Radiative corrections in Dalitz decays of π^0, η and η' mesons

Tomáš Husek[1,*]

[1]IFIC, Universitat de València – CSIC, Apt. Correus 22085, E-46071 València, Spain

Abstract. We briefly summarize current experimental and theoretical results on the two important processes of the low-energy hadron physics involving neutral pions: the Dalitz decay of π^0 and the rare decay $\pi^0 \to e^+e^-$. As novel results we present the complete set of radiative corrections to the Dalitz decays $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ beyond the soft-photon approximation, i.e. over the whole range of the Dalitz plot and with no restrictions on the energy of a radiative photon. The corrections inevitably depend on the $\eta^{(\prime)} \to \gamma^*\gamma^{(*)}$ transition form factors.

1 Introduction

The rare decay of the neutral pion, i.e. the process $\pi^0 \to e^+e^-$, is loop- and helicity-suppressed compared to the two-photon decay, which makes it potentially sensitive to effects of new physics. That is why it drew attention of theorists during last years due to the precise measurement of its branching ratio done by KTeV experiment at Fermilab [1]:

$$\mathcal{B}(\pi^0 \to e^+e^-(\gamma),\ x > 0.95)\big|_{\mathrm{KTeV}} = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}\,. \tag{1}$$

Subsequent comparison with the Standard Model prediction [2] was interpreted as a $3.3\,\sigma$ discrepancy between theory and experiment. One could immediately think that this might be a sign for new physics. However, one should look first for a more conventional solution. In what follows we will investigate in detail the radiative corrections for neutral-pion decays in general. Another possibility could be introducing a new model for the π^0 electromagnetic transition form factor [3].

Two-loop virtual radiative corrections for the π^0 rare decay were calculated in [4] and the bremsstrahlung beyond the soft-photon approximation was discussed in [5]. The final next-to-leading-order (NLO) correction was found to be $\delta^{\mathrm{NLO}}(0.95) = -5.5(2)\,\%$, which differs significantly from the previous approximate results also used in [1]. When the exactly calculated radiative corrections are taken into account, the original discrepancy reduces down to the inconclusive $2\,\sigma$ level or below [3, 5].

2 Neutral-pion Dalitz decay

We see that correct incorporation of radiative corrections is crucial in order to provide relevant experimental results. In the rare-pion-decay search performed by the KTeV experiment, the

*e-mail: thusek@ific.uv.es

Dalitz decay was used as the normalization channel. It is thus important to have the radiative corrections for this decay under control, also in order to correctly extract valuable information about the singly-virtual transition form factor.

Radiative corrections to the total decay rate were first (numerically) addressed by Joseph [6]. A pioneering study of the corrections to the differential decay rate was done in [7], although only in the soft-photon approximation. This was extended later in the classical work [8], where the corrections to the Dalitz plot in the form of a table of values were presented.

The new investigation [9] of this topic was motivated by needs of NA48/NA62 experiments at CERN, the aim of which, among other goals, was to measure the slope a_π of the singly-virtual transition form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,q^2)$. Unlike before, the one-photon-irreducible (1γIR) contribution, which was considered negligible for a long time, was added. Additionally, no approximation regarding masses of the particles involved was used during the calculation, so the results are applicable for related decays like $\eta \to \ell^+\ell^-\gamma$. Finally, the C++ code was developed, which returns the radiative correction for any given kinematically allowed point. This code became a part of the Monte Carlo event generator in the NA62 experiment.

3 Dalitz decays of $\eta^{(\prime)}$

Unlike in the neutral-pion case, due to the higher $\eta^{(\prime)}$ rest masses the $\eta^{(\prime)}$ Dalitz decays do not belong to those with the highest branching ratio: the hadronic decay channels are open. Nevertheless, studying the Dalitz decays provides a way to access the electromagnetic transition form factors and consequently information about the structure of related mesons. The form factors in turn represent a valuable input for the anomalous magnetic moment $(g-2)$ of the muon.

Naïve radiative corrections for the $\eta \to e^+e^-\gamma$ process were published in [10]: compared to the earlier work [8], only the numerical value of the physical mass of the decaying pseudoscalar was changed. Ref. [11] completes the list of the NLO corrections in the QED sector and improves the previous approach [10]. Compared to [10], which relates only to the case of the $\eta \to e^+e^-\gamma$ decay, we took into account muon loops and hadronic corrections as a part of the vacuum-polarization contribution, 1γIR contribution at one-loop level, higher-order final-state-lepton-mass corrections and form-factor effects. Moreover, we provide a complete systematic study of the NLO radiative corrections to the differential decay widths to three additional processes including η' decays: $\eta \to \mu^+\mu^-\gamma$, $\eta' \to e^+e^-\gamma$ and $\eta' \to \mu^+\mu^-\gamma$. Sufficiently dense tables of values suitable for interpolation are submitted together with Ref. [11] in a form of ancillary files.

