Optical-Mechanical Analogy Approach for the Purposes of Detection of IR-MW Radiation

A.V. Bogatskaya[1,2,3], N.V. Klenov[1,3,4,5], M.V. Tereshonok[3,5] and A.M. Popov[1,2,4]

[1]D. V. Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow, Russia, annabogatskaya@gmail.com
[2]P. N. Lebedev Physical Institute, RAS, Moscow Russia
[3]Moscow Technical University of Communication and Informatics, 111024, Moscow, Russia
[4]Department of Physics, Moscow State University, 119991, Moscow, Russia
[5] Moscow Technological University (MIREA), 119454, Moscow, Russia

Here we propose a generalized physical approach providing possible technical realizations of a number of urgent problems related to the interaction of electromagnetic radiation with plasma and conducting media. Proposed approach is based on the use of well-known optical-mechanical analogy which exploit mathematical identity of the stationary Schroedinger equation in quantum mechanics for the particle motion in potential field and Helmholtz equation in wave theory.

Let us consider spatially inhomogeneous non-magnetic medium characterized by the susceptibility $\chi(\vec{r})$, or permittivity $\varepsilon(\vec{r}) = 1 + 4\pi\chi(\vec{r})$. In the case of monochromatic field $\vec{E} = \vec{E}_0(\vec{r})\exp(-i\omega t)$ ($\vec{H} = \vec{H}_0(\vec{r})\exp(-i\omega t)$, ω is the radiation frequency) Maxwell equations for electric and magnetic field strength can be written as:

$$rot\,\vec{E} = \frac{1}{c}i\omega\vec{H}, \quad div(\varepsilon\vec{E}) = 0,$$
$$rot\,\vec{H} = -\frac{\varepsilon}{c}i\omega\vec{E}, \quad div\,\vec{H} = 0. \quad (1)$$

From (1) one can obtain the following equation for electric field strength $\vec{E}$:

$$\Delta\vec{E} + \nabla\left(\frac{1}{\varepsilon}(\vec{E}\nabla)\varepsilon\right) + \frac{\varepsilon\omega^2}{c^2}\vec{E} = 0. \quad (2)$$

For the case when permittivity depends only on one spatial coordinate $\varepsilon = \varepsilon(z)$ and wave field propagates alone this direction the equation (2) transforms to the well-known Helmholtz equation for the spatial distribution of electric field strength E:

$$\frac{d^2 E}{dz^2} + k_0^2(1 + 4\pi\chi(z))E = 0. \quad (3)$$

with $k_0^2 = \omega^2/c^2$. Here electric field propagates in the direction perpendicular to z-axis.

Equation (3) is mathematically equivalent to the stationary Schrödinger equation in quantum mechanics for the particle wave function $\psi(z)$ in the potential field $V(z)$:

$$\frac{d^2\psi}{dz^2} + \kappa_0^2\left(1 - \frac{V(z)}{\zeta}\right)\psi = 0, \quad (4)$$

where $\kappa_0^2 = 2m\zeta/\hbar^2$ is the wave vector of the particle with energy ζ. Direct comparison of eq. (3) and (4) leads to the conclusion that potential function $V(z)$ in quantum mechanics is similar to the susceptibility in electromagnetic theory $(2m/\hbar^2)V(z) \to (1-\varepsilon)\cdot(\omega/c)^2 = -4\pi\chi(z)\cdot(\omega/c)^2$.

Thus the eigenvalue problem for the Hamiltonian in quantum theory turns out to be mathematically identical to the problem of calculating the stationary distribution of the electric field strength in a wave. The medium with $\varepsilon > 0$ can be associated with an attractive potential $V(z) < 0$ (potential well) while the medium with $\varepsilon < 0$ acts as potential barrier $V(z) > 0$. In particular, the transport of the electron flux in heterostructures is mathematically identical to the problem propagation of electromagnetic waves through inhomogeneous media.

If the potential curve $V(z)$ has the piecewise-continuous structure (Fig. 1), both the ψ-function and its derivative $d\psi/dx$ should be continuous functions in the potential breaking points. Similar boundary conditions appear to exist in electromagnetic theory: the tangential components of $\vec{E}$, $\vec{H}$ should also be continuous functions at the interface regions. Using Maxwell equations one can rewrite the boundary conditions as the continuity of tangential components of $\vec{E}$ and its derivative. For the normal incidence when only tangential components of $\vec{E}$, $\vec{H}$ have the non-zero values these boundary conditions are equivalent to boundary conditions for the wave function in quantum mechanics. The above conclusion known as an optical-mechanical analogy in quantum theory gives rise to a lot of practical applications and transfer the quantum theory problem solutions to optics and vice versa.

We perform an analysis of the possibility of penetration of electromagnetic waves through opaque media using the analogy of tunneling of a quantum-particle flux through a potential barrier with a height greater than its kinetic energy. As an example, we consider plasma sheath surrounding the hypersonic vehicle as a potential barrier and analyze the overcoming of radiocommunication blackout problem [1].

Really, for the collisionless plasma the permittivity reads $\varepsilon_p = 1 - \omega_p^2/\omega^2$, where $\omega_p^2 = 4\pi e^2 n_e/m$ is the plasma frequency squared and n_e is the electron density. From this point of view the plasma sheath appearing around the hypersonic vehicle during the

flight looks like a potential barrier for the target transmission frequencies less than plasma frequency.
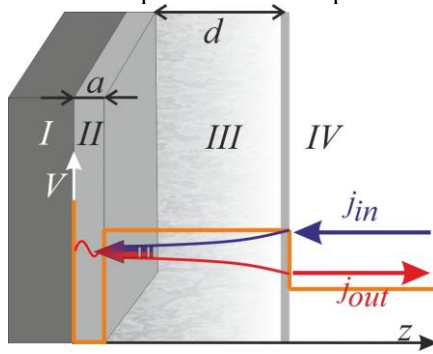

Fig. 1. The concept of overcoming of radio communication blackout: profile of the "potential barrier" $V(z) \leftrightarrow (1 - \varepsilon(z))$ containing a vehicle surface (I), dielectric layer (thickness a) with embedded antenna (II) and plasma sheath (III). (IV) corresponds to the region of infinite motion of the electromagnetic wave (atmospheric air).

The main idea is to embed a «resonator» (it can be a dielectric layer with rather large value of permittivity ε_d) between the surface on the vehicle and plasma sheath which is supposed to provide an effective tunnelling of the signal to the receiving antenna. Calculations (see Fig.2) show sharp increase of tunnelling probability of electromagnetic signal if the frequency of incident radiation coincides with the eigen-frequency of resonator (dielectric layer) which is determined by formula $\omega_n \approx \dfrac{\pi c}{a\sqrt{\varepsilon_d}} n$,

$n = 1,2,3...$. Here we introduce the filling factor $F(f)$ which represents the degree of resonator filling by the incoming radiation flux:

$$F(f) = \max\left\{|E_d|^2 / |E_{a+}|^2\right\}, \qquad (5)$$

where $\left|\vec{E}_d\right|^2$ and $\left|\vec{E}_{a+}\right|^2$ are the squared absolute values of electric field strength in dielectric layer and air correspondingly.
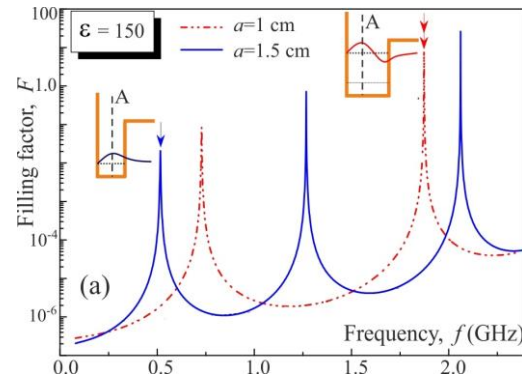

Fig. 2. The filling factor $F(f)$ in dependence on the transmitted signal frequency (normal incidence on vehicle surface). Calculations are made for $d = 10$ cm and $\varepsilon_d = 150$. The inset to figure schematically represents parts of the corresponding wave functions of stationary states within the framework of the optical-mechanical analogy.

The role of collisions in plasma as well as cases of normal and oblique incidence of radiofrequency waves on the vehicle surface are studied in [1].

One more promising application of the suggested approach implies a method for increasing the efficiency of bolometric photodetectors. The results of calculations show that by choosing thickness and dielectric constant of the substrate-resonator under a thin layer of metal (superconductor) one could bring the fraction of absorbed radiation in the detector to a value close to unity at a certain frequency of the infrared range [2].

References

1. Bogatskaya A. V., Klenov N. V., Tereshonok M. V., Adjemov S. S., Popov A. M. // J. Phys. D. 2018. V. 51, P. 185602.

2. Bogatskaya A. V., Klenov N. V., Tereshonok M. V., Popov A. M. // *Technical Physics Letters* 2018. V.44, № 8, p. 667–670