U(6) dynamical and quasi-dynamical symmetry in strongly deformed heavy nuclei

H. G. Ganev[1,2,*]

[1]Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, Russia
[2]Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria

Abstract. The low-lying collective states of the ground, β and γ bands in ^{154}Sm and ^{238}U are investigated within the framework of the microscopic proton-neutron symplectic model (PNSM). For this purpose, the model Hamiltonian is diagonalized in a $U(6)$-coupled basis, restricted to the symplectic state space spanned by the fully symmetric $U(6)$ vectors. A good description of the energy levels of the three bands under consideration, as well as the intraband $B(E2)$ transition strengths between the states of the ground band is obtained for the two nuclei without the use of an effective charge. The calculations show that when the collective quadrupole dynamics is covered already by the symplectic bandhead structure, as in the case of ^{154}Sm, the results show the presence of a very good $U(6)$ dynamical symmetry. In the case of ^{238}U, when we have an observed enhancement of the intraband $B(E2)$ transition strengths, then the results show small admixtures from the higher major shells and a highly coherent mixing of different irreps which is manifested by the presence of a good $U(6)$ quasi-dynamical symmetry in the microscopic structure of the collective states under consideration.

1 Introduction

Experimental spectra in heavy nuclei show the emergence of simple collective patterns represented primarily by the nuclear collective rotation. The microscopic shell-model structure of these low-lying rotational states is still a challenge for the microscopic many-particle nuclear theory. This is particularly so because the model space dimensionality rules out the use of standard shell-model theory. As a consequence, different algebraic models which capitalize on symmetries, exact or approximate, have been developed to reduce the model space in manageable size.

The first microscopic, algebraic, model of nuclear collective motion in light nuclei is the Elliott's $SU(3)$ model [1] which showed how states with rotational properties could emerge within the framework of the nuclear shell model. It defined a relevant coupling scheme for identifying the collective dynamics and performing a large shell-model calculations. Further, a natural multi-shell generalization of the Elliott model has been incorporated in the one-component $Sp(6,R)$ symplectic model [2, 3], which together with valence shell also includes monopole and quadrupole giant vibrational degrees of freedom. From the hydrodynamic perspective, it has been shown that the $Sp(6,R)$ symplectic model is a microscopic generalization of the Bohr-Mottelson [4] collective model, augmented by the intrinsic vortex spin degrees of freedom, which is also compatible with the microscopic nucleon structure of nucleus [5]. The vortex spin degrees of freedom allows for the presence of full range of collective flows in nuclei – from irrotational to the rigid-rotor rotations. The internal vortex degrees of freedom play also an important role in the construction of the microscopic wave functions with the proper antisymmetric properties and are responsible for the appearance of low-lying collective states in nuclear spectra.

Recently, the fully microscopic proton-neutron symplectic model (PNSM) of nuclear collective motion with $Sp(12,R)$ dynamical symmetry was introduced by considering the symplectic geometry and possible collective flows in the two-component many-particle nuclear system [6]. Further, it was shown that, in its hydrodynamic limit, it reduces to the $U(6)$-phonon model with the semi-direct product structure $[HW(21)]U(6)$ which unifies both the two-fluid irrotational-flow collective model of Bohr-Mottelson type and a microscopically based $U(6)$ model [7]. The latter naturally generalizes the $SU(3)$ model of Elliott [1] for the case of two-component many-particle nuclear system and is related to the valence proton-neutron degrees of freedom. From the hydrodynamic perspective, the $U(6)$-phonon model therefore includes the irrotational collective flows and their coupling to the intrinsic vortex degrees of freedom. In this way the extra degrees of freedom contained in this larger $U(6)$ algebraic structure will therefore embrace the basic $SU(3)$ rotor as well as the low-lying vibrational degrees of freedom.

The appearance of an $U(6)$ intrinsic structure in both the PNSM and $U(6)$-phonon model is of significant importance for the microscopic theory of nuclear collective excitations. In this regard, we recall that the popular Interacting Boson Model [8] has clearly demonstrated that simple algebraic ways exist to get collective spectra within $U(6)$-based scheme. Then, within the framework of the

*e-mail: huben@theor.jinr.ru

PNSM (or its macroscopic hydrodynamic limit), the full range of low-lying states could be described by microscopically based $U(6)$ structure along the lines of the IBM, albeit in contrast to the latter, renormalized by their coupling to the giant resonance vibrations. This result could not be overestimated recalling also that in order to obtain the low-lying excited collective bands (e.g., beta bands) within the framework of the one-component symplectic model [2] one needs to involve a representation mixing caused by, e.g., pairing, spin-orbit and other symplectic-breaking components of the nuclear interaction (cf. Ref. [9]).

