Calculations of η-nuclear quasi-bound states in few-body systems

M. Schäfer[1,2,*], N. Barnea[3], E. Friedman[3], A. Gal[3], and J. Mareš[2]

[1]Czech Technical University in Prague, Faculty of Nuclear Sciences and Physical Engineering, Břehová 7, 11519 Prague 1, Czech Republic
[2]Nuclear Physics Institute, Academy of Sciences of Czech Republic, 250 69 Řež, Czech Republic
[3]Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel

Abstract. We report on our Stochastic Variational Method (SVM) calculations of η-nuclear quasi-bound states in s-shell nuclei as well as the very recent calculation of the p-shell nucleus ^{6}Li. The ηN potentials used were constructed from ηN scattering amplitudes obtained within coupled-channel models that incorporate $N^\star(1535)$ resonance. We found that η^6Li is bound in the ηN interaction models that yield $\mathrm{Re}\,a_{\eta N} \geq 0.67$ fm. Additional repulsion caused by the imaginary part of ηN potentials shifts the onset of η-nuclear binding to η^4He, yielding very likely no quasi-bound state in η^3He.

1 Introduction

The current status of our theoretical studies of η-nuclear quasi-bound states, including discussion of the self-consistent treatment of the strong energy dependence of ηN scattering amplitudes derived from coupled-channel meson-baryon interaction models have been discussed thoroughly in Refs. [1–3]. So far, few-body calculations of η-nuclear quasi-bound states have been restricted to s-shell nuclei up to η^4He. In this contribution, we present our first SVM calculation of the η-nuclear quasi-bound state in the p-shell nuclear system η^6Li, taking into account all possible spin-isospin configurations. Moreover, we focus on the effect of the imaginary part of the complex $V_{\eta N}$ potential on the η binding energy B_η. We show that the effect could be considerable in light η-nuclear systems and must be taken into account in the study of the onset of η-nuclear binding.

2 Theoretical approach

Properties of η-nuclear quasi-bound states are studied within the SVM with a correlated Gaussian basis [4]. This approach was successfully applied in our previous calculations of s-shell η-nuclei and proved itself as highly accurate method with straightforward extension to systems with the number of particles $N \geq 5$.

The wave function of an η-nuclear system with orbital momentum $L = 0$ is expanded as a linear combination of correlated Gaussians

*e-mail: m.schafer@ujf.cas.cz

$$\Psi = \sum_k c_k \mathcal{A} \left\{ \chi_S^k \xi_{TT_z}^k \exp\left(-\frac{1}{2} \mathbf{x}^T A_k \mathbf{x} \right) \right\}, \tag{1}$$

where $\mathbf{x}$ stands for Jacobi coordinates and χ_S^k ($\xi_{TT_z}^k$) are corresponding spin (isospin) parts of a given spin (isospin) configuration. The matrix A_k is symmetric positive definite and includes $N(N-1)/2$ variational parameters. The SVM optimizes the variational basis step-by-step in a random trial and error procedure (details can be found in Ref. [5]).

SVM calculations of η-nuclear quasi-bound states in p-shell nuclei represent a rather challenging task. First, the computational complexity scales with $N!$, second, the amount of different spin-isospin configurations starts to increase quite rapidly. Preliminary results [6] showed that taking into account only one configuration underestimated binding of the nuclear core ^{6}Li by approximately 1.8 MeV. This led to development of a new high-performance SVM code which was used in the very recent fully self-consistent calculation of η^6Li, taking into account all posible spin-isospin configurations.

In our study of η-nuclear quasi-bound states we use the Minnesota NN central potential [7] which reproduces well properties of the ground states of s-shell and light p-shell nuclei. The interaction of the η meson with nucleons is described by a complex two-body energy dependent effective potential derived from the coupled-channel meson-baryon interaction models GW [8] and CS [9]. The form of ηN potential is taken according to [1] as

$$V_{\eta N}(\delta\sqrt{s}, r) = -\frac{4\pi}{2\mu_{\eta N}} b(\delta\sqrt{s}) \rho_\Lambda(r), \quad \rho_\Lambda(r) = \left(\frac{\Lambda}{2\sqrt{\pi}} \right)^3 \exp\left(-\frac{\Lambda^2 r^2}{4} \right), \tag{2}$$

where $\mu_{\eta N}$ stands for the ηN reduced mass, $\delta\sqrt{s} = \sqrt{s} - \sqrt{s_{th}}$ is the energy shift with respect to the ηN threshold, Λ is a scale parameter which is inversely proportional to the range of $V_{\eta N}$, and $b(\delta\sqrt{s})$ is an energy dependent complex amplitude.

The value of Λ is connected to EFT momentum cut-off; its upper bound corresponds to vector-meson exchange $\Lambda \leq 3.9\mathrm{fm}^{-1}$ or more restrictively to $\Lambda \leq 3.0\ \mathrm{fm}^{-1}$ excluding ρN channel from dynamical generation of the $N^\star(1535)$ resonance [3].

For given Λ, $b(\delta\sqrt{s})$ is fitted to the phase shifts derived from subthreshold $\delta\sqrt{s} < 0$ scattering amplitude of the corresponding ηN interaction model. See Ref. [3] for details.

The energy dependence of $V_{\eta N}$ is treated self-consistently: we search for a SVM solution that fulfills $\delta\sqrt{s_{sc}} = \left\langle \delta\sqrt{s_{sc}} \right\rangle$ where $\delta\sqrt{s}$ enters $V_{\eta N}$ and $\left\langle \delta\sqrt{s} \right\rangle$ is obtained from the SVM solution for a given value of $\delta\sqrt{s}$ [3]:

$$\left\langle \delta\sqrt{s} \right\rangle = -\frac{B}{A} - \xi_N \frac{1}{A} \langle T_N \rangle + \frac{A-1}{A} E_\eta - \xi_A \xi_\eta \left(\frac{A-1}{A} \right)^2 \left\langle T_\eta \right\rangle, \tag{3}$$

where B is the total binding energy, T_N (T_η) denotes the kinetic energy of nucleons (η), and A is the number of nucleons. The energy $E_\eta = \langle\psi| H - H_N |\psi\rangle$ where H_N is Hamiltