Generation of vortex beamlet lattices via diffraction of Bessel vortex beams on 2D hole arrays: analytical and numerical calculations and comparison with experiments

O. E. Kameshkov[1, 2], B. A. Knyazev[1, 2], I.A. Kotelnikov[1, 2]

[1]Budker Institute of Nuclear Physics of Siberian Branch Russian Academy of Sciences, Novosibirsk, Russia, o.kameshkov@g.nsu.ru
[2]Physics Department, Novosibirsk State University, Novosibirsk, Russia

The Novosibirsk Free Electron Laser (NovoFEL) [1] is a source of monochromatic and frequency-tunable terahertz radiation of high average power. At the entrance to a workstation, the beam of the laser is a Gaussian beam within good accuracy. However, it is necessary to transform the beam mode composition (focusing radiation with a defined intensity distribution). Diffraction optical elements (DOEs) were shown to excel refractive elements in the manipulation of high-power laser radiation in the experiments at the NovoFEL; DOEs can be considered as amplitude-phase masks (APMs) within the framework of physical optics [2-3]. Objects in some experiments, such as terahertz holography [4] and beam diffraction of complex mode composition on lattices [5], are also APMs.

For the experiments, a program in the Matlab environment with an easy-to-use interface has been written to simulate radiation transmission through optical systems consisting of a sequence of amplitude-phase elements. The calculations were performed within the framework of the scalar theory of diffraction. The software calculates the Rayleigh-Sommerfeld integral in the Fresnel approximation [6] using a combination of the impulse response method and the transfer function method, which ensures the solution correctness in the entire Fresnel diffraction region (see eq. (1)) [7].

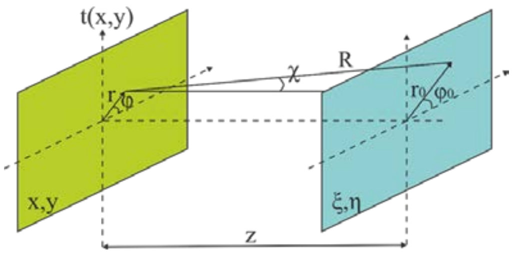

Fig. 1. Calculation geometry. The left rectangular is the amplitude-phase mask plane and the right rectangular is the observation plane.

$$E(z,\xi,\eta)=\frac{1}{i\lambda}\begin{cases}F^{-1}\left\{F\left[u(x,y)\right]F\left[h_f(z,x,y)\right]\right\} & z\geq\dfrac{L\cdot\Delta x}{\lambda}\\ F^{-1}\left\{F\left[u(x,y)\right]\cdot H_f\right\} & z\leq\dfrac{L\cdot\Delta x}{\lambda}\end{cases} \quad (1)$$

where

$$h_f(z,x-\xi,y-\eta)=\frac{\exp(ikz)}{z}\cdot\exp\left(ik\frac{(x-\xi)^2+(y-\eta)^2}{2z}\right)$$

is the impulse response function and its Fourier image is $H_f(\nu_x,\nu_y)=i\lambda\cdot\exp(ikz)\cdot\exp\left[i\pi\lambda z\left(\nu_x^2+\nu_y^2\right)\right]$, which is a transfer function, $u(x,y)=E(0,x,y)\cdot t(x,y)$, F is the Fourier transformation, L is the size of grid, and Δx is the grid spacing.

During a year, the program was widely applied to modeling of beam propagation in different optical systems used in experiments at the NovoFEL such as formation of beams of different mode compositions with DOEs (Hermite-Gaussian and Laguerre-Gaussian beams and Bessel beams with an orbital angular momentum), diffraction of beams on amplitude and phase gratings etc. In all cases, the results of experiments and modeling were in good agreement. Experiments on the diffraction of Bessel beam with an orbital angular momentum on periodic gratings (the quasi-Talbot effect) turned out to be the most interesting among these explorations. They were conducted for the first time.

We have investigated transmission of vortex Bessel beam formed by binary phase axicons through two-dimensional periodic gratings of round holes. It has been found that behind the grating, in the planes corresponding to the planes of self-images of the classical Talbot effect (see eq. (2)), a lattice of vortex ring beams appears, the topological charge of which corresponds to the charge of the incident beam:

$$L_T=\frac{2p^2}{\lambda}\left(N-1+\frac{n}{m}\right)=Z_T\cdot\left(N-1+\frac{n}{m}\right), \quad (2)$$

where N and $n<m$ are integers and p is the period of the grating.