2 The proton-neutron symplectic model

Collective observables of the proton-neutron symplectic model, which span the $Sp(12,R)$ algebra, are given by the following one-body operators [6]:

$$Q_{ij}(\alpha,\beta) = \sum_{s=1}^{m} x_{is}(\alpha)x_{js}(\beta), \tag{1}$$

$$S_{ij}(\alpha,\beta) = \sum_{s=1}^{m}\Big(x_{is}(\alpha)p_{js}(\beta) + p_{is}(\alpha)x_{js}(\beta)\Big), \tag{2}$$

$$L_{ij}(\alpha,\beta) = \sum_{s=1}^{m}\Big(x_{is}(\alpha)p_{js}(\beta) - x_{js}(\beta)p_{is}(\alpha)\Big), \tag{3}$$

$$T_{ij}(\alpha,\beta) = \sum_{s=1}^{m} p_{is}(\alpha)p_{js}(\beta), \tag{4}$$

where $i,j = 1,2,3; \alpha,\beta = p,n$ and $s = 1,\ldots,m = A-1$. In Eqs. (1)–(4), $x_{is}(\alpha)$ and $p_{is}(\alpha)$ denote the coordinates and corresponding momenta of the translationally-invariant Jacobi vectors of the m-quasiparticle two-component nuclear system and A is the number of protons and neutrons.

In terms of the harmonic oscillator creation and annihilation operators

$$b^{\dagger}_{i\alpha,s} = \sqrt{\frac{m_\alpha\omega}{2\hbar}}\Big(x_{is}(\alpha) - \frac{i}{m_\alpha\omega}p_{is}(\alpha)\Big),$$
$$b_{i\alpha,s} = \sqrt{\frac{m_\alpha\omega}{2\hbar}}\Big(x_{is}(\alpha) + \frac{i}{m_\alpha\omega}p_{is}(\alpha)\Big), \tag{5}$$

the many-particle realization of the $Sp(12,R)$ Lie algebra is given by [10]:

$$F_{ij}(\alpha,\beta) = \sum_{s=1}^{m} b^{\dagger}_{i\alpha,s}b^{\dagger}_{j\beta,s}, \tag{6}$$

$$G_{ij}(\alpha,\beta) = \sum_{s=1}^{m} b_{i\alpha,s}b_{j\beta,s}, \tag{7}$$

$$A_{ij}(\alpha,\beta) = \frac{1}{2}\sum_{s=1}^{m}(b^{\dagger}_{i\alpha,s}b_{j\beta,s} + b_{j\beta,s}b^{\dagger}_{i\alpha,s}). \tag{8}$$

An $Sp(12,R)$ unitary irreducible representation is characterized by the $U(6)$ quantum numbers $\sigma = [\sigma_1,\ldots,\sigma_6]$ of its lowest-weight state $|\sigma\rangle$, i.e. $|\sigma\rangle$ satisfies

$$G_{ab}|\sigma\rangle = 0;$$
$$A_{ab}|\sigma\rangle = 0, \quad a < b;$$
$$A_{aa}|\sigma\rangle = \Big(\sigma_a + \frac{m}{2}\Big)|\sigma\rangle \tag{9}$$

for the indices $a \equiv i\alpha$ and $b \equiv j\beta$ taking the values $1,\ldots,6$. If we introduce the $U(6)$ tensor product operators $P^{(n)}(F) = [F\times\ldots\times F]^{(n)}$, where $n = [n_1,\ldots,n_6]$ is a partition with even integer parts, then by an $U(6)$ coupling of these tensor products to the lowest-weight state $|\sigma\rangle$, one constructs the whole basis of states for an $Sp(12,R)$ irrep

$$|\Psi(\sigma n\rho E\eta)\rangle = [P^{(n)}(F)\times|\sigma\rangle]^{\rho E}_{\eta}, \tag{10}$$

where $E = [E_1,\ldots,E_6]$ indicates the $U(6)$ quantum numbers of the coupled state, η labels a basis of states for the coupled $U(6)$ irrep E and ρ is a multiplicity index. In this way we obtain a basis of $Sp(12,R)$ states that reduces the subgroup chain $Sp(12,R) \supset U(6)$. To fix the basis η one has to consider further the reduction of the $U(6)$ to the 3-dimensional rotational group $SO(3)$. Thus, in order to completely classify the basis states, we use the following reduction chain [10]:

$$\begin{array}{ccc} Sp(12,R) & \supset & \\ \sigma & & n\rho \end{array}$$
$$\begin{array}{ccc} \supset U(6) \supset & SU_p(3)\otimes SU_n(3) & \\ E\ \ \gamma & (\lambda_p,\mu_p)\ \ (\lambda_n,\mu_n) & \end{array}$$
$$\begin{array}{cccc} \supset SU(3) \supset & SO(3) \supset & SO(2), & \\ \varrho\ \ (\lambda,\mu)\ \ K & L & M & \end{array} \tag{11}$$

which defines a shell-model coupling scheme. The chain (11) corresponds to the following choice of the index $\eta = \gamma(\lambda_p,\mu_p)(\lambda_n,\mu_n)\varrho(\lambda,\mu)KLM$, labeling the basis states (10) of an $Sp(12,R)$ irrep. Each $Sp(12,R)$ irreducible representation is determined by a symplectic bandhead or an intrinsic $U(6)$ space, which in turn is fixed by the underlying proton-neutron shell-model structure. So, the theory becomes completely compatible with the Pauli principle.

