin-scalar isovector, (b) – spin-vector isoscalar, (c) – spin-vector isovector. Arrows inside ellipses show the direction of spin projections; p (n) corresponds to protons (neutrons).

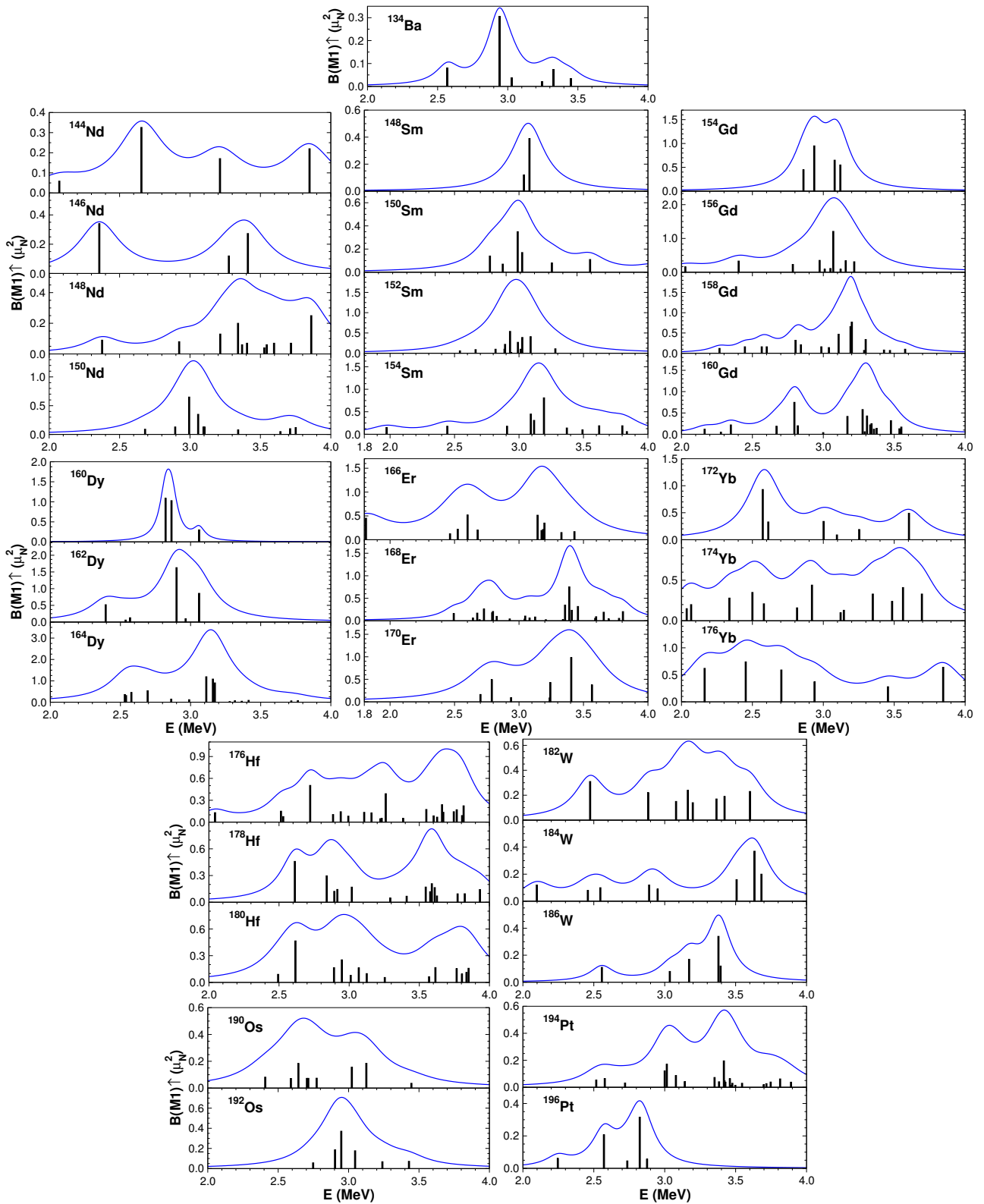
It would be interesting to find the examples of experimental spectra of  $M1$  excitations demonstrating the similar fine structure – the more or less pronounced division of levels in three separate groups. The available experimental spectra for rare earth nuclei are shown in figure 3. For the sake of convenience we folded these spectra with Lorentzians of increasing widths. The widths were increased until only three humps (if possible) remained. The careful analysis of this figure allows one to find 10 nuclei where the three humps structure of spectra is seen:  $^{134}\text{Ba}$ ,  $^{144}\text{Nd}$ ,  $^{166}\text{Er}$ ,  $^{172}\text{Yb}$ ,  $^{176,178,180}\text{Hf}$ ,  $^{186}\text{W}$ ,  $^{194,196}\text{Pt}$ .

The preliminary results of calculations for Hf isotopes are compared with experimental data in table 2. For  $^{176}\text{Hf}$  the agreement between theoretical and experimental values of energies  $E$  and  $B(M1)$  for all three humps can be characterized as satisfactory. Only the distribution of  $M1$

strength between the top and middle humps spoils the impression: the theory predicts the middle hump stronger than the top hump that contradicts to the experiment. On the other hand the summed  $B(M1)$  of top and middle humps predicted by theory  $2.72 \mu_N^2$  is quite close to the respective experimental value  $2.46 \mu_N^2$ . It is necessary to emphasize that the energy centroid of all three theoretical humps is in excellent agreement with experimental value, whereas the respective summed  $B(M1)$  value is disposed just in between the minimal (Experiment  $1^+$  in table 2) and maximum (Experiment  $1^+ + 1^\pi$ ) possible experimental results. The situation in  $^{180}\text{Hf}$  looks quite similar, if to compare the theoretical  $B(M1)$  with maximum possible experimental  $B(M1)$  values. It is difficult to estimate the degree of agreement (or disagreement) of experimental data with theoretical predictions for  $^{178}\text{Hf}$ , because here the theory strongly overestimate ( $0.85 \mu_N^2$ ) the summed  $B(M1)$  value. One can not exclude that some  $1^+$  levels of this nucleus are not observed yet.

Taking into account isovector-isoscalar coupling allows one to resolve the old standing problem – puzzle of the  $^{164}\text{Dy}$  scissors mode. The best introduction into this problem is the citation from the paper of J. Margraf *et al* [16]: "There are clearly two groups of strong  $M1$  excitations around 2.5 and 3.1 MeV, respectively. The upper group is attributed to the scissors mode. ... The group of  $M1$  excitations in  $^{164}\text{Dy}$  around 2.5 MeV is not included in the systematics of the scissors mode, because a  $^{165}\text{Ho}(t,\alpha)^{164}\text{Dy}$  experiment [25] reported a nearly pure two-quasiparticle  $M1$  excitation in this energy region." The additional, even stronger, argument to exclude the lower group of  $M1$  excitations from the scissors mode systematics is the remarkable spin contribution into the magnetic strength of this group, observed by D. Frekers *et al* [26]:  $B_\sigma(M1) = 0.72 \mu_N^2$  at  $E = 2.6$  MeV. The moderate spin contribution  $B_\sigma(M1) = 0.50 \mu_N^2$  to the higher group also is excluded from systematics.

The solution of dynamical equations for second order moments of TDHFB equations with the isovector-isoscalar coupling taken into account (see table 3) allows



**Figure 3.** The experimentally observed spectra of  $1^+$  excitations:  $^{134}\text{Ba}$  – [9],  $^{144-150}\text{Nd}$  – [10, 11],  $^{148-154}\text{Sm}$  – [12],  $^{154-160}\text{Gd}$  – [13, 14],  $^{160-164}\text{Dy}$  – [15, 16],  $^{166-170}\text{Er}$  – [17],  $^{172-176}\text{Yb}$  – [18],  $^{176-180}\text{Hf}$  – [19, 20],  $^{182-186}\text{W}$  – [21],  $^{190-192}\text{Os}$  – [22],  $^{194-196}\text{Pt}$  – [23, 24]. The lines of spectra are allotted by artificial widths and joined by Lorentzian curves.

**Table 2.** WFM – theory, the values of all model parameters are taken from our paper [8]. Experimental data are taken from [19, 20]:  $1^+$  – only levels with  $J^\pi = 1^+$  are considered,  $1^+ + 1^\pi$  – the levels with  $J = 1$  but  $\pi$  unknown are added to  $1^+$  levels. See text for detailed explanations.

