

## Pion assisted dibaryons: the $d^*(2380)$ resonance

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**Abstract.** The structure and width of the recently established  $d^*(2380)$  resonance are discussed, confronting the consequences of a Pion Assisted Dibaryons hadronic model with those of quark motivated calculations. In particular, the small width  $\Gamma_{d^*} \approx 70$  MeV favors hadronic structure for the  $d^*(2380)$  dibaryon rather than a six-quark structure.

### 1 Pion assisted $N\Delta$ and $\Delta\Delta$ dibaryons

Nonstrange  $s$ -wave dibaryon resonances  $\mathcal{D}_{IS}$  with isospin  $I$  and spin  $S$  were predicted by Dyson and Xuong in 1964 [1] as early as SU(6) symmetry proved successful, placing the nucleon  $N(939)$  and its  $P_{33}$   $\pi N$  resonance  $\Delta(1232)$  in the same **56** multiplet which reduces to a **20** SU(4) spin-isospin multiplet for nonstrange baryons. For SU(3)-color singlet and spatially symmetric  $L = 0$   $6q$  configuration, the spin-isospin  $6q$  configuration ensuring a totally antisymmetric color-spin-isospin-space  $6q$  wavefunction is a **50** dimensional SU(4) representation, denoted by its  $[3,3,0,0]$  Young tableau, which is the lowest-dimension SU(4) multiplet in the  $\mathbf{20} \times \mathbf{20}$  direct product [2]. This **50** SU(4) multiplet includes the deuteron  $\mathcal{D}_{01}$  and  $NN$  virtual state  $\mathcal{D}_{10}$ , plus four more nonstrange dibaryons, with masses listed in Table 1 in terms of SU(4) mass-formula constants  $A$  and  $B$ .

**Table 1.** Predicted masses of non-strange  $L = 0$  dibaryons  $\mathcal{D}_{IS}$  with isospin  $I$  and spin  $S$ , using the Dyson-Xuong [1] SU(6)→SU(4) mass formula  $M = A + B[I(I + 1) + S(S + 1) - 2]$ .

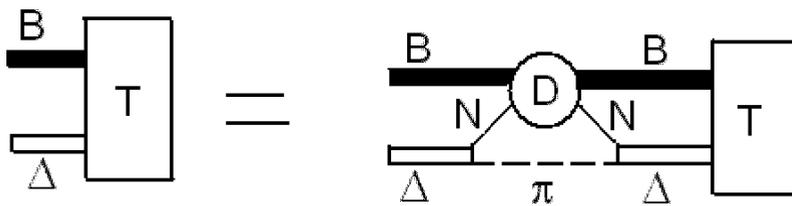
$\mathcal{D}_{IS}$	$\mathcal{D}_{01}$	$\mathcal{D}_{10}$	$\mathcal{D}_{12}$	$\mathcal{D}_{21}$	$\mathcal{D}_{03}$	$\mathcal{D}_{30}$
$BB'$	$NN$	$NN$	$N\Delta$	$N\Delta$	$\Delta\Delta$	$\Delta\Delta$
SU(3) <sub>f</sub>	$\overline{\mathbf{10}}$	$\mathbf{27}$	$\mathbf{27}$	$\mathbf{35}$	$\overline{\mathbf{10}}$	$\mathbf{28}$
$M(\mathcal{D}_{IS})$	$A$	$A$	$A + 6B$	$A + 6B$	$A + 10B$	$A + 10B$

Identifying  $A$  with the  $NN$  threshold mass 1878 MeV, the value  $B \approx 47$  MeV was derived by assigning  $\mathcal{D}_{12}$  to the  $pp \leftrightarrow \pi^+d$  coupled-channel resonance behavior noted then at 2160 MeV, near the  $N\Delta$  threshold (2.171 MeV). This led in particular to a predicted mass  $M = 2350$  MeV for the  $\Delta\Delta$  dibaryon candidate  $\mathcal{D}_{03}$  assigned at present to the recently established  $d^*(2380)$  resonance [3]. Since the  $\mathbf{27}$  and  $\overline{\mathbf{10}}$  flavor-SU(3) multiplets accommodate  $NN$   $s$ -wave states that are close to binding ( $^1S_0$ ) or weakly bound ( $^3S_1$ ), we focus here on the  $\mathcal{D}_{12}$  and  $\mathcal{D}_{03}$  dibaryon candidates assigned to these flavor-SU(3) multiplets.