In the present paper, we consider the Hilbert space spanned by the fully symmetric $U(6)$ irreducible representations only, which are multiplicity free, i.e. $\rho = 1$. As a consequence, in this case, all the multiplicity indices, except the quantum number K, in the basis (11) are equal to 1 and can be dropped in our further considerations.

3 Application

A more general Hamiltonian of the PNSM can be represented in the form

$$H = H_0 + H_{head} + V(A,F,G), \tag{12}$$

consisting of the spherical harmonic oscillator part H_0, a term which determines the bandhead energies of different bands, and a collective potential $V = V(A,F,G)$, which is a rotational scalar function of the $Sp(12,R)$ generators (6)-(8). Such a Hamiltonian contains the main components of nuclear interaction and covers simultaneously both the single particle and collective aspects of nuclear dynamics.

In particular, we use the following Hamiltonian

$$H = N\hbar\omega - \xi C_2[SU(3)] - \frac{1}{2}\chi \widetilde{Q}_p \cdot \widetilde{Q}_n - \kappa \sum_{\alpha\neq\beta} \Big(A^2(\alpha,\alpha)\cdot G^2(\beta,\beta) + G^2(\alpha,\alpha)\cdot G^2(\beta,\beta) + h.c.\Big) + aL^2, \quad (13)$$

where $N = N_p + N_n$ and where $\widetilde{Q}_{\alpha,m} = A^{2m}(\alpha,\alpha)$ with $\alpha = p, n$ are the in-shell truncated Elliott $SU(3)$ quadrupole operators for the proton and neutron subsystems, respectively. The $SU(3)$ second-order Casimir operator of the combined proton-neutron system is given by

$$C_2[SU(3)] = \widetilde{Q}\cdot\widetilde{Q} + \frac{1}{2}L^2 \quad (14)$$

and has an eigenvalue $\langle C_2[SU(3)]\rangle = \frac{2}{3}(\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu)$. The forth term provides a vertical coupling of states from different major shells. Its role will be clarified further. Finally, the last term in (13), which represents a residual rotor part, allows the experimentally observed moment of inertia to be reproduced without altering the wave functions. The Hamiltonian (13) preserves the symplectic symmetry, thus having $Sp(12,R)$ as its dynamical symmetry. The full dynamics for it therefore occurs within a single irreducible representation of $Sp(12,R)$.

The first point in the practical application of the theory for description of the low-lying collective states in strongly deformed nuclei is the determination of the relevant irreducible representations of $Sp(12,R)$. Different approaches exist to determine these symplectic irreps by fixing the shell-model structure of the ground state using isotropic or anisotropic harmonic oscillator with or without spin-orbit interaction. It is well known that, for heavy mass nuclei from the rare-earth and actinide regions, the latter is strong and destroys the oscillator structure. Due to this, we use the pseudo-$SU(3)$ scheme to determine the relevant irreducible representations of $Sp(12,R)$. The shell-model considerations based on the pseudo-$SU(3)$ thus give the symplectic irreps $\langle\sigma\rangle = \langle 72+\frac{153}{2}, 42+\frac{153}{2}, 42+\frac{153}{2}, 42+\frac{153}{2}, 42+\frac{153}{2}, 42+\frac{153}{2}\rangle$ and $\langle\sigma\rangle = \langle 111+\frac{237}{2}, 57+\frac{237}{2}, 57+\frac{237}{2}, 57+\frac{237}{2}, 57+\frac{237}{2}, 57+\frac{237}{2}\rangle$, which are determined by the intrinsic $U(6)$ structure of the corresponding lowest-weight states $\sigma = [72,42,42,42,42,42]_6 \equiv [30]_6$ and $\sigma = [111+57,57,57,57,57,57]_6 \equiv [54]_6$, as relevant for ^{154}Sm and ^{238}U, respectively. The latter are fixed by the proton-neutron shell-model structure of the corresponding ground states. More details about the construction and structure of the shell-model representations of the PNSM can be found in Ref. [10].