$^A X$	$E$ , MeV			$B(M1), \mu_N^2$			$\bar{E}$ , MeV			$\sum B(M1), \mu_N^2$		
	WFM	Experiment		WFM	Experiment		WFM	Experiment		WFM	Experiment	
		$1^+$	$1^+ + 1^\pi$		$1^+$	$1^+ + 1^\pi$		$1^+$	$1^+ + 1^\pi$		$1^+$	$1^+ + 1^\pi$
$^{176}\text{Hf}$	2.66	2.57	2.54	1.08	0.86(06)	1.02(09)						
	3.36	3.14	3.17	1.96	1.13(09)	1.45(13)	3.26	3.22	3.25	3.79	3.32(28)	4.40(45)
	3.86	3.70	3.69	0.76	1.33(13)	1.93(23)						
$^{178}\text{Hf}$	2.62	2.61	2.61	0.96	0.46(03)	0.46(03)						
	3.33	2.91	2.91	1.89	0.73(08)	0.73(08)	3.21	3.21	3.26	3.42	2.38(33)	2.57(37)
	3.82	3.64	3.67	0.58	1.19(22)	1.38(26)						
$^{180}\text{Hf}$	2.60	2.60	2.56	0.93	0.56(07)	0.62(09)						
	3.34	3.01	3.02	2.04	0.82(10)	0.96(14)	3.22	3.16	3.30	3.51	2.13(30)	3.00(49)
	3.83	3.75	3.80	0.54	0.75(13)	1.43(27)						

**Table 3.** The nuclear scissors fine structure. The results of calculations are compared with experimental data for  $^{164}\text{Dy}$  [16], the values of all model parameters are taken from our paper [8]. The numbers in square brackets are obtained with  $g_s = 0$ . See text for detailed explanations.

Theory (WFM)					Experiment	
$E$ , MeV	$B(M1), \mu_N^2$	$\bar{E}$ , MeV	$\sum B(M1), \mu_N^2$	$\bar{E}$ , MeV	$\sum B(M1), \mu_N^2$	
2.20	1.76 [0.53]	2.20	1.76	2.60	1.67(14)	
2.87	2.24 [1.52]	3.17	3.80	3.17	3.85(31)	
3.59	1.56 [6.63]					

one to explain the nature of both groups of  $1^+$  levels. The calculations produce three  $1^+$  states which can be classified approximately as spin-vector isovector state (lowest one), spin-vector isoscalar state (middle) and spin-scalar isovector state (highest). It is easy to see that the energy centroid  $\bar{E}$  and summarized  $B(M1)$  of the lower group of the experimental  $1^+$  states agree very well with the calculated  $E$  and  $B(M1)$  of the spin-vector isovector level, whereas the respective values of the higher group are in excellent agreement with the energy centroid and summed  $B(M1)$  of the calculated spin-vector isoscalar and spin-scalar isovector levels. So, the lower group is practically pure spin scissors and the higher group is the mixture of two kinds of scissors: spin and orbital ones. Nevertheless one has to emphasize here that both, spin and orbital scissors, are equally sensitive to the spin dependent part of nuclear forces, that is confirmed by the results of calculations of  $B(M1)$  with ( $g_s = 0.7g_s^{\text{free}}$ ) and without ( $g_s = 0$ ) the spin part of a perturbing dipole magnetic operator. Really, comparing second and third columns of table 3 one observes the moderate constructive interference of the orbital and spin contributions in the case of the spin scissors mode and their very strong destructive interference in the case of the orbital scissors mode.

#### 4 Concluding remarks

The most important cognitive result of the exact (without the artificial isovector-isoscalar decoupling) solution of dynamical equations for second order moments of TD-HFB equations is the understanding of the remarkable fact: three different types of the scissors mode can exist in nuclei. Being understood this result can be obtained quite

easy from the simple combinatorics: there are only three ways to divide four objects (spin up and spin down protons and neutrons) in pairs!

The calculations have shown that the scissors mode is not pure isovector!

Another inference is that all three scissors modes are not pure orbital and not pure spin modes.

Nevertheless one has to be careful with this statement because it is true only in the sense, that all they are equally sensitive to the spin part of the external field. On the other side one has to remember that both spin scissors exist only due to the existence of spin. It becomes easily seen, if to remove arrows from figures 2 (a, b, c) – in this case figures (b) and (c) become identical and senseless, because the division of neutrons and protons in two parts becomes questionable. Only in this sense both spin scissors can be called by the pure spin modes.

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