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The idea behind the concept of pion assisted dibaryons [4] is that since the  $\pi N$   $p$ -wave interaction in the  $P_{33}$  channel is so strong as to form the  $\Delta(1232)$  baryon resonance, acting on two nucleons it may assist in forming  $s$ -wave  $N\Delta$  dibaryon states, and subsequently also in forming  $s$ -wave  $\Delta\Delta$  dibaryon states. This goes beyond the major role played by a  $t$ -channel exchange low-mass pion in binding or almost binding  $NN$   $s$ -wave states.

As discussed below, describing  $N\Delta$  systems in terms of a stable nucleon ( $N$ ) and a two-body  $\pi N$  resonance ( $\Delta$ ) leads to a well defined  $\pi NN$  three-body model in which  $IJ = 12$  and  $21$  resonances identified with the  $\mathcal{D}_{12}$  and  $\mathcal{D}_{21}$  dibaryons of Table 1 are generated. This relationship between  $N\Delta$  and  $\pi NN$  may be generalized into relationship between a two-body  $B\Delta$  system and a three-body  $\pi NB$  system, where the baryon  $B$  stands for  $N, \Delta, Y$  (hyperon) etc. In order to stay within a three-body formulation one needs to assume that the baryon  $B$  is stable. For  $B = N$ , this formulation relates the  $N\Delta$  system to the three-body  $\pi NN$  system. For  $B = \Delta$ , once properly formulated, it relates the  $\Delta\Delta$  system to the three-body  $\pi N\Delta$  system, suggesting to seek  $\Delta\Delta$  dibaryon resonances by solving  $\pi N\Delta$  Faddeev equations, with a stable  $\Delta$ . The decay width of the  $\Delta$  resonance is considered then at the penultimate stage of the calculation. In terms of two-body isobars we have then a coupled-channel problem  $B\Delta \leftrightarrow \pi D$ , where  $D$  stands generically for appropriate dibaryon isobars: (i)  $\mathcal{D}_{01}$  and  $\mathcal{D}_{10}$ , which are the  $NN$  isobars identified with the deuteron and virtual state respectively, for  $B = N$ , and (ii)  $\mathcal{D}_{12}$  and  $\mathcal{D}_{21}$  for  $B = \Delta$ .



**Figure 1.** Diagrammatic representation of the  $B\Delta T$ -matrix integral equation from  $\pi NB$  Faddeev equations with separable pairwise interactions where  $B = N, \Delta$  [5, 6].

Within this model, and using separable pairwise interactions, the coupled-channel  $B\Delta - \pi D$  eigenvalue problem reduces to a single integral equation for the  $B\Delta T$  matrix shown diagrammatically in Fig. 1, where starting with a  $B\Delta$  configuration the  $\Delta$ -resonance isobar decays into  $\pi N$ , followed by  $NB \rightarrow NB$  scattering through the  $D$ -isobar with a spectator pion, and ultimately by means of the inverse decay  $\pi N \rightarrow \Delta$  back into the  $B\Delta$  configuration. The interaction between the  $\pi$  meson and  $B$  is neglected for  $B = \Delta$ , for lack of known  $\pi\Delta$  isobar resonances in the relevant energy range.

The  $\mathcal{D}_{12}$  dibaryon of Table 1 shows up clearly in the Argand diagram of the  $NN$   $^1D_2$  partial wave which is coupled above the  $NN\pi$  threshold to the  $I = 1$   $s$ -wave  $N\Delta$  channel. Its  $S$ -matrix pole position  $W = M - i\Gamma/2$  was given by 2148–i63 MeV in  $NN$  phase shift analyses [7] and by 2144–i55 MeV in dedicated  $pp \leftrightarrow np\pi^+$  coupled-channels analyses [8]. Values of  $\mathcal{D}_{12}$  and  $\mathcal{D}_{21}$  pole positions from our hadronic-model three-body  $\pi NN$  Faddeev calculations [5, 6] described in the previous subsection are listed in Table 2. The  $\mathcal{D}_{12}$  mass and width calculated in the Faddeev hadronic model version using  $r_\Delta \approx 1.3$  fm are remarkably close to these phenomenologically derived values. As for the  $\mathcal{D}_{21}$  dibaryon, recent  $pp \rightarrow pp\pi^+\pi^-$  production data [9] place it almost degenerate with the  $\mathcal{D}_{12}$ . Our  $\pi NN$  Faddeev calculations produce it about 10–20 MeV higher than the  $\mathcal{D}_{12}$ , see Table 2. The widths of these near-threshold  $N\Delta$  dibaryons are, naturally, close to that of the  $\Delta$  resonance. We note that

only  ${}^3S_1 NN$  enters the calculation of the  $\mathcal{D}_{12}$  resonance, while for the  $\mathcal{D}_{21}$  resonance calculation only  ${}^1S_0 NN$  enters, both with maximal strength. Obviously, with the  ${}^1S_0$  interaction the weaker of the two, one expects indeed that the  $\mathcal{D}_{21}$  resonance lies above the  $\mathcal{D}_{12}$  resonance. Moreover, these two dibaryon resonances differ also in their flavor-SU(3) classification, see Table 1, which is likely to push up the  $\mathcal{D}_{21}$  further away from the  $\mathcal{D}_{12}$ . Finally, the  $N\Delta$   $s$ -wave states with  $IJ = 11$  and  $22$  are found not to resonate in the  $\pi NN$  Faddeev calculations [6].

**Table 2.**  $\mathcal{D}_{IS}$  dibaryon  $S$ -matrix pole positions  $M - i\Gamma/2$  (in MeV) obtained by solving the  $N\Delta$  and  $\Delta\Delta$   $T$ -matrix integral equation Fig. 1 are listed for  $\pi N P_{33}$  form factors specified by radius parameter  $r_\Delta$  [5, 6].

$r_\Delta$ (fm)	$N\Delta$		$\Delta\Delta$	
	$\mathcal{D}_{12}$	$\mathcal{D}_{21}$	$\mathcal{D}_{03}$	$\mathcal{D}_{30}$
1.3	2147-i60	2165-i64	2383-i41	2411-i41
0.9	2159-i70	2169-i69	2343-i24	2370-i22

The  $\mathcal{D}_{03}$  dibaryon of Table 1 is best demonstrated by the relatively narrow peak observed in  $pn \rightarrow d\pi^0\pi^0$  by the WASA-at-COSY Collaboration [10] about 80 MeV above the  $\pi^0\pi^0$  production threshold and 80 MeV below the  $\Delta\Delta$  threshold, with  $\Gamma_{d^*} \approx 70$  MeV. Its  $I = 0$  isospin assignment follows from the isospin balance in  $pn \rightarrow d\pi^0\pi^0$ , and the  $J^P = 3^+$  spin-parity assignment follows from the measured deuteron angular distribution. The  $d^*(2380)$  was also observed in  $pn \rightarrow d\pi^+\pi^-$  [11], with cross section consistent with that measured in  $pn \rightarrow d\pi^0\pi^0$ , and studied in several  $pn \rightarrow NN\pi\pi$  reactions [12]. Recent measurements of  $pn$  scattering and analyzing power [13] have led to a  $pn$   ${}^3D_3$  partial-wave Argand plot fully supporting the  $\mathcal{D}_{03}$  dibaryon resonance interpretation.

Values of  $\mathcal{D}_{03}$  and  $\mathcal{D}_{30}$  pole positions  $W = M - i\Gamma/2$  from our hadronic-model three-body  $\pi N\Delta$  Faddeev calculations [5, 6] are also listed in Table 2. The  $\mathcal{D}_{03}$  mass and width values calculated in the Faddeev hadronic model version using  $r_\Delta \approx 1.3$  fm are remarkably close to the experimentally determined ones. The  $\mathcal{D}_{30}$  dibaryon resonance is found in our  $\pi N\Delta$  Faddeev calculations to lie about 30 MeV above the  $\mathcal{D}_{03}$ . These two states are degenerate in the limit of equal  $D = \mathcal{D}_{12}$  and  $D = \mathcal{D}_{21}$  isobar propagators in Fig. 1. Since  $\mathcal{D}_{12}$  was found to lie lower than  $\mathcal{D}_{21}$ , we expect also  $\mathcal{D}_{03}$  to lie lower than  $\mathcal{D}_{30}$  as satisfied in our Faddeev calculations. Moreover, here too the difference in their flavor-SU(3) classification will push the  $\mathcal{D}_{30}$  further apart from the  $\mathcal{D}_{03}$ . The  $\mathcal{D}_{30}$  has not been observed and only upper limits for its production in  $pp \rightarrow pp\pi^+\pi^+\pi^-\pi^-$  are available [14].

Finally, we briefly discuss the  $\mathcal{D}_{03}$  mass and width values from two recent quark-based resonating-group-method (RGM) calculations [15, 16] that add  $\Delta_8\Delta_8$  hidden-color (CC) components to a  $\Delta_1\Delta_1$  cluster. The two listed calculations generate mass values that are close to the mass of the  $d^*(2380)$ . The calculated widths, however, differ a lot from each other: one calculation generates a width of 150 MeV [15], exceeding substantially the reported value  $\Gamma_{d^*(2380)} = 80 \pm 10$  MeV [13], the other one generates a width of 72 MeV [16], thereby reproducing the  $d^*(2380)$  width. While the introduction of CC components has moderate effect on the resulting mass and width in the chiral version of the first calculation, lowering the mass by 20 MeV and the width by 25 MeV, it leads to substantial reduction of the width in the second (also chiral) calculation from 133 MeV to 72 MeV. The reason is that the dominant CC  $\Delta_8\Delta_8$  components, with 68% weight [16], cannot decay through single-fermion transitions  $\Delta_8 \rightarrow N_1\pi_1$  to asymptotically free color-singlet hadrons. However, as argued in the next section, these quark-based width calculations miss important kinematical ingredients that make the width of a single compact  $\Delta_1\Delta_1$  cluster considerably smaller than  $\Gamma_{d^*(2380)}$ . The introduction of substantial  $\Delta_8\Delta_8$  components only aggravates the disagreement.

## 2 The width of $d^*(2380)$ , small or large?

The width derived for the  $\mathcal{D}_{03}$  dibaryon resonance  $d^*(2380)$  by WASA-at-COSY and SAID,  $\Gamma_{d^*(2380)}=80\pm 10$  MeV [13], is dominated by  $\Gamma_{d^*\rightarrow NN\pi\pi} \approx 65$  MeV which is much smaller than twice the width  $\Gamma_\Delta \approx 115$  MeV [17, 18] of a single free-space  $\Delta$ , expected naively for a  $\Delta\Delta$  quasibound configuration. However, considering the reduced phase space,  $M_\Delta = 1232 \Rightarrow E_\Delta = 1232 - B_{\Delta\Delta}/2$  MeV in a bound- $\Delta$  decay, where  $B_{\Delta\Delta} = 2 \times 1232 - 2380 = 84$  MeV is the  $\Delta\Delta$  binding energy, the free-space  $\Delta$  width gets reduced to 81 MeV using the in-medium single- $\Delta$  width  $\Gamma_{\Delta\rightarrow N\pi}$  expression obtained from the empirical  $\Delta$ -decay momentum dependence

$$\Gamma_{\Delta\rightarrow N\pi}(q_{\Delta\rightarrow N\pi}) = \gamma \frac{q_{\Delta\rightarrow N\pi}^3}{q_0^2 + q_{\Delta\rightarrow N\pi}^2}, \quad (1)$$

with  $\gamma = 0.74$  and  $q_0 = 159$  MeV [19]. Yet, this simple estimate is incomplete since neither of the two  $\Delta$ s is at rest in a deeply bound  $\Delta\Delta$  state, as also noted by Niskanen [20]. To take account of the  $\Delta\Delta$  momentum distribution, we evaluate the bound- $\Delta$  decay width  $\bar{\Gamma}_{\Delta\rightarrow N\pi}$  by averaging  $\Gamma_{\Delta\rightarrow N\pi}(\sqrt{s_\Delta})$  over the  $\Delta\Delta$  bound-state momentum-space distribution [21],

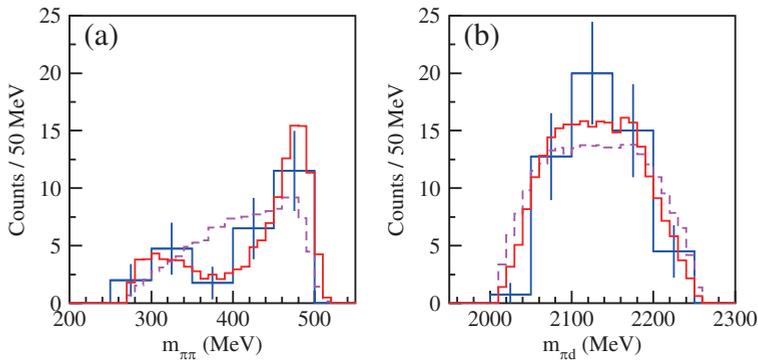
$$\bar{\Gamma}_{\Delta\rightarrow N\pi} \equiv \langle \Psi^*(p_{\Delta\Delta}) | \Gamma_{\Delta\rightarrow N\pi}(\sqrt{s_\Delta}) | \Psi(p_{\Delta\Delta}) \rangle \approx \Gamma_{\Delta\rightarrow N\pi}(\sqrt{s_\Delta}), \quad (2)$$