After the experiments and numerical calculations, it became clear to us that the diffraction patterns observed in the Talbot planes had a beautiful geometry, which suggested existence of a beautiful analytical solution for their description. The problem was also solved analytically. The analytical expression obtained so far for the main Talbot plane is as follows:

$$E(\boldsymbol{x},z_N)=\frac{2\pi^2R^2}{\kappa}\sum_{m,n}\iint\frac{d^2k_\perp}{(2\pi)^2}\exp\left(i\boldsymbol{k}_\perp\cdot\boldsymbol{x}+i\ell\varphi_k\right)\cdot i^{-\ell}\times \quad (3)$$
$$\times\delta(k_\perp-\kappa)\cdot\delta\left(\boldsymbol{x}-\boldsymbol{p}_{mn}-\frac{k_\perp z_N}{k}\right)$$

where $\boldsymbol{x}=\{x,y\}$, $\boldsymbol{p}_{mn}=\{mp,np\}$, R is the hole radius, k is the wavenumber, $z_N=Z_T\cdot N$, ℓ is the orbital angular momentum, and κ is the radial wavenumber.

Hence, it is clear that the image has a form of circles with centers at the points $\boldsymbol{x}=\boldsymbol{p}_{mn}$ and radii $a_N=\frac{\kappa\cdot z_N}{k}=\kappa\frac{p^2}{\pi}N$. The results of the analytical studies of the problem in more detail will be published elsewhere.

Funding Information

The study of the vortex beams was carried out with the support of the RFBR grant (project 15-02-06444). The NovoFEL radiation transport beamline to the workstation was constructed with the support of the Russian Science Foundation (grant 14-50-00080). The work was carried out at the collective research center supported by the Ministry of Education and Science of the Russian Federation (project RFMEFI62117X0012).

Acknowledgments

The authors are grateful to G.N. Kulipanov, V. G. Serbo, and V. A. Soifer for stimulating discussions, V.S. Pavelyev and B. O. Volodkin for technical support, Yu. Yu. Choporova and N.D. Osintseva for participating in the experiments and useful discussions, and Ya. V.Getmanov, V. V. Kubarev, T. V.Salikova, M. A. Scheglov, O. A. Shevchenko, D. A. Skorokhod, and other members of the NovoFEL team for the invaluable support of the experiments.

References

1. *Kulipanov G.N., Bagryanskaya E.G., Chesnokov E.N., Choporova Yu.Yu., Gerasimov V.V.,Getmanov Ya.V., Kiselev S.L., Knyazev B.A., Kubarev V.V., Peltek S.E., Popik V.M., Salikova T.V.,Scheglov M.A., Seredniakov S.S., Shevchenko O.A., Skrinsky A.N., Veber S.L., Vinokurov N.A.* Novosibirsk free electron laser as a user facility *// IEEE Trans. on THz Sci. and Techn.* 2015. V. 5. №. 5. P. 798.

2. *Knyazev B.A., Cherkassky V.S., Choporova Yu.Yu., Gerasimov V.V., Vlasenko M.G., Dem'yanenko M.A., Esaev D.G.* Real-time imaging using a high-power monochromatic terahertz source: comparative description of imaging techniques with examples of application *// J Infrared Milli Terahz Waves.* 2011. V. 32, P. 1053.

3. *Agafonov A.N., Choporova Yu.Yu., Kaveev A.K., Knyazev B.A., Kropotov G.I., Pavelyev V.S.,Tukmakov K.N., Volodkin B.O.* Control of transverse mode spectrum of Novosibirsk free electron laser radiation *// Appl. Optics. 2015. V. 54, № 12. P. 3635.*

4. *Choporova Y.Y., Knyazev B.A., Mitkov M.S.* Classical holography in the terahertz range: Recording and reconstruction techniques *// IEEE Trans. on THz Sci. and Techn. 2015. V. 5. №. 5. C. 836.*

5. *Knyazev B.A., Kameshkov O.E., Vinokurov N.A., Cherkassky V.N., Choporova Yu.Yu., Pavelyev V.S.* Quasi-Talbot effect with vortex beams and formation of vortex beamlet arrays // Opt. Expr. 2018. V. 26. №. 11. P. 14174.

6. *Goodman, Joseph W.* Introduction to Fourier optics. *Roberts and Company Publishers,* 2005.

7. *Voelz, D.G.* Computational Fourier optics: a MATLAB tutorial. *Bellingham, WA: SPIE press,* 2011.