In the case of η decays, we could conveniently and extensively draw from the previous work [9], which governed the neutral-pion Dalitz decay, since it was already written in a sufficiently general way. What really brings the current topic to a different level of difficulty is a desire to tackle the radiative corrections for the η' decays. The resulting framework of [11] is, of course, directly applicable for the η and π^0 cases. Let us only mention that the numerical results obtained for the π^0 Dalitz decay using the new framework are indeed compatible with the form-factor slope correction suggested at the end of Section V of [9]. There is thus no particular need to use this generalized framework for the pion case: one gains a correction to the correction at the level of 1 %.

Let us briefly discuss the subtleties and difficulties which one encounters and needs to deal with when facing the Dalitz decays of $\eta^{(\prime)}$ mesons and associated NLO radiative corrections and which are mainly driven by the properties of the η' meson. The main differences compared to the pion case stem from the following facts. First, it is the higher rest mass, which in the case of η is above the muon-pair production threshold and in the case of η' even above the lowest-lying resonances ρ and ω, the former of which is a broad resonance in $\pi\pi$ scattering. This is connected to the fact that the form-factor slope parameter is not negligible as it was in the pion case: the form factor cannot be scaled out anymore and its particular model is required to be taken into account. We then need to distinguish between two separate cases.

3.1 Bremsstrahlung

Similarly to the leading-order decay width, in the case of the bremsstrahlung correction the singly-virtual transition form factor appears. The calculation of this contribution includes integration over angles and energies of the bremsstrahlung photon. For these integrals to be well-defined in order to obtain reasonable results, including the width of the lowest-lying vector-meson resonances becomes necessary. Due to the fact that such a calculation will be unavoidably sensitive to the width of the broad ρ resonance, we have decided to incorporate the recent dispersive calculations [12, 13]. In the Källén–Lehmann spectral representation, the form factor has the following form:

$$\frac{\mathcal{F}(q^2)}{\mathcal{F}(0)} \simeq 1 + q^2 \int_{4m_\pi^2}^{\Lambda^2} \frac{\mathcal{A}(s)\,\mathrm{d}s}{q^2 - s + i\epsilon}\,, \tag{2}$$

where we have used a common spectral density function

$$\mathcal{A}(s) = w_\omega \mathcal{A}_\omega(s) + w_\phi \mathcal{A}_\phi(s) - \frac{\kappa}{96\pi^2 F_\pi^2}\left[1 - \frac{4m_\pi^2}{s}\right]^{3/2} P(s)\,R(s)\,|\Omega(s)|^2\,. \tag{3}$$

3.2 One-photon-irreducible correction

In the case of the 1γIR correction, one needs to take into account the doubly-virtual transition form factor beyond effective approach. Since the quark content of the $\eta^{(\prime)}$ physical states is not equal to the U(3) isoscalar states, there is a mixing between η and η' mesons. In the quark-flavor basis [14, 15], $j^\ell \equiv \frac{i}{2}[\bar{u}\gamma_5 u + \bar{d}\gamma_5 d]$, $j^{\mathrm{s}} \equiv \frac{i}{\sqrt{2}}[\bar{s}\gamma_5 s]$, this mixing occurs (for $A \in \{\ell, \mathrm{s}\}$) among the states $|\eta^A\rangle$ defined as $\langle 0|j^A|\eta^B\rangle = B_0 F_\pi f_A \delta^{AB}$ together with the orthonormality relation $\langle\eta^A|\eta^B\rangle = \delta^{AB}$. In the quark-flavor basis, the mixing can be written as

$$|\eta\rangle = \cos\phi\,|\eta^\ell\rangle - \sin\phi\,|\eta^{\mathrm{s}}\rangle\,, \tag{4}$$

$$|\eta'\rangle = \sin\phi\,|\eta^\ell\rangle + \cos\phi\,|\eta^{\mathrm{s}}\rangle\,. \tag{5}$$

We do not expect any substantial dependence of the result on the vector-meson decay widths and we use a simple VMD-inspired model, which incorporates the strange-flavor content of $\eta^{(\prime)}$ mesons and the η-η' mixing. In the case of η meson, the form factor reads

$$e^2\mathcal{F}^{\mathrm{VMD}}_{\eta\gamma^*\gamma^*}(p^2,q^2) = -\frac{N_{\mathrm{c}}}{8\pi^2 F_\pi}\frac{2e^2}{3}\left[\frac{5}{3}\frac{\cos\phi}{f_\ell}\frac{M^4_{\omega/\rho}}{(p^2-M^2_{\omega/\rho})(q^2-M^2_{\omega/\rho})} - \frac{\sqrt{2}}{3}\frac{\sin\phi}{f_{\mathrm{s}}}\frac{M^4_\phi}{(p^2-M^2_\phi)(q^2-M^2_\phi)}\right]. \tag{6}$$