where  $\Psi(p_{\Delta\Delta})$  is the  $\Delta\Delta$  momentum-space wavefunction and the dependence of  $\Gamma_{\Delta\rightarrow N\pi}$  on  $q_{\Delta\rightarrow N\pi}$  for on-mass-shell nucleons and pions was replaced by dependence on  $\sqrt{s_\Delta}$ . The averaged bound- $\Delta$  invariant energy squared  $\bar{s}_\Delta$  is defined by  $\bar{s}_\Delta = (1232 - B_{\Delta\Delta}/2)^2 - P_{\Delta\Delta}^2$  in terms of a  $\Delta\Delta$  bound-state r.m.s. momentum  $P_{\Delta\Delta} \equiv \langle p_{\Delta\Delta}^2 \rangle^{1/2}$  inversely proportional to the r.m.s. radius  $R_{\Delta\Delta}$ .

The  $d^*(2380)$  in the quark-based RGM calculations of Ref. [16] appears quite squeezed compared to the diffuse deuteron. Its size,  $R_{\Delta\Delta}=0.76$  fm [22], leads to unacceptably small upper limit of about 47 MeV for  $\Gamma_{d^*\rightarrow NN\pi\pi}$  [21]. This drastic effect of momentum dependence is missing in quark-based width calculations dealing with pionic decay modes of  $\Delta_1\Delta_1$  components, e.g. Ref. [16] and as presented here at MESON2018 by Fei Huang. Practitioners of quark-based models ought therefore to ask “what makes  $\Gamma_{d^*(2380)}$  so much larger than the width calculated for a compact  $\Delta\Delta$  dibaryon?” rather than “what makes  $\Gamma_{d^*(2380)}$  so much smaller than twice a free-space  $\Delta$  width?”

The preceding discussion of  $\Gamma_{d^*(2380)}$  suggests that quark-based model findings of a tightly bound  $\Delta\Delta$   $s$ -wave configuration are in conflict with the observed width. Fortunately, hadronic-model calculations [5, 6] offer resolution of this insufficiency by coupling to the tightly bound and compact  $\Delta\Delta$  component of the  $d^*(2380)$  dibaryon’s wavefunction a  $\pi N\Delta$  resonating component dominated asymptotically by a  $p$ -wave pion attached loosely to the near-threshold  $N\Delta$  dibaryon  $\mathcal{D}_{12}$  with size about 1.5–2 fm. Formally, one can recouple spins and isospins in this  $\pi\mathcal{D}_{12}$  system, so as to assume an extended  $\Delta\Delta$ -like object. This explains why a discussion of  $\Gamma_{d^*\rightarrow NN\pi\pi}$  in terms of a  $\Delta\Delta$  constituent model requires a size  $R_{\Delta\Delta}$  considerably larger than provided by quark-based RGM calculations [16] to reconcile with the reported value of  $\Gamma_{d^*(2380)}$ . We recall that the width calculated in our diffuse-structure  $\pi N\Delta$  model [5, 6], as listed in Table 2, is in good agreement with the observed width of the  $d^*(2380)$  dibaryon resonance.

Support for the role of the  $\pi\mathcal{D}_{12}$  configuration in the decay of the  $d^*(2380)$  dibaryon resonance is provided by a recent ELPH  $\gamma d \rightarrow d\pi^0\pi^0$  experiment [23] looking for the  $d^*(2380)$ . The cross section data agree with a relativistic Breit-Wigner resonance shape with mass of 2370 MeV and width of 68 MeV, but the statistical significance of the fit is low, particularly since most of the data are from the energy region above the  $d^*(2380)$ . Invariant mass distributions from this experiment at  $\sqrt{s} = 2.39$  GeV, recorded in Fig. 2, are more illuminating. The  $\pi\pi$  mass distribution shown in (a)



**Figure 2.** Invariant mass distributions in ELPH experiment [23]  $\gamma d \rightarrow d\pi^0\pi^0$  at  $\sqrt{s} = 2.39$  GeV.

suggests a two-bump structure, fitted in solid red. The lower bump around 300 MeV is perhaps a manifestation of the ABC effect [24], already observed in  $pn \rightarrow d\pi^0\pi^0$  by WASA-at-COSY [10, 19] and interpreted in Ref. [21] as due to a tightly bound  $\Delta\Delta$  decay with reduced  $\Delta \rightarrow N\pi$  phase space. The upper bump in (a) is consistent then with the  $d^*(2380) \rightarrow \pi\mathcal{D}_{12}$  decay mode, in agreement with the  $\pi d$  mass distribution shown in (b) that peaks slightly below the  $\mathcal{D}_{12}(2150)$  mass.

