

Bridge motion of liquid in a gap between the adjacent coils of axisymmetric tubular heat exchanger

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Abstract. A model of a liquid bridge, which is formed between adjacent turns of an axisymmetric tubular heat exchanger, has been developed. The shape of the free surface of the bridge cross-section is determined taking into account the forces of liquid ponderability, surface tension, and centrifugal forces. The equations for the coordinates of the points of bridge free surface are presented in an integral form based on the problem solution in the analytical form. The effect of centrifugal forces on the liquid bridge shape and its flow characteristics were analyzed for an axisymmetric heat exchanger, basing on calculations of the shape of the bridge and known data on the speed of the bridge.

The motion of a liquid in a form of a bridge flowing in a gap between two inclined tubes is observed in different types of heat exchangers. This raises questions about the bridge shape, maximal amount of liquid held in it, volumetric flow rate of liquid in the bridge, its average velocity. The determination of equilibrium shapes of liquid bridges between adjacent straight tubes is based on the hydrostatic equation, which is reduced to the nonlinear differential equation relative to the coordinates of the free surface [1,2]. Geshev [3] used approach based on direct calculation of the coordinate of the upper contact point by the Newton method from some non-linear integral equation. Equations for the coordinates of points of the bridge free surface were obtained in the integral form based on solving the problem in the analytical form in [4, 5]. When turning from heat exchangers with the straight working tubes to the version of heat exchanger with helical tubes, there are the questions related to the appearance of centrifugal forces. When analyzing the work of an axisymmetric tubular heat exchanger, where the tubes are laid along the helical lines, the following questions arise: how does the curvature of the working tubes and velocity of the liquid bridge motion through the gap between adjacent tubes affect the cross-sectional shape of the bridge, the coefficient of liquid flow from one row to another, and maximal liquid flow rate in the bridge.

Results of calculations of velocity of bridge motion between the straight tubes are presented in [5] depending on the angle of tube inclination to the horizon under the following conditions: liquid is water, tube radius is $r = 5$ mm, distance between the tubes is 2 mm. Here are the experimental data. It turned out that at small angles of tube inclination to the horizon of up to 4.7 degrees, the experimental data correspond better to the laminar regime of liquid flow in the bridge, and at the angles of above 4.7 degrees data correspond better to the turbulent one. It is obvious that at small ratios of the working tube diameter to

the diameter of a cylindrical heat exchanger, the velocity of helical bridge motion can be determined by the method described in [5]. Thus, in order to evaluate the effect of centrifugal forces on liquid bridge formation, it is necessary to find the shape of bridge cross-sectional. With the purpose of simplification, we assume that: 1) the tubes have the shape of the circular rings in a horizontal plane; 2) all points of the bridge cross-section have the same velocity v ; 3) the velocity of bridge motion along the spiral tubes is determined solving the problem of their direct motion with other things being equal (see **Fig.1**). We will also neglect a change in centrifugal acceleration at different points of the bridge because r is significantly less than radius of tube turn R .

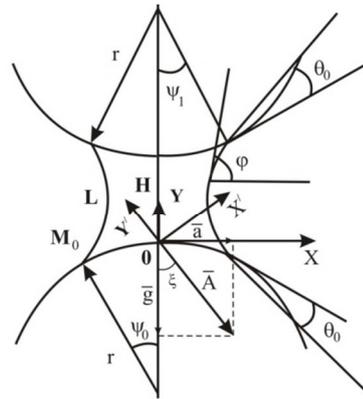


Fig. 1. The scheme of bridge cross-section

Hence, the gravity forces with gravitational acceleration g and centrifugal forces caused by centrifugal acceleration $a = v^2/R$, directed respectively vertically down and horizontally from the heat exchanger axis of symmetry, affect the elementary volumes of the bridge. Summarizing vectors g and a , we obtain that the points of the bridge cross-section are exposed to the forces caused by united constant acceleration with module $A = \sqrt{g^2 + v^4/R^2}$, directed at angle $\xi = \arctan(v^2/(R \cdot g))$ to the vertical, where g is gravitational acceleration, v is the velocity of liquid bridge motion, r is the working tube radius, and R is the radius of tube coil in the projection on the horizontal plane.

