Evidence of synchronization of large Josephson-junction arrays by traveling electromagnetic waves

M.A. Galin\textsuperscript{1,2}, E. A. Borodianskyi\textsuperscript{2}, V. V. Kurin\textsuperscript{1}, I. A. Shereshevskiy\textsuperscript{1}, N. K. Vdovicheva\textsuperscript{1}, V. M. Krasnov\textsuperscript{2}, and A.M. Klushin\textsuperscript{1}

\textsuperscript{1}Institute for Physics of Microstructures RAS, Nizhny Novgorod, Russia, a_klushin@ipnras.ru
\textsuperscript{2}Department of Physics, Stockholm University, AlbaNova University Center, Stockholm, Sweden

The Josephson effect can be used for the generation of high-frequency electromagnetic radiation \cite{1}. The frequency is limited only by the superconducting energy gap. For achieving a practically important mW level of emission power the arrays containing \(N \approx 10^3-10^5\) JJs would be needed. Such arrays would have a typical size of \(L \approx 1\) cm, which is much larger than the wavelength \(\lambda\) even at sub-THz frequencies. The substantial phase delays between JJs are the significant problem impeding the synchronization of large JJ arrays.

It was recently suggested \cite{2, 3} that the synchronization of large JJ arrays can be mediated by a unidirectional traveling wave along the array. The main fingerprint of the traveling-wave regime in an oscillating system is a strong forward-backward asymmetry of the radiation pattern. This is qualitatively different from the resonant, standing-wave case, which per definition has a symmetric radiation pattern. Thus, the shape of the radiation pattern allows a clear distinction of the two scenarios.

In our work we have studied angular dependence of electromagnetic wave emission from two large Nb/NbSi/Nb JJ series arrays having different design. For both arrays we observe a significant forward-backward asymmetry of the measured radiation patterns which is interpreted by the involvement of the traveling-wave mechanism of synchronization. We have also performed numerical simulations of the Josephson traveling-wave antenna which have supported our conclusions.

The studied arrays contain 6972 and 9000 JJs which are located on silicon substrates, on the area of 5\times5 mm. The first sample was named as linear array while the second was named as meander array. More details about characterization of the arrays can be found in Ref. \cite{4}. Electromagnetic radiation was detected by a high-purity \(n\)-doped InSb bolometer which was preliminarily calibrated. Measurements are performed in a closed-cycle cryostat with a sample-in-gas cooling and the base temperature \(T = 1.8\) K. Array can be rotated by special gear mechanism while the detector is permanently fixed. The detector is located at approximately 1 cm from the geometrical center of the array. Position of the detector face-to-face on top of the array corresponds to the angle \(\alpha = 90^\circ\). The relative accuracy of the rotator is 0.02°.

We performed measurements of current-voltage characteristics (IVCs) at different \(\alpha\) simultaneously with measurements of the detected power. It was obtained that critical current of both JJ arrays is \(I_c \sim 3\) mA and the steps in the IVCs appear in the range \(U \sim 2-2.5\) V.

The angular measurements were performed in the stable steps of IVCs. An analysis of the angular dependencies of the emission power reveals a forward-backward asymmetry of the radiation patterns for both samples. In the Fig.1 the radiation pattern measured for the meander array was shown. It represents decreases of zero-bias resistance \(-\Delta R_{\det}\) relative to the case of unbiased array. Measurement was made at \(U = 2.223\) V corresponding to oscillation frequency \(f = 119.3\) GHz. As seen, the radiation pattern demonstrates a pronounced asymmetry in emission between forward (\(\alpha < 90^\circ\)) and backward (\(\alpha > 90^\circ\)) directions. Similar, but less prominent, asymmetry of radiation power is demonstrated by the linear array. Such asymmetry is inconsistent with the resonant mechanism of synchronization of the arrays by standing waves because standing waves per definition should have forward-backward symmetric radiation patterns.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{angular_dependence.png}
\caption{Measured angular dependence of the ac-detected signal \(-\Delta R_{\det}(\Omega)\) for meander array.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{emission_power.png}
\caption{The emission power reached the maximum values \(P_{em} \sim 80-90\) \(\mu\)W for both arrays. For the linear array it occurs at the step in the resistive branch of IVC. However for the meander array it occurs outside the step in the reverse branch of IVC near \(U = 1.8\) V (see Fig.2, inset). This suggests that traveling waves play a more prominent role in the emitted electromagnetic field for the meander, than for the linear array.}
\end{figure}