**Table 3.**  $d^*(2380)$  decay width branching ratios (BRs) calculated in Ref. [21], for a total decay width  $\Gamma_{d^*(2380)}=75$  MeV, are compared with BRs derived from experiment [25, 26].

%	$d\pi^0\pi^0$	$d\pi^+\pi^-$	$pn\pi^0\pi^0$	$pn\pi^+\pi^-$	$pp\pi^-\pi^0$	$nn\pi^+\pi^0$	$NN\pi$	$NN$	total
BR(th.)	11.2	20.4	11.6	25.8	4.7	4.7	8.3	13.3	100
BR(exp.)	14 $\pm$ 1	23 $\pm$ 2	12 $\pm$ 2	30 $\pm$ 5	6 $\pm$ 1	6 $\pm$ 1	$\leq$ 9	12 $\pm$ 3	103

Recalling the  $\Delta\Delta - \pi\mathcal{D}_{12}$  coupled channel nature of the  $d^*(2380)$  in our hadronic model [5, 6], one may describe satisfactorily the  $d^*(2380)$  total and partial decay widths in terms of an incoherent mixture of these relatively short-ranged ( $\Delta\Delta$ ) and long-ranged ( $\pi\mathcal{D}_{12}$ ) channels. This is demonstrated in Table 3 where weights of  $\frac{5}{7}$  and  $\frac{2}{7}$  for  $\Delta\Delta$  and  $\pi\mathcal{D}_{12}$ , respectively, are assigned to an assumed value of  $\Gamma_{d^* \rightarrow NN\pi\pi}=60$  MeV [21]. This choice yields a branching ratio for  $\Gamma_{d^* \rightarrow NN\pi\pi}$  which does not exceed the upper limit of  $\text{BR} \leq 9\%$  determined recently from *not* observing the single-pion decay branch [26]. A pure  $\Delta\Delta$  description leads, as expected, to  $\text{BR} \ll 1\%$  [27].

### 3 Discussion

We end with a brief discussion of possible 6q admixtures in the essentially hadronic wavefunction of the  $d^*(2380)$  dibaryon resonance. For this we refer to the recent 6q non-strange dibaryon variational calculation in Ref. [2] which depending on the assumed confinement potential generates a  ${}^3S_1$  6q dibaryon about 550 to 700 MeV above the deuteron, and a  ${}^7S_3$  6q dibaryon about 230 to 350 MeV above the  $d^*(2380)$ . Taking a typical 20 MeV potential matrix element from deuteron structure calculations and 600 MeV for the energy separation between the deuteron and the  ${}^3S_1$  6q dibaryon, one finds admixture amplitude of order 0.03 and hence 6q admixture probability of order 0.001 which is compatible with that discussed recently by Miller [28]. Using the same 20 MeV potential matrix

element for the  $\Delta\Delta$  dibaryon candidate and 300 MeV for the energy separation between the  $d^*(2380)$  and the  ${}^7S_3$  6q dibaryon, one finds twice as large admixture amplitude and hence four times larger 6q admixture probability in the  $d^*(2380)$ , altogether smaller than 1%. These order-of-magnitude estimates demonstrate that long-range hadronic and short-range quark degrees of freedom hardly mix also for  $\Delta\Delta$  configurations, and that the  $d^*(2380)$  is extremely far from a pure 6q configuration. This conclusion is at odds with the conjecture made recently by Bashkanov, Brodsky and Clement [29] that 6q CC components dominate the wavefunctions of the  $\Delta\Delta$  dibaryon candidates  $\mathcal{D}_{03}$ , identified with the observed  $d^*(2380)$ , and  $\mathcal{D}_{30}$ . Unfortunately, most of the quark-based calculations discussed in the present work combine quark-model input with hadronic-exchange model input in a loose way [30] which discards their predictive power.

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