Once the appropriate symplectic irreps are fixed, the model Hamiltonian (13) is further used to determine the microscopic structure of the low-lying collective states in the two isotopes ^{154}Sm and ^{238}U, respectively. For this purpose, we diagonalize the model Hamiltonian (13) in a $U(6)$-coupled basis, restricted to state space spanned by the fully symmetric $U(6)$ irreps.

Consider first the ^{154}Sm. The theoretical energy levels of the states of ground, β and γ bands in this nucleus, obtained by the Hamiltonian with $\kappa = 0$ are compared with experiment [11] in Fig. 1. From the latter one sees a good agreement with the experimental data.

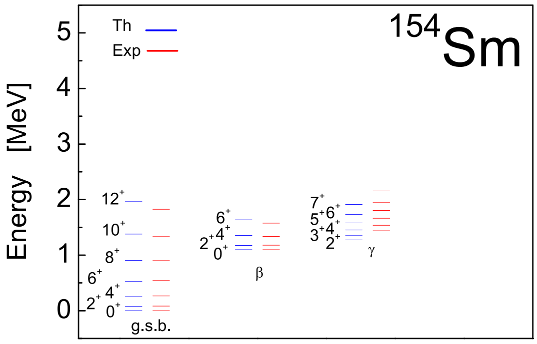

Figure 1. (Color online) Comparison of the theoretical and experimental energy levels for the ground, β, and γ bands in ^{154}Sm. The values for the model parameters are as follows (in MeV): $\chi = 0.011$, $\xi = 0.0073$, $\kappa = 0$, and $a = 0.014$.

In Fig. 2, we show the $SU(3)$ probability distributions, obtained in the calculations, for the 0^+ states of the ground and β bands,as well as the 2^+ state of the γ band in ^{154}Sm. From the figure we see that the $SU(3)$ dynamical symmetry is slightly broken due to the mixing. In particular, for the states of ground and β bands we see a comparatively simple structure in which several $SU(3)$ multiplets contribute. For the 2^+ states of the γ band one sees almost a pure $SU(3)$ structure, determined by the $SU(3)$ irrep (26, 2) which exhausts 99,589%. Since $\kappa = 0$ (there is no vertical mixing), all $SU(3)$ states, contributing to the structure of the collective states under consideration, belong to a single $U(6)$ irrep, namely that of the symplectic bandhead. The same picture is obtained for the other collective states.

We also calculate the reduced intraband $E2$ electromagnetic transition strengths between the states of the ground state band

$$B(E2; L_i \to L_f) = \frac{2L_f+1}{2L_i+1}\left(\frac{5}{16\pi}\right)\left(\frac{eZ}{A-1}\right)^2 \left|\langle f\|Q(p,p)\|i\rangle\right|^2. \quad (15)$$

Note that in the definition of the operator $Q(p,p)$ (cf. Eq. (1)), the summation is over the $(A-1)$ Jacobi quasi-particles. Thus, in order to obtain the proton charge quadrupole operator, the $Q(p,p)$ operator is multiplied by the factor $Z/(A-1)$. The calculated reduced intraband $E2$ electromagnetic transition strengths in ^{154}Sm are compared with experiment [11] in Fig. 3. We note that no effective charge is used in the calculation, i.e. $e = 1$.

From the results for the energy levels and $B(E2)$ transition strengths, shown in Figs. 1-3, it follows that the collective dynamics in ^{154}Sm is already reproduced at the level of $Sp(12,R)$ bandhead intrinsic structure. Something more, the calculations with the Hamiltonian (13) (with $\kappa = 0$), in which the collective potential $\widetilde{Q}_p \cdot \widetilde{Q}_n$ is replaced by the full major-shell mixing $Q_p \cdot Q_n$ interaction, show that its eigenvectors obtained as a reasult of the diagonalization in a model space up to $40\hbar\omega$ belong prac-

