Numerical simulation on the development of periodic three-dimensional disturbances in a supersonic boundary layer

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Abstract. The results of a numerical study of the development of periodic pulsations in a supersonic boundary layer on a flat plate are presented at a Mach number of 2.5 and a unit Reynolds number of 8×10^{6} m^{-1}. Using the software complex ANSYS, the complete Navier-Stokes equations were solved. Periodic mass flow disturbances with a frequency of 20 kHz were introduced into the boundary layer through a small-diameter hole on the surface of the model. Downstream the profiles of the longitudinal mass flow pulsations were recorded, and spectral analysis of the data was carried out. The main characteristics of the development of unstable disturbances in both physical and wave spaces are obtained.

1 Introduction

One of the most informative methods of experimental investigation of the laminar-turbulent transition in boundary layers is the controlled excitation of periodic disturbances [1-5]. This approach allows one to obtain the wave characteristics of individual modes with specified frequencies. In addition to studies of the laminar-turbulent transition, the excitation of periodic disturbances inside the boundary layer is possible method for controlling high-speed flows.

To excite controlled disturbances in a supersonic boundary layer, a glow discharge technique was developed at ITAM SB RAS. First of all, experiments with such a source of disturbances were performed in a flat plate [5, 6]. Good agreement between experimental and numerical results was obtained for both linear and nonlinear wave train development [7]. Note, that the calculations were performed with using the linear theory of hydrodynamic stability. However, it is also necessary to carry out calculations in the framework of direct numerical simulation (DNS). The goal of this work is to obtain the main wave characteristics of the development of unstable disturbances in supersonic boundary layer on a flat plate by using DNS.

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2 Problem statement and numerical method

The motion of gas is described by the known Navier-Stokes, continuity, energy and state equations [8]. In this calculations c_p = 1006.43 j/kg-k (c_p – specific heat at a constant pressure) and thermal conductivity was taken in according to the kinetic theory. The temperature of the main flow was set equal T_r = 128.8 K. Mach number was taken M = 2.5, pressure was equal to 4800 Pa and it corresponded to unit Reynolds number $Re_1 = (U\rho/\mu)_\infty = 8 \cdot 10^6$/m.

The computational domain is schematically presented in Figure 1. A`D`BC is the plate with the disturbance source – a hole with a diameter of 1 mm, which is located on the center of the plate at a distance of 37 mm from the leading edge. Length of a plate equaled 140 mm, before a plate the area of 5 mm was set. Conditions of an adiabatic wall were realized on a plate. Height of the computational domain approximately corresponded to about 20 mm, and on the upper border (EFGH) nonreflecting boundary conditions were laid down. Width of the domain was set equal 40 mm, and it was enough that controlled disturbances from the source extinguished on lateral borders. Moreover, on the sides ABHE and DCGF there are non-reflecting boundary conditions, which means that the fluctuations on the walls were set equal to zero during the calculation. In this case, the side walls were installed at such a distance along the z-axis, so that they did not affect the disturbances excitation and development downstream. Conditions of the oncoming flow were laid down on border AEFD. Exit conditions were set on the border BCGH.

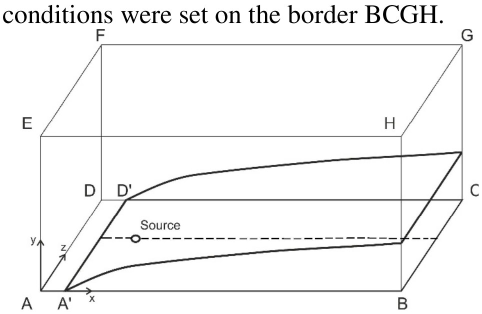

Fig. 1. Computational domain

In this work, the following structured grid was chosen: the quantity of cells on x-coordinate was equal 1200, on z – 200 and on y – 400. Also, the condensation at the surface of the model on y-coordinate was used. For the solution of the task the program complex ANSYS was used. To solve the three-dimensional Navier-Stokes equations in the framework of this problem, we used a density-based solver and an implicit scheme of MUSCL of the third order. The splitting of convective flows was made with using the AUSM method. The problem was solved in two stages. At the first stage, the stationary problem was solved. At the second stage, the task was solved in the presence of a periodic disturbances, which were created by the air injection through a hole. Disturbances were a normal component of the mass flow, whose amplitude was distributed in time by law $|\sin(2\pi f t)|$, where f=10 kHz. Duration of calculation equaled 1000 microseconds, the time step was equal 10^{-2} µs.

3 Results

As a result of calculations, value of mass flow fluctuations depending on the time and coordinates, $m' = m'(t,x,y,z)$, have been received. It was observed that the maximum of disturbances is located at $y_{\max}$, where $\rho U(y_{\max})/(\rho U)_\infty \approx 0.6\text{-}0.8$. Below, the results of the development of unstable disturbances are shown at this value of $y_{\max}$.