In Fig.2 the dependence of detected emission power of meander array along the reverse branch of the IVC is demonstrated. Here the dc-voltage response was registered \(-\Delta U_{\det}\) and the data was obtained at \(\alpha = 40^\circ\) when the maximal power was attained (Fig.2, inset). During the sweeping from a large bias current downward to zero the junctions switch in sequence from the resistive into the zero-voltage state. Thus, variation of the emitted power with bias voltage

© The Authors, published by EDP Sciences. This is an open access article distributed under the terms of the Creative Commons Attribution License 4.0 (http://creativecommons.org/licenses/by/4.0/).
Fig. 2. Emitted electromagnetic power (dc-detector response, $-\Delta U_{\text{det}}$) along the reverse branch of the IVCs of meander arrays at $\alpha = 40^\circ$. In the inset the corresponding IVCs obtained by repeated back-and-forth sweeping of bias current is shown. The gray bar indicates the detected power (dc-detector response)

is primarily due to variation of the number of oscillating junctions $N_o$, approximately proportional to voltage $U$. It is seen that a rapid superlinear enhancement of the emission power occurs at $1.3$ V. It is known that for the case, when all the oscillating junctions are in the exactly identical state, the emission power should increase as $P_{\text{em}} \sim N^2$. Thus the data in Fig.2 indicate the regime of superradiant emission in the meander array. It is suggested that a nonresonant traveling-wave mechanism of synchronization contributes to the observed superradiant emission in the meander array, along with persisting standing-wave resonances that is indicated by the presence of steps in the IVCs.

As suggested earlier [2, 3] large JJ arrays may act as Josephson traveling-wave antennas. Such antennas have asymmetric radiation patterns with a maximum in the direction of propagation of the wave at an angle $\alpha = \arccos(h/k)$, where $h$ is the wave number of current oscillations in the antenna and $k$ is the wave number of the emitted wave in vacuum. For analysis of nonlinear dynamics of the Josephson traveling-wave antennas, we develop a numerical code for solving Maxwell equations by the FDTD method in combination with self-consistent solution of junction dynamics within the RSJ model.

Our simulated array contains 10 JJs, linear lumped passive elements and two voltage sources (Fig.3a). Elements in the arrays are connected by perfectly conducting wires. The array is placed on dielectric substrate a dielectric permittivity 10. Junction parameters used in simulations are $I_{\text{th}}=2.5$ mA, normal resistance 0.5 $\Omega$, and the McCumber parameter 0.2. We inserted passive elements (resistances and inductances with the values of 25 $\Omega$ and 200 pH, respectively) which are different at the opposite sides of the circuits. These elements are needed for simulating the dynamical violation of the symmetrical state that has to take place in real JJ arrays.

Numerically simulated radiation pattern are shown in Fig.3b. The chosen radiation frequency $f=136.6$ GHz provides the wavelength of the current wave considerably smaller than the dimension of the circuits.

It is seen that array exhibit a pronounced forward-backward asymmetry of radiation patterns with maximum in the direction of positive $x$-axis direction. The value of directivity in the maximum power radiation is 4.6 dBi. The radiation pattern has two large lobes with equal amplitudes. One of them is directed strictly along the $x$-axis that indicates a surface plasmon excited in the array. The relation $h/k > 1$ is realized in this case. For another large lobe $h/k < 1$, i.e. an ordinary traveling wave exists with the finite angular deviation $\alpha$ from the $x$-axis which is $\alpha = 42^\circ$ in this case.

The aim of these simulations was rather to demonstrate that the main evidence for the traveling-wave regime is the forward-backward asymmetry of the radiation pattern with a maximum in the traveling-wave direction. This is qualitatively consistent with the observed forward-backward asymmetry of emission from the studied large JJ arrays.

The work was supported by the Russian Foundation for Basic Research (Grants No. 16-32-00686 and No. 18-02-00912), the Swedish Foundation for International Cooperation in Research and Higher Education (Grant No. IG2013-5453), and the Swedish Research Council (Grant No. 621-2014-4314). V. K. and A. M. K. would also like to acknowledge the partial support of this research by the Russian Science Foundation (Grant No. 15-12-10020).

References