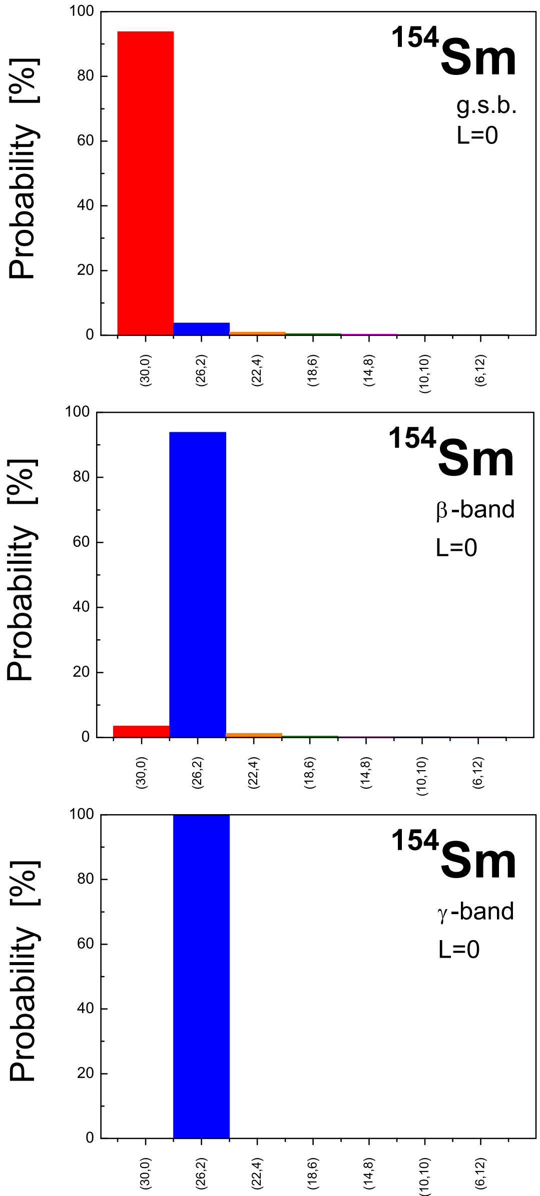

Figure 2. (Color online) Calculated $SU(3)$ probability distributions for the wave functions of the 0^+ states of the ground and β bands, as well as the 2^+ state of the γ band. The values for the model parameters are as follows (in MeV): $\chi = 0.011$, $\xi = 0.0073$, $\kappa = 0$, and $a = 0.014$.

tically to the $Sp(12, R)$ bandhead state space only. (Actually, this was the reason to introduce an additional term (the κ-term) to the Hamiltonian (13), which provides more stronger vertical mixing than that provided by the full major-shell mixing quadrupole-quadrupole driving force, $Q_p \cdot Q_n$.) All this reveals the very good $U(6)$ dynamical symmetry, present in the spectra of ^{154}Sm.

Next, we consider the isotope ^{238}U. The latter is a typical rotational nucleus and, in addition, clearly shows enhanced quadrupole collective dynamics through the measured $B(E2)$ transition strengths between collective states of the ground band. In Fig. 4, we show the $E2$ transition probability between the ground and first excited states of the ground band in ^{238}U as a function of the parameter

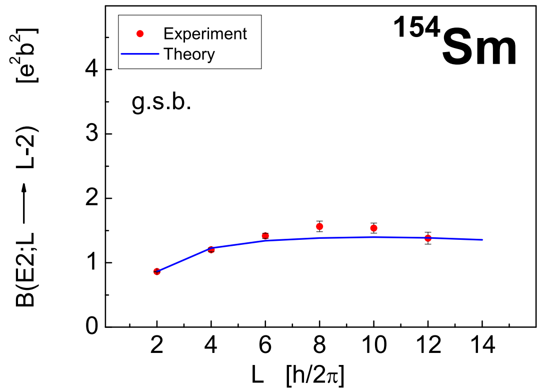

Figure 3. (Color online) Calculated and experimental intraband $B(E2)$ values between the states of the ground band in ^{154}Sm. No effective charge is used. The values for the model parameters are as follows (in MeV): $\chi = 0.011$, $\xi = 0.0073$, $\kappa = 0$, and $a = 0.014$.

χ, changing in a certain interval of physically meaningful values and $\kappa = 0$. From the latter we see a reduction of the $B(E2)$ strength with the increase of χ.

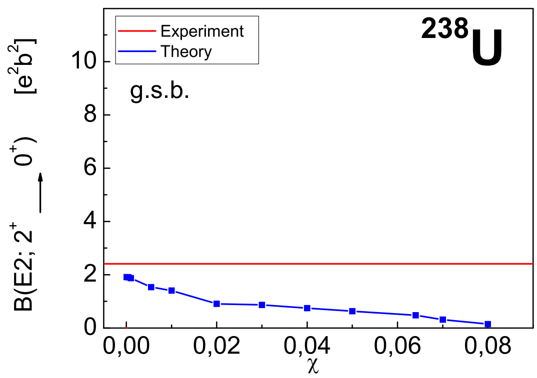

Figure 4. (Color online) Calculated $B(E2; 2_1^+ \to 0_1^+)$ transition strength in ^{238}U as a function of the model parameter χ and $\kappa = 0$. No effective charge is used.

Figure 4 shows clearly that the observed collective dynamics in ^{238}U is not covered by the $Sp(12, R)$ bandhead structure and that in order to build up the required collectivity, in contrast to the $Sp(6, R)$ case, one needs to introduce a vertical mixing term which is different from the full major-shell quadrupole-quadrupole mixing interaction. Such a mixing can be obtained by switching on the forth term in the Hamiltonian (13), allowing κ to be varied.