3.3 Photon self-energy

In the pion case, the vacuum polarization was dominated by the electron loop. It turns out though that in the high-invariant-mass region of the photon propagator the hadronic effects become significant, which should be taken into account for the $\eta^{(\prime)}$ decays. Thus, in general, we shall deal with the photon self-energy in the form $\Pi(s) = \Pi_{\rm L}(s) + \Pi_{\rm H}(s)$. For the lepton loops (electrons and muons) we take

$$\Pi_{\rm L}(M_P^2 x) = \frac{\alpha}{\pi} \sum_{\ell'=e,\mu} \left\{ \frac{8}{9} - \frac{\beta_{\ell'}^2}{3} + \left(1 - \frac{\beta_{\ell'}^2}{3}\right) \frac{\beta_{\ell'}}{2} \log\left[-\gamma_{\ell'} + i\epsilon\right] \right\}. \tag{7}$$

The hadronic contribution to the photon self-energy can be expressed via a dispersive integral [16]

$$\Pi_{\rm H}(s) = -\frac{s}{4\pi^2\alpha} \int_{4m_\pi^2}^{\infty} \frac{\sigma_{\rm H}(s')\,{\rm d}s'}{s - s' + i\epsilon}\,. \tag{8}$$

Here, $\sigma_{\rm H}$ is the total cross section of the e^+e^- annihilation into hadrons and is related to the ratio $R(s)$ as $\sigma_{\rm H}(s) = (4\pi\alpha^2/3s)R(s)$.

For completeness, let us revise at this point what we mean by the virtual correction. In agreement with Eq. (16) in [9], we use

$$\delta^{\rm virt}(x,y) = \frac{1}{|1 + \Pi(M_P^2 x)|^2} - 1 + 2\,{\rm Re}\left\{ F_1(x) + \frac{2F_2(x)}{1 + y^2 + \frac{\nu^2}{x}} \right\}, \tag{9}$$

where $\Pi(s)$ contains not only electron and muon loops, but also the whole hadronic contribution. For the form factors F_1 and F_2 the reader is referred to seek the expressions in [9].

Acknowledgments

This contribution is based on collaboration with K. Kampf, J. Novotný and S. Leupold.

It has been supported in part by the Agencia Estatal de Investigación (AEI, ES) and the European Regional Development Fund (ERDF, EU) [Grants No. FPA2014-53631-C2-1-P, FPA2017-84445-P and SEV-2014-0398], by Generalitat Valenciana [Grant No. PROMETEO/2017/053], by the Czech Science Foundation grant GAČR 18-17224S and by the ERC starting grant 336581 "KaonLepton".

References

[1] E. Abouzaid et al. (KTeV), Phys. Rev. D **75**, 012004 (2007)
[2] A. E. Dorokhov, M.A. Ivanov, Phys. Rev. D **75**, 114007 (2007)
[3] T. Husek, S. Leupold, Eur. Phys. J. C **75**, 586 (2015)
[4] P. Vasko, J. Novotný, JHEP **1110**, 122 (2011)
[5] T. Husek, K. Kampf, J. Novotný, Eur. Phys. J. C **74**, 3010 (2014)
[6] D. Joseph, Nuovo Cim. **16**, 997 (1960)
[7] B. Lautrup, J. Smith, Phys. Rev. D **3**, 1122 (1971)
[8] K. O. Mikaelian, J. Smith, Phys. Rev. D **5**, 1763 (1972)
[9] T. Husek, K. Kampf, J. Novotny, Phys. Rev. D **92**, 054027 (2015)
[10] K. O. Mikaelian, J. Smith, Phys. Rev. D **5**, 2890 (1972)
[11] T. Husek, K. Kampf, S. Leupold, J. Novotný, Phys. Rev. D **97**, 096013 (2018)
[12] C. Hanhart et al., Eur. Phys. J. C **73**, 2668 (2013), [Erratum: Eur. Phys. J. C **75**, no. 6, 242(2015)]
[13] C. Hanhart et al., Eur. Phys. J. C **77**, 98 (2017)
[14] T. Feldmann, P. Kroll, B. Stech, Phys. Lett. B **449**, 339 (1999)
[15] R. Escribano, J.M. Frere, JHEP **06**, 029 (2005)
[16] F. Jegerlehner, Z. Phys. C **32**, 195 (1986)