Unitarity, analyticity and duality constraints in η and π photoproduction

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Abstract. We report an update of the isobar model EtaMAID. A new approach is proposed to avoid double counting in the overlap region of Regge and resonances. Dispersion relation is applied on top of the isobar model, and both models describe the data equally well. Application of these ideas to pion photoproduction is discussed.

The isobar model EtaMAID is part of the Mainz MAID project [1, 2] with online programs performing real-time calculations of observables, amplitudes and multipoles. EtaMAID was introduced in 2001 [3] as a model with 8 prominent nucleon resonances, Born terms and t-channel ρ, ω-exchanges. The 2003 update of EtaMAID featured a reggeized isobar model for $p(\gamma, \eta)p$, and an extension to $p(\gamma, \eta')p$ in the threshold region. Recently, high-intensity polarized photon beams with modern 4π detectors and spin-polarized targets at Mainz [4], Bonn [5], and Jlab i[6] have provided new information about η and η' photoproduction. At the GRAAL [7] and the LEPS [8] facilities, photon beams with high linear polarization are available. Eta photoproduction on the nucleon has been studied in various theoretical approaches, e.g. isobar models [3, 9–13], dispersion theoretical calculations [13–15], and coupled channels partial wave analyses (PWA) [16–18].

Here, we report an update of the isobar EtaMAID, and apply dispersion relations to study the effect of the analyticity, unitarity and crossing constraints.

1 Formalism

For η photoproduction on the nucleon, we consider the reaction $\gamma(k)+N(p_i) \to \eta(q)+N'(p_f)$. The Mandelstam variables are $s = W^2 = (p_i + k)^2$, $t = (q - k)^2$, $u = (p_f - q)^2$, and their sum is fixed by $s + t + u = 2m_N^2 + m_\eta^2$, where m_N and m_η are masses of proton and η meson, respectively. The crossing symmetrical variable is $\nu = (s - u)/4m_N$. The nucleon electromagnetic current for pseudoscalar meson photoproduction can be expressed in terms of four invariant amplitudes A_i [19], $J^\mu = \sum_{i=1}^4 A_i(\nu, t)\, M_i^\mu$, with the gauge-invariant four-vectors M_i^μ given by

$$M_1^\mu = -\frac{1}{2} i\gamma_5 \left(\gamma^\mu \not{k} - \not{k}\gamma^\mu\right) , \qquad M_2^\mu = 2i\gamma_5 \left(P^\mu\, k\cdot(q - \frac{1}{2}k) - (q - \frac{1}{2}k)^\mu\, k\cdot P\right) ,$$
$$M_3^\mu = -i\gamma_5 \left(\gamma^\mu\, k\cdot q - \not{k} q^\mu\right) , \qquad M_4^\mu = -2i\gamma_5 \left(\gamma^\mu\, k\cdot P - \not{k} P^\mu\right) - 2m_N\, M_1^\mu , \tag{1}$$

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with $P^\mu = (p_i^\mu + p_f^\mu)/2$. The invariant amplitudes are written in terms of nucleon resonance contributions, and background consisting of Born terms and t-channel vector meson exchanges,

$$A_i = A_i^{Res} + A_i^{Born} + A_i^{VM}. \tag{2}$$

1.1 Isobar model

In the isobar model, the resonances are introduced on the level of partial waves in the direct channel. To that end, the multipole decomposition of the invariant amplitudes in the center-of-mass frame of the pion and the final nucleon is performed [20, 21], $A_i^I = \sum_\ell \left\{a_{i,\ell\pm}^I E_{\ell\pm}^I(W) + b_{i,\ell\pm}^I M_{\ell\pm}^I(W)\right\}$, with $E_{\ell\pm}^I$ and $M_{\ell\pm}^I$ the multipoles describing the electric and magnetic transition to the πN-state with the angular orbital momentum ℓ and the total orbital momentum $j = l \pm 1/2$, and isospin I. The multipoles are functions of W only, and the angular dependence in terms of Legendre polynomials and their derivatives is contained in the coefficients a, b. We generically denote the quantum numbers of partial waves by $\alpha = \alpha\{j, \ell, I\}$. For a given partial wave α, a set of N_α nucleon resonances are added as generalized Breit-Wigner functions with a unitarity phase Φ for each resonance,

$$t_{\gamma,\eta}^{\alpha,r}(W) = \sum_{j=1}^{N_\alpha} t_{\gamma,\eta}^{\alpha,BW,j}(W)\, e^{i\Phi_j}. \tag{3}$$

The (energy-independent) phase Φ_j, is new for our EtaMAID models but has always been applied in pion production. While in γ, π the Watson theorem determines the phase Φ_j at least below the $\pi\pi$ threshold, in η and η' production we have no theoretical guideline and use Φ_j as a fit parameter.