In Fig. 5, the calculated intraband $B(E2)$ transition strengths between the collective states of the ground band in ^{238}U are compared with experiment [11]. The theoretical results are obtained with the following model parameters: $\chi = 0.0055$, $\xi = 0.0034$, $a = 0.006$, and $\kappa = 0.0101$. From the figure one sees that the enhanced $B(E2)$ strengths, observed in ^{238}U, are already well reproduced by switching on the κ vertical mixing term. Additionally, in Fig. 6, the energy levels of the ground, β, and γ bands in ^{238}U are also shown. One sees a good description of the three collective bands under consideration.

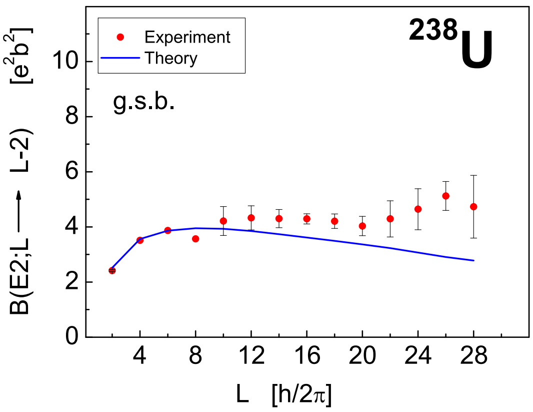

Figure 5. (Color online) Calculated and experimental intraband $B(E2)$ values between the states of the ground band in ^{238}U. No effective charge is used. The values for the model parameters are as follows (in MeV): $\chi = 0.0055$, $\xi = 0.0034$, $a = 0.006$, and $\kappa = 0.0101$.

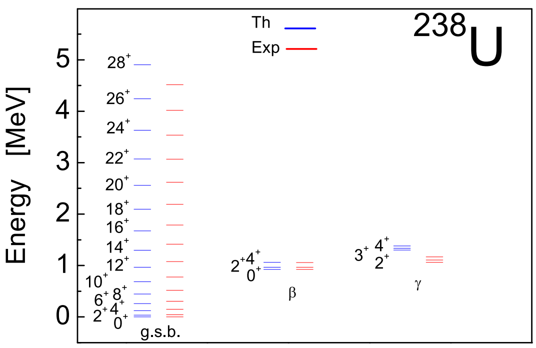

Figure 6. (Color online) Comparison of the theoretical and experimental energy levels for the ground, β, and γ bands in ^{238}U. The values for the model parameters are as follows (in MeV): $\chi = 0.0055$, $\xi = 0.0034$, $a = 0.006$, and $\kappa = 0.0101$.

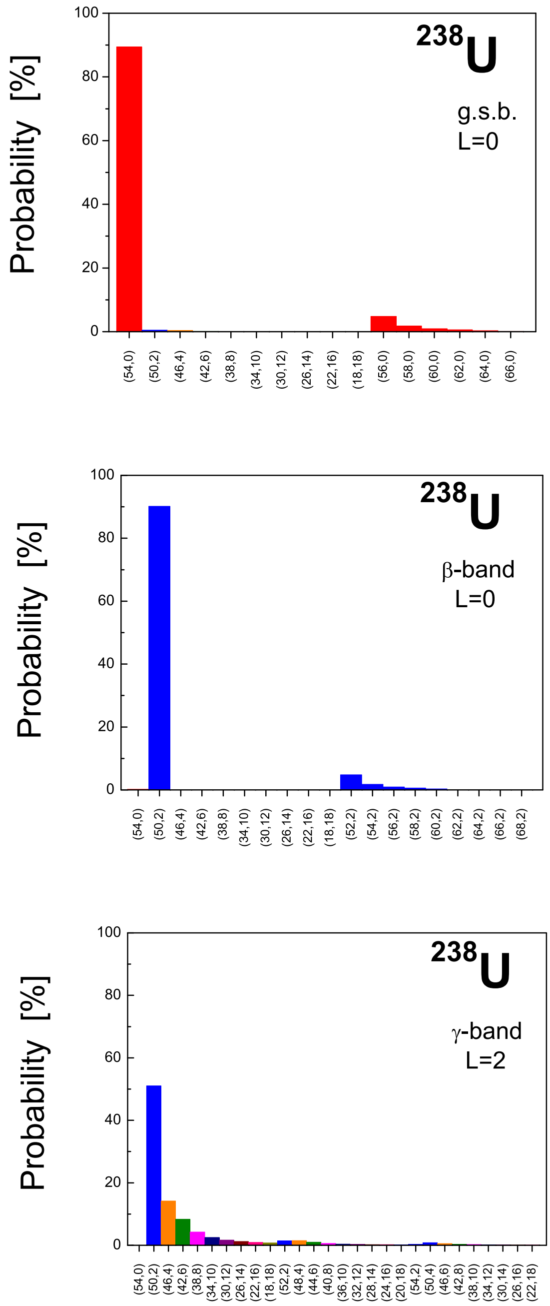

Figure 7. (Color online) Calculated $SU(3)$ probability distributions for the wave functions of the ground, β, and γ bands. The values for the model parameters are as follows (in MeV): $\chi = 0.0055$, $\xi = 0.0034$, $a = 0.006$, and $\kappa = 0.0101$.