To plot the shape of the liquid bridge between the circular tubes, we used the method suggested for construction of a similar bridge between the straight lines.

We introduce a new coordinate system, where axis Y coincides with the direction of total acceleration, i.e., the previous coordinate system is turned by angle ξ .

The formulas of coordinate transformation take form:

$$X' = X \cos \xi - Y \sin \xi$$

$$Y' = X \sin \xi + Y \cos \xi$$

where X, Y are previous coordinates, X', Y' are new coordinates.

Due to these transformations, we came to the conditions, typical of the problem with the straight tubes. The parametrical equations for the shapes of free boundaries of the bridge cross-section take form:

$$x' = x'_0 + \frac{1}{Bo} \int_{\varphi'_0}^{\varphi'} \frac{\cos \mu}{\text{sign}(G + y'_0) \sqrt{G^2 - 2 \frac{-C + \cos \mu}{Bo}}} d\mu$$

$$y' = -G + \text{sign}(G + y'_0) \sqrt{G^2 - 2 \frac{-C + \cos \mu}{Bo}}$$

where $x' = X/r$ and $y' = Y/r$ are dimensionless coordinates in the new reference system, x'_0 and y'_0 are initial coordinates in the new reference system, φ'_0 is the initial angle in the new reference system, equal to $\varphi'_0 = \theta + \psi + \xi$ (symbols are kept), G is parameter, obtained by selection of solution by the boundary conditions, $Bo = \frac{\rho r^2 \cdot A}{\sigma} = Bo_0 \sqrt{1 + (a/g)^2}$ is the Bond number, $Bo_0 = \frac{\rho g r^2}{\sigma}$, $C = \frac{Bo}{2} y'_0 + G \cdot Bo \cdot y'_0 + \cos \varphi'_0$ - is the integration constant.

The algorithm for constructing the shape of free boundaries of the bridge is similar to the algorithm used previously [5], when constructing the bridge without centrifugal effects.

By setting some value of ψ_0 , we select such value of G , to make the plotted curve, which deviates at a predetermined angle θ_0 from the surface of the bottom tube, approaching the upper tube at the same angle θ_0 .

At that, we do not forget that we are finding the solution in the rotated coordinate system, and to check the second boundary condition, we should use the reverse transformation.

As a result, the second boundary condition takes form:

$$\theta_0 = \pi - \varphi - \arcsin|x/r|$$

where r is the tube radius, x is coordinate of the point of contact in the previous coordinate system, $\varphi = \varphi' - \xi$.

Among all values of ψ , at given v and R , we choose the maximal one, when we can select G , meeting the boundary conditions.

The second branch of solution we find in the reverse order; i.e., using G , determined before, we find ψ , meeting the boundary conditions. Thus, instead of two independent curves, we have an integral associated bridge.

To find the second branch of solution, we should note that the only factor affecting system symmetry is the direction of acceleration \bar{A} , and we can just replace angle ζ by $-\zeta$. Then, finding the solution and turning it to the previous coordinate system, we replace coordinate x for this branch by $-x$.

The examples of calculations of the shape of free boundaries of liquid bridges, moving with indicated velocities along the gaps between the circular tubes, are shown in **Fig. 2**. In figure, the circle center is in the right.

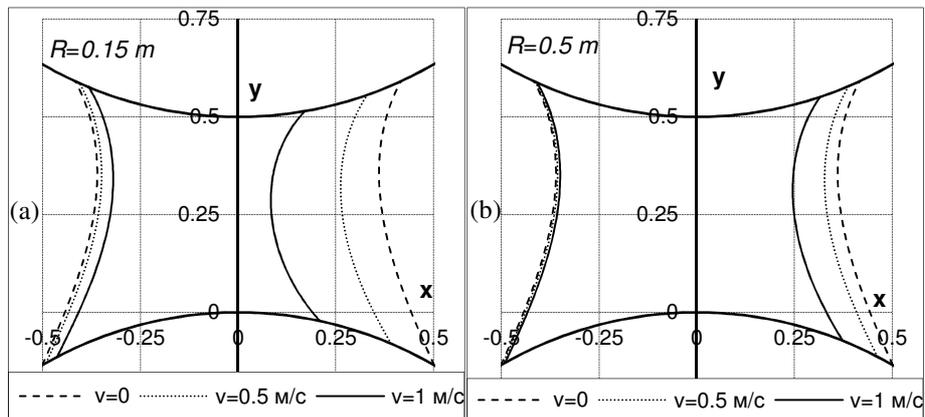


Fig. 2(a, b) Examples of calculations of the liquid bridge shapes between the coils of working tubes with consideration of centrifugal forces. Water, $\theta_0=36^\circ$, $\sigma=0.073$ N/m.

We find the cross-sectional area of the bridge.

The area of each of two bridge regions, lying on opposite sides of the ordinate axis, can be calculated by formula

$$s = \int_{y_0}^0 (|x(y)| - \sqrt{r^2 - y^2}) dy + \int_0^h |x(y)| dy + \int_h^{y_{max}} (|x(y)| - \sqrt{r^2 - (h + r - y)^2}) dy$$

The areas of the total cross-section of the bridges (maximally possible) are shown in Fig. 3 depending on the Bond number at the given values of the wetting angle.

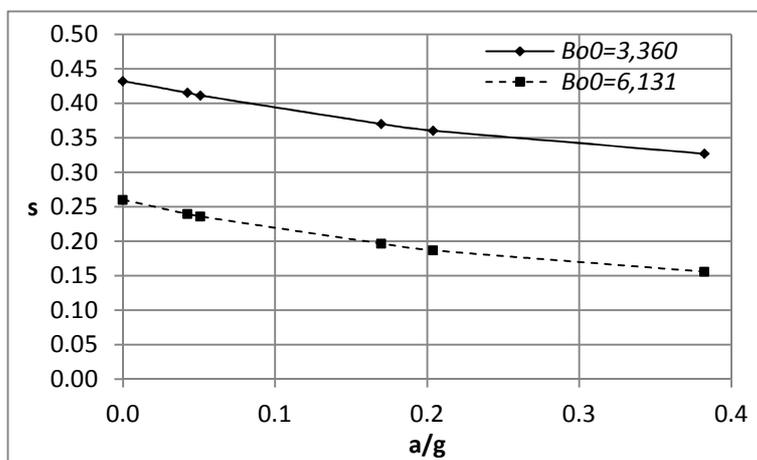


Fig. 3. Area of bridge cross-section depending on parameters a and Bo_0 at $\theta_0 = 36^\circ$, $h = 0.5$

We should note again that in this problem, the Bond number reflects the total action of the gravity and centrifugal forces.

As we can see, consideration of the centrifugal forces leads to a significant decrease in the area of the bridge cross-section and, as a sequence, to a decrease in the liquid flow rate in the bridge.

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References

1. E. Pitts, J. F. M., **59**, 4 (1973)
2. S. V. Alekseenko and P. A. Kuibin, Proc. XXV Int. Siberian Seminar in Memory of S.S. Kutateladze, Novosibirsk, Russia, pp. 19–31 (1996)
3. P. I. Geshev, IPHT, **5**, 4 (2017)
4. L. I. Maltsev and V. I. Zuikova, T. & A., **11**, 1 (2004)
5. L. I. Maltsev, P. A. Houghton and D. V. Kulikov, IPHT, **4**, 4 (2016).