On the width of the $\Delta(1232)$ in $N\Delta$ and $\Delta\Delta$ states

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Abstract. Due to the finite kinetic energy in the intermediate $N\Delta$ state the (internal) energy available for mesonic decay is decreased and consequently the effective $N\Delta$ width is suppressed in NN scattering. The same can happen also in $\Delta\Delta$ case. Also the $N\Delta$ angular momentum suppresses the width as well, while the effect of the initial NN angular momentum is more subtle. The state dependence affects e.g. pion production observables and can also be seen as the origin of $T = 1$ "dibaryons".

1 Introduction

$N\Delta$ configurations arise by coupled channels in various contexts (e.g. pion production and absorption) as intermediate excited states of the externally given NN states. As such, below the nominal $N\Delta$ threshold the channel is naturally closed (virtual) and of finite range. Also at and above threshold the $N\Delta$ wave function is confined due to the finite pionic decay width of the Δ. In both cases the expectation value of the kinetic energy would be finite in the channel. As seen from Fig. 1, apparently the relative kinetic energy $E(p)$ does not participate in the decay as the invariants s_i do, but should rather be subtracted from the total overall energy [1].

For nonzero angular momenta another aspect of kinetic energy, the centrifugal barrier, can obviously act as repulsion in the $N\Delta$ channels strongly suppressing the corresponding wave functions. This, in turn, causes strong state dependence to their effects [2] further conveyed to observables. It turns out that the state dependence goes also into the widths giving each $N\Delta$ channel an *effective* width $\Gamma_{\rm eff}$. This leads to an improved agreement with experiment in $pp \leftrightarrow d\pi^+$ [1].

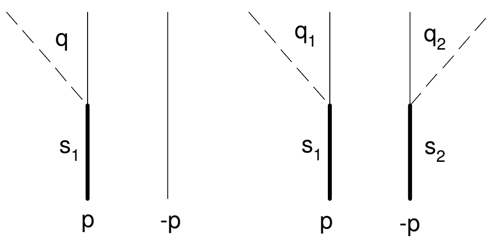

Figure 1. Kinematics of the $N\Delta$ (left) and $\Delta\Delta$ (right) decays. The relative momenta q_i between the pion and nucleon are given by $s_i = \sqrt{q_{(i)}^2 + \mu^2} + \sqrt{q_{(i)}^2 + M^2}$ with $\sqrt{s_i}$ the internal energy of the Δ.

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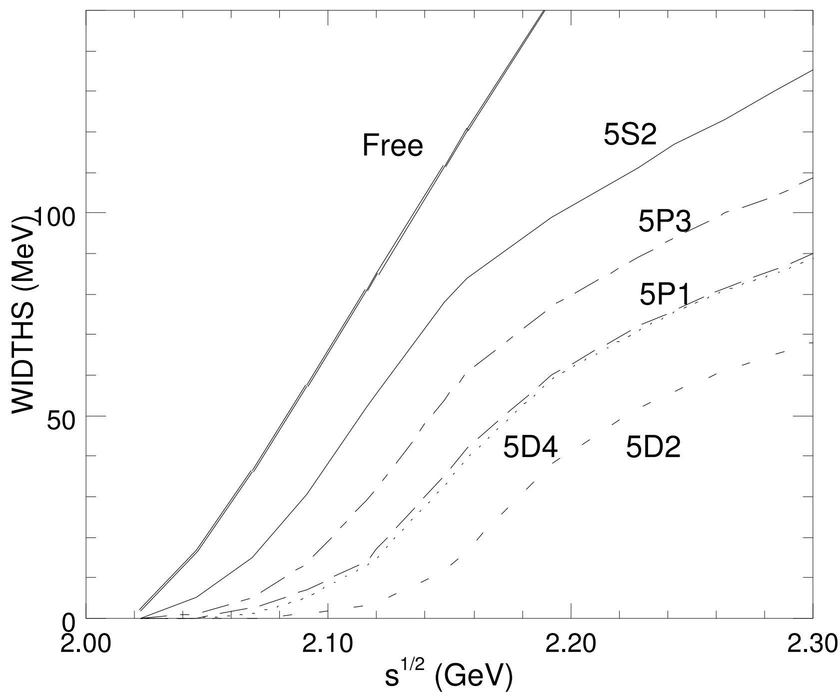

Figure 2. The widths of a representative selection of $N\Delta$ states in NN scattering: Curves as described in the text. The free width is the thick line above the others.

2 $N\Delta$ states

The effective decay width of the $N\Delta$ configuration, associated with NN scattering, into the three-body final state of Fig. 1 can be calculated explicitly as an average over kinematically allowed momenta [1, 3]

$$\Gamma_{3,\mathrm{eff}} = \frac{2}{\pi}\,\frac{\int_0^{p_{\max}} |\Psi_{N\Delta}(p)|^2\,\Gamma(q)\,p^2\,dp}{\int_0^{\infty} |\Psi_{N\Delta}(r)|^2\,r^2\,dr}. \tag{1}$$

Here $\Psi_{N\Delta}(p)$ is the Fourier transform of the appropriate partial wave component and $\Gamma(q)$ the free $\Delta \to N\pi$ width with q as the relative $N\pi$ momentum. The kinematics is determined by first subtracting the kinetic part from the c.m.s. energy. The effect of this decrease of available decay energy (and angular momentum barrier) is shown in Fig. 2. The half-width is used as an imaginary constant potential input in the relevant $N\Delta$-channel Schrödinger equation. Clearly, there is a remarkable reduction from free width values for any $N\Delta$ angular momentum. The ${}^5S_2(N\Delta)$ configuration originates from ${}^1D_2(NN)$ scattering. The ${}^5D_2(N\Delta)$ (short dashes) from the same initial state is much more suppressed, as anticipated earlier. Further, an interesting interplay with NN angular momenta shows up. The ${}^5D_4(N\Delta)$ (from ${}^1G_4(NN)$, dotted) is less suppressed. Similarly, ${}^5P_3(N\Delta)$ from ${}^3F_3(NN)$ (dash-dot) is less suppressed than ${}^5P_1(N\Delta)$ from ${}^3P_1(NN)$ (long dashes). In contrast the width of ${}^5D_0(N\Delta)$ from ${}^1S_0(NN)$ is totally negligible below $E_{\mathrm{lab}} \approx 600$ MeV and above 800 MeV only about 15–20 MeV. Apparently higher NN centrifugal barriers "push" the baryonic state to $N\Delta$. This can be understood considering that the most influential range for the interaction is ≈ 1 fm, and actually the $N\Delta$ wave function peaks around this range. Once the particles are at that distance, the barrier is $\approx 40 \times L(L+1)$ MeV and the loss of the mass barrier $M_\Delta - M_N$ in transition is partly regained from the diminished centrifugal barrier. However, the angular momentum barrier in the $N\Delta$ state is the dominant effect and the NN secondary. Still, it is remarkable that by these arguments one gets the quantum numbers of the isovector dibaryons of Ref. [4] correctly and the mass values reasonably well from the above rotational series [5].

3 $\Delta\Delta$ states

In the case of two decaying particles it may be necessary to specify how the lifetime is defined. The decay rate for particles 1 and 2 with widths Γ_1 and Γ_2 starting from time zero is taken to be $\Gamma_1 \exp(-\Gamma_1 t_1) \times \Gamma_2 \exp(-\Gamma_2 t_2)$. The total transition probability at time t is then (integrating over different time orderings)

$$\begin{aligned} P(t) = & \quad \Gamma_1\Gamma_2 \Big(\int_0^t e^{-\Gamma_1 t_1} dt_1 \int_0^{t_1} e^{-\Gamma_2 t_2} dt_2 + \int_0^t e^{-\Gamma_2 t_2} dt_2 \int_0^{t_2} e^{-\Gamma_1 t_1} dt_1 \Big) \\ = & \quad 1 - e^{-\Gamma_1 t} - e^{-\Gamma_2 t} + e^{-(\Gamma_1+\Gamma_2)t} \, . \end{aligned} \tag{2}$$

and the survival probability $1 - P(t) = \exp(-\Gamma t)[\exp(-\delta t) + \exp(+\delta t) - \exp(-\Gamma t)]$ (with the notation $\Gamma = (\Gamma_1 + \Gamma_2)/2$ and $\delta = (\Gamma_1 - \Gamma_2)/2$). So, the dominant part is consistent with the decay width being the average Γ, or the single width in the case $\Gamma_1 = \Gamma_2 = \Gamma$. In view of the kinematic results of Sec. 2 it may be possible that even this is further decreased.

Now the two-Δ decay width into $NN\pi\pi$ is calculated as the double integral

$$\Gamma_{4,\mathrm{eff}} = \frac{2}{\pi} \frac{\int |\Psi_{\Delta\Delta}(p)|^2 [\Gamma(q_1) + \Gamma(q_2)]/2 \, p^2 dp \, dq_1}{q_{\max} \int_0^\infty |\Psi_{\Delta\Delta}(r)|^2 \, r^2 dr} \, . \tag{3}$$

Here the maximum limit of the free variable p is obviously from the kinematics of Fig. 1 $p_{\max} = \sqrt{s/4 - (M+\mu)^2}$ and the upper limit of the pion momentum as a function of p is obtained from the maximum internal energy of particle one

$$s_{1\max} = [\sqrt{s} - \sqrt{(M+\mu)^2 + p^2}\,]^2 - p^2 \tag{4}$$

as

$$q^2_{1\max} = \frac{(s_{1\max} - M^2 - \mu^2)^2 - 4\mu^2 M^2}{4\, s_{1\max}} \, . \tag{5}$$

In the pion integration the second dependent momentum q_2 in turn is obtained from

$$q_2^2 = \frac{(s_2 - M^2 - \mu^2)^2 - 4\mu^2 M^2}{4\, s_2} \tag{6}$$

with $s_2 = [\sqrt{s} - \sqrt{s_1 + p^2}\,]^2 - p^2$ and $s_1 = (M^2 + q_1^2) + (\mu^2 + q_1^2)$.

Presently the $\Delta\Delta$ width in calculated for NN scattering in isospin zero 3^+ (*i.e.* ${}^3D_3$ and ${}^3G_3$) state(s) reported as a resonance $d'(2380)$ discovered in WASA@COSY experiments [6] and speculated as a possible dibaryon. The result is shown in Fig. 3. Both curves indicate a width less than a single free Δ and relatively well agreeing with the experimental value. The narrowness may be considered surprising, though it is in line with the results of Sec. 2 and also of Gal and Garcilazo [7, 8].

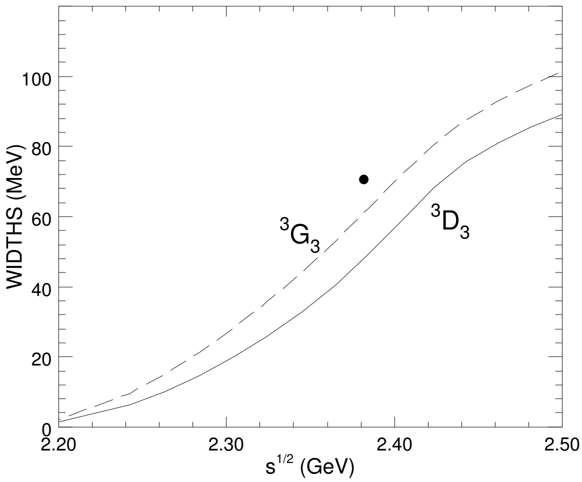

Figure 3. The widths of the $^7S_3(\Delta\Delta)$ state in $I = 0$ NN scattering: The solid curve arises from $^3D_3(NN)$ and the dashed one from $^3G_3(NN)$. The bullet shows the energy and width of the resonance reported e.g. in Ref. [6].

4 Critique

The present calculation does not purport to be a genuine dynamic *ab initio* field theory starting from interaction vertices. Rather the "vertices" in Fig. 1 illustrate the actual observable width for the decay $\Delta \to N\pi$. A deeper work would involve also complex meson exchanges [9], which would probably increase inelasticity. On the other hand these may be strongly attractive and long-ranged bringing the mass from the $\Delta\Delta$ threshold cusp at 2440 MeV down to the $d'(2380)$ region. Also inelasticities due to heavier particles ρ or $N'(1440)$, the Roper resonance, are absent. Further, one might question the validity of the extension of the parametrization of the experimental free width to the high momenta needed in Eq. 3.

In spite of these shortcomings and the lack of a quark calculation here, the present results may throw some doubt on the inevitability of $d'(2380)$ being the "smoking gun" manifestly demonstrating quark exotics in intermediate energy NN scattering based mainly on its narrowness. A similar conclusion may be implied also from Ref. [7]. An interesting extension of these calculations would be to isospin 2 or 3, which are not directly coupled with two-nucleon scattering states.

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