In Fig. 7, we show the $SU(3)$ probability distribution for the ground state, 0^+ state of the β, and 2^+ state of the γ bands. One sees a simple structure which is dominated by the so called $SU(3)$ stretched states, defined as the set of $SU(3)$ states $(\lambda_0 + 2n, \mu_0)$ [5], where (λ_0, μ_0) is the leading irreducible representation for the combined proton-neutron nuclear system and $n = 0, 1, 2, 3, \ldots$. This is in agreement with the results for the ground band within the framework of the one-component $Sp(6, R)$ symplectic model [12]. The present calculations show that the stretched $SU(3)$ states built on the leading $SU(3)$ irreducible representation $(54, 0)$ of the $Sp(12, R)$ bandhead exhaust $\sim 98.52\%$ of the structure for the ground state in ^{238}U. Similar picture was obtained within the contracted version of the $Sp(6, R)$ model, in which the stretched states give rise up to 93,7% to the structure of the ground state in ^{238}U [12]. Similar structure is obtained for the wave function of the 0^+ state of β band, in which the stretched $SU(3)$ states built on the $SU(3)$ irrep $(50, 2)$ make up $\sim$ 99%. From Fig. 7, one sees a wider $SU(3)$ decomposition for the 2^+ state of the γ band, in which the corresponding contribution of stretched $SU(3)$ states built on the $SU(3)$ irrep $(50, 2)$ is $\sim$ 53%.

In order to obtain a more generalized picture of the microscopic structure of the rotational states in ^{238}U, we plot the $U(6)$ wave function decomposition for the ground state, 0^+ state of the β, and 2^+ state of the γ bands, respectively, in Fig. 8. From the latter, one sees that the $U(6)$ symmetry is broken due to the mixing of different irreps. Nevertheless, one observes a simple structure to which only a few $U(6)$ irreducible representations contribute. From the figure, one sees also that although the

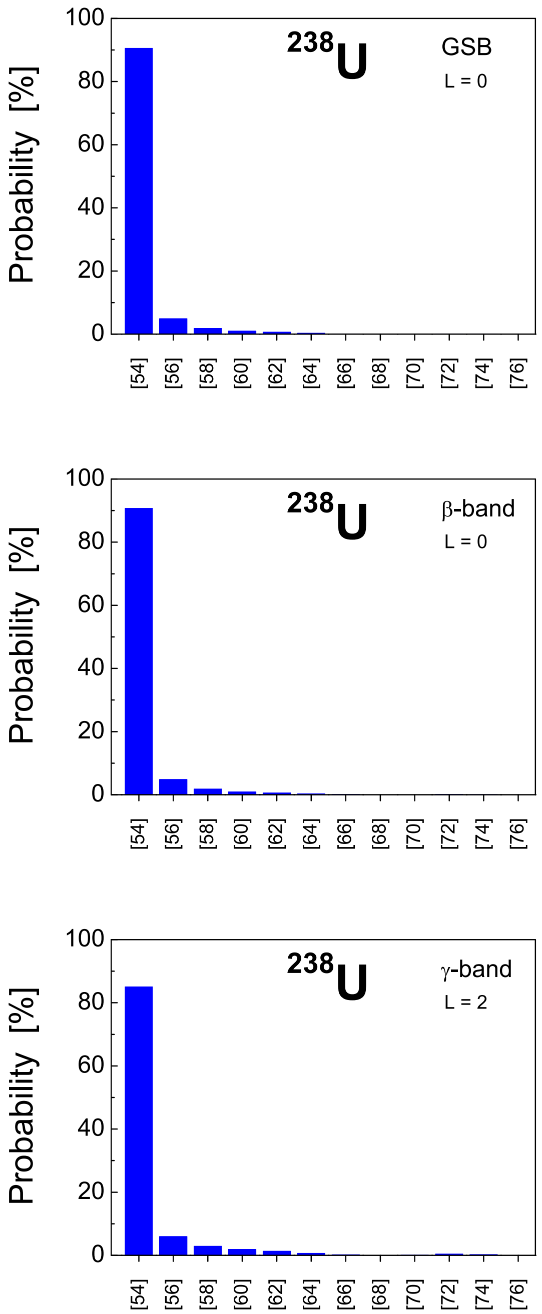

Figure 8. (Color online) Calculated $U(6)$ probability distributions for the wave functions of the 0^+ states of the ground and β bands, as well as the 2^+ state of the γ band. The values for the model parameters are as follows (in MeV): $\chi = 0.0055$, $\xi = 0.0034$, $a = 0.006$, and $\kappa = 0.0101$.