The Born terms for η and η' photoproduction play a minor role: while the πNN coupling is very large, $g_{\pi NN}^2/4\pi \approx 14$, for η and η' photoproduction $g_{\eta NN}^2/4\pi \sim g_{\eta' NN}^2/4\pi \lesssim 0.1$ [22].

Unlike in pion production, the physical region for η, η' production starts at considerably high energy. Already at $\nu \sim 2$ GeV the low-t data are well-represented by Regge exchanges. The reggeization is achieved by replacing the meson propagator by the Regge propagator

$$\frac{1}{t - M^2} \Rightarrow D(s,t) = \left(\frac{\nu}{\nu_0}\right)^{\alpha(t)-1} \frac{\pi\,\alpha'}{\sin[\pi\alpha(t)]}\, \frac{\mathcal{S} + e^{-i\pi\alpha(t)}}{2}\, \frac{1}{\Gamma(\alpha(t))}\,, \tag{4}$$

where M is the mass of the Reggeon, $\mathcal{S}$ is the signature of the Regge trajectory ($\mathcal{S} = -1$ for vector and axial-vector mesons), and $\nu_0 = 1$ GeVis a mass scale. The Gamma function $\Gamma(\alpha(t))$ is introduced to suppress unphysical poles at integer negative t. The parameters of the Regge amplitudes are taken from a recent Ref. [23]. To make use of all the data available for η photoproduction we use a background function that is a continuation of the Regge amplitude in the resonance region. However, adding Regge and resonances together one runs into the well-known double-counting problem: when projected on the s-channel partial waves, Regge amplitude generates resonance-like structures seen as the so-called Schmid loops on the Argand diagram for each partial wave [24], and extraction of resonance parameters becomes ambiguous. Here we propose a new method to avoid double-counting by introducing a damping factor $F_d(W)$ that vanishes at the threshold and approaches unity above some energy,

$$A_i^{Regge} \to F_d(W) A_i^{Regge} \text{ with } F_d(W) = (1 - Exp[(W_{thr} - W)/\Lambda_R])\,\theta(W - W_{thr})\,. \tag{5}$$

The scale Λ_R describes at which energy Regge description fully sets in and is obtained from a fit. The way this damping factor cures the double counting problem can be seen as follows.

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The duality principle states that the full amplitude can be obtained by summing an infinite tower of either s- or t-channel resonances, $A = \sum_{i=1}^{\infty} A_s^{Res_i} = \sum_{i=1}^{\infty} A_t^{Res_i}$. The t-channel sum can actually be performed, and we identify it with the Regge amplitude M^{Regge}. In isobar models it is only possible to account for the lowest s-channel resonances, so the s-channel sum runs up to $i = N$. Then, identically

$$A = \sum_{i=1}^{N} A_s^{Res_i} + \left[\sum_{i=1}^{\infty} A_t^{Res_i} - \sum_{i=1}^{N} A_s^{Res_i} \right] \approx \sum_{i=1}^{N} A_s^{Res_i} + F_d(W) A^{Regge}. \tag{6}$$

1.2 Fixed-t dispersion relations

Isobar model has been quite successful in describing data in various photoproduction processes. However it has shortcomings, such as lack of analyticity (real and imaginary parts of the amplitudes are independent) and crossing symmetry (resonances are added in the direct channel only). To include these important physics constraints we opt to study fixed-t dispersion relations [15]. These take the form

$$\mathrm{Re}\, A_i(\nu, t) \quad = \quad A_i^B + \frac{1}{\pi} \mathcal{P} \int_{\nu_\pi}^{\infty} d\nu' \mathrm{Im}\, A_i(\nu', t) \left[\frac{1}{\nu' - \nu} + \frac{\xi_i^I}{\nu' + \nu} \right], . \tag{7}$$

with $\xi_i^I = \pm 1$ isospin-dependent crossing phases. The isobar model described in the previous section allows to obtain the real and imaginary parts of each amplitude. We use the imaginary parts of the amplitudes (these have resonance and Regge contributions) obtained at this first step as input in the dispersion relation. The real part obtained from a dispersion relation will generally differ from the isobar model fit, and the fit has to be reiterated. It was found that this process converges already after two iterations. We present the results for differential cross sections in the next section.

2 Results

In Fig. 1 we display MAMI data for the angular distributions in η photoproduction [25] in comparison with the isobar and dispersion fits. It is seen that both fits give a good description of the data. Dispersion fit seems slightly favorable at higher energies and forward angles. The difference between the two is small as evidenced by the χ^2 values indicated on the plot.

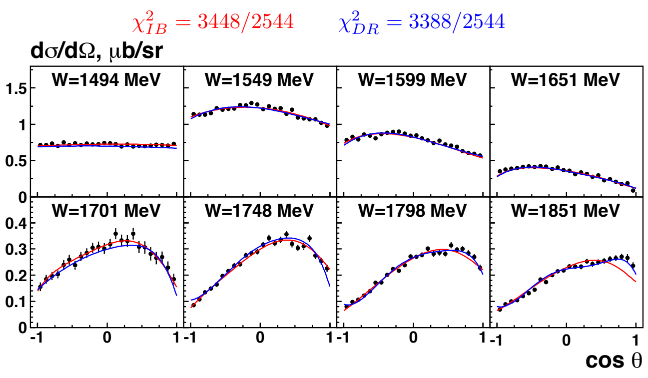

Figure 1. Isobar model (red curves) and dispersion relation (blue curves) fits compared to MAMI [25] differential cross section data on η photoproduction at eight values of W indicated in each panel as function of $\cos\theta$. Plot adopted from [15]

3 Outlook: application to pion photoproduction

We plan to extend the model with the modified Regge background to pion photoproduction. There, Watson theorem requires that in each partial wave, the phase of the pion photoproduction amplitude should be equal to the pion-nucleon scattering phase. To implement Watson theorem, thus it is necessary to perform multipole expansion not only of resonance and Born contributions, but also the Regge background. However, expanding a Regge amplitude shown in Eq. (4) with the damping factor of Eq. (5) leads to an oscillating behavior, as can be seen in the middle panel of Fig. 2. The origin of these oscillations are the zeroes of the inverse Γ-function in Eq. (4) needed to suppress unphysical poles at integer negative t. As a practical and elegant solution to the problem one can require that the trajectory remains bounded $\alpha \geq -1$ at asymptotic momentum transfer [26], the two trajectories displayed in the left panel of Fig. 2. The same multipole projected out of the Regge amplitude with the saturated trajectory has no oscillations, as shown in the right panel of Fig. 2, and can be used for partial wave expansion. The middle and right panels of Fig. 2 also illustrate the double-counting problem: it is seen that a minimum of the real part (dashed curves) almost always coincides with a maximum of the imaginary part, which is a typical resonance behavior in partial waves. This effect is a pure artifact since the original Regge amplitude clearly has no poles in W. The right panel of Fig. 2 demonstrates that saturating the Regge trajectory minimizes the double-counting leaving only one oscillation. The application of these ideas to the PWA of pion photoproduction is work in progress.

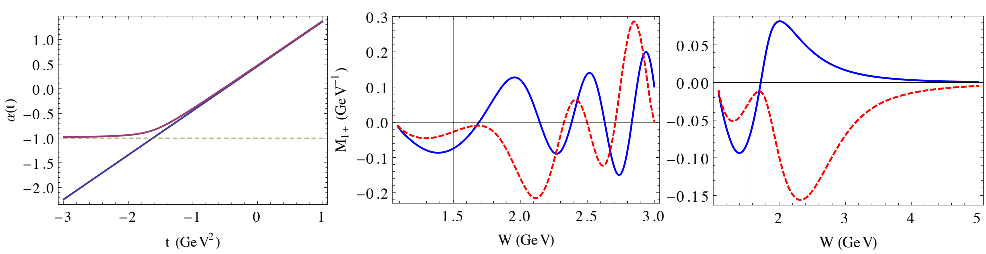

Figure 2. Left panel: linear and saturated Regge trajectory. Middle panel: M_{1+} multipole obtained from a Regge amplitude with a linear trajectory. Right panel: same for a saturated trajectory. Solid curves show the imaginary part, dashed curves show the real part.

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