The frequency-wave disturbance spectra at the fixed $y_{\max}$ and x were determined using the discrete Fourier transform in the form:

$$\mathrm{A}_{f\beta}(x) = A_{f\beta}(x, y_{\max})\exp(i\Phi_{f\beta}(x)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m'(x, y_{\max}, t, z)e^{(-i(\beta z - 2\pi ft)}dtdz. \tag{1}$$

Thus, after the Fourier transform with respect to time, the amplitude distributions in space for a frequency of 20 kHz were obtained. These results are shown in Figure 2. One can see the amplification of the wave packet, as well as a spreading of the packet along the z-coordinate. From amplitude β-spectra (see Figure 2) it is possible to notice, that the maximum value of the amplitude corresponds to wave number $\beta \approx 1$ rad/mm.

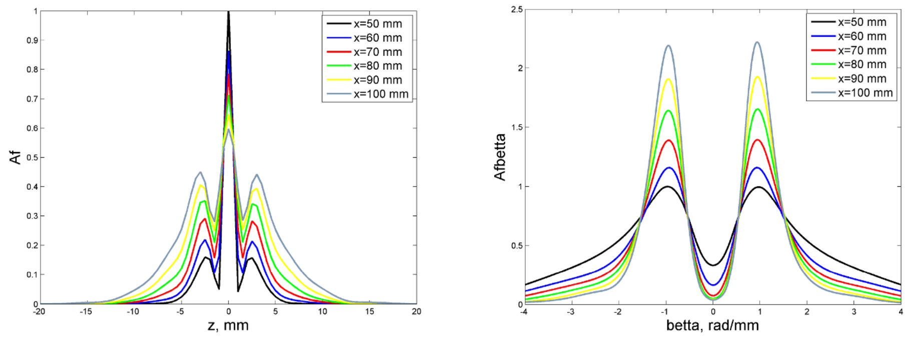

Fig. 2. Amplitude distribution in space (left) and amplitude β-spectra (right) for 20 kHz

Knowing the dependence of the phase on longitudinal coordinate, $\Phi_{f\beta}(x)$, it is possible to estimate the wave number α_r in the x-direction. Then, the angle of inclination χ of the wave vector to the direction of the oncoming flow is calculated. The corresponding results are shown in Figure 3. It is seen, that the value of the longitudinal wave number α_r is in the range of 0.3-0.4 rad/mm. Also, it was established that the greatest growth of disturbances corresponds to angles $\chi > 45°$ that, in general, is in accordance with data, based on stability theory of parallel flows. It has been received, that for frequency 20 kHz the maxima in amplitude β-spectra is located at $\chi \approx 70°$. Note, that starting from a section of 60 mm downstream, the amplitude maximum is located at one value of the angle of inclination χ and does not shift.

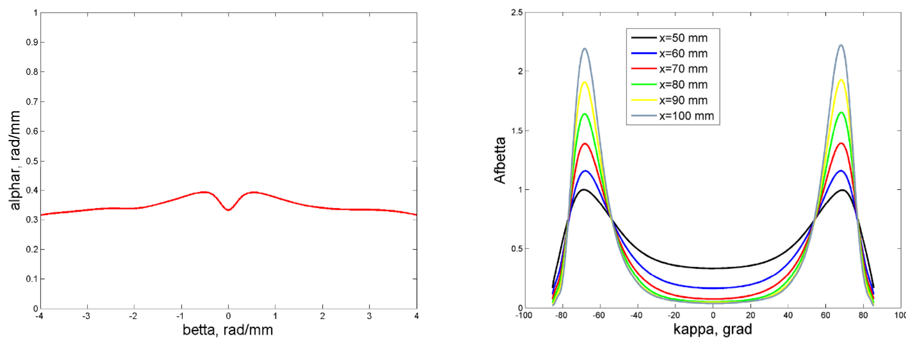

Fig. 3. Dispersion relation $\alpha_r(\beta)$ (left) and dependence $A_{f\beta}(\chi)$ (right) for 20 kHz

4 Conclusion

Direct numerical simulation of the development of unstable disturbances with frequency 20 kHz in supersonic flow (Mach number M=2.5) on a flat plate is carried out.

It is established that inside boundary layer the maximum of a mass flow disturbances is located at $y_{\max}$, where $\rho U(y_{\max})/(\rho U)_\infty \approx 0.6\text{-}0.8$.

In the process of the wave train development downstream, its amplitude is increasing and its boundaries are spreading in transverse direction.

In amplitude β-spectra the maximum belongs to waves with wave numbers $\beta \approx 1$ rad/mm, and it corresponds to an inclination angle of a wave vector to the oncoming flow $\chi \approx 70°$.

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