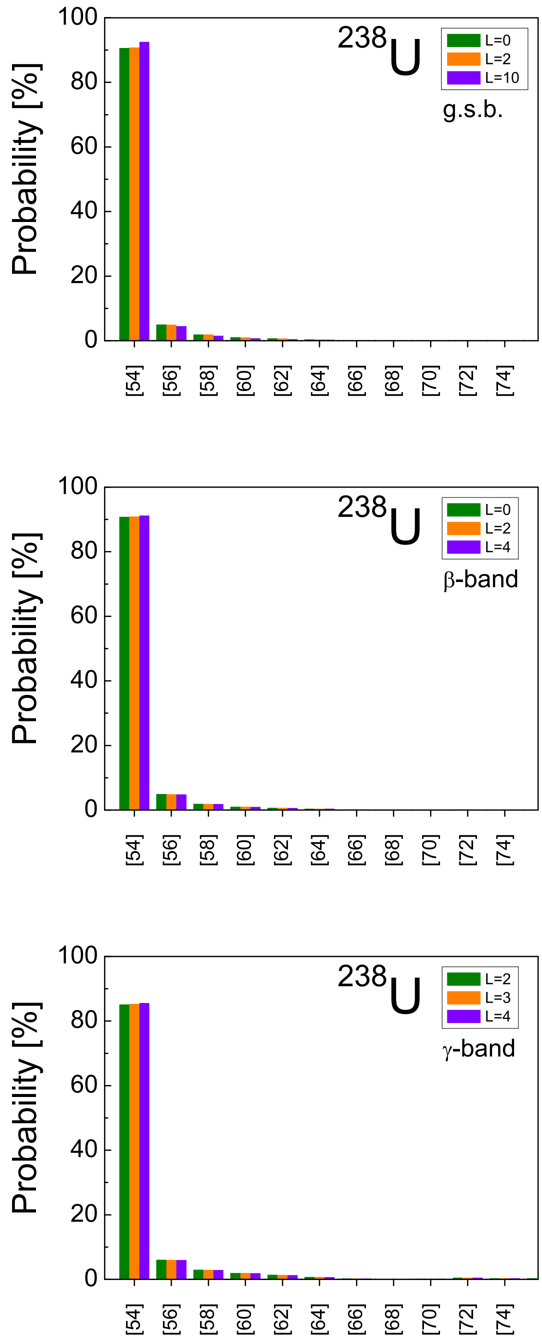

Figure 9. (Color online) Calculated $U(6)$ probability distributions for the wave functions of the ground, β, and γ bands for three different angular momentum values.

$SU(3)$ decomposition of the 2^+ state of the γ band looks differently from those of 0^+ states of ground and β bands, the wave functions of the three states share a similar $U(6)$ decomposition, in which the lowest-grade $U(6)$ irrep of the $Sp(12, R)$ bandhead predominates the structure making up $\sim 85-90\%$, plus small admixtures from the next few higher major shells.

Additionally, in Fig. 9, the $U(6)$ decomposition of the wave functions of ground, β, and γ bands, respectively, for three different values of the angular momentum in each band is shown. From the figure, one sees a highly coherent mixing in which the squared amplitudes are practically L-independent, at least for low angular momenta for which the Coriolis and centrifugal forces are not so strong. The latter means that the low-lying collective states of the three bands under consideration in ^{238}U reveal the presence of a good $U(6)$ quasi-dynamical symmetry, in the sense given in Refs. [13, 14]. Then the obtained linear combinations of $U(6)$ irreducible representations can be represented by an average effective irrep.

4 Conclusions

In the present paper, the low-lying collective states of the ground, β and γ bands in the two strongly deformed nuclei ^{154}Sm and ^{238}U are investigated within the framework of the microscopic proton-neutron symplectic model. For this purpose, the model Hamiltonian is diagonalized in a $U(6)$-coupled basis, restricted to the symplectic state space spanned by the fully symmetric $U(6)$ vectors. A good description of the energy levels of the three bands under consideration, as well as the intraband $B(E2)$ transition strengths between the states of the ground band is obtained for the two nuclei without the use of an effective charge. The calculations show that when the collective quadrupole dynamics is already covered by the symplectic bandhead structure, as in the case of ^{154}Sm, the results reveal the presence of a very good $U(6)$ dynamical symmetry. In the case of ^{238}U, when we have an observed enhancement of the intraband $B(E2)$ transition strengths, then the results show small admixtures from the higher major shells and a highly coherent mixing of different irreps which is manifested by the presence of a good $U(6)$ quasi-dynamical symmetry in the microscopic structure of the collective states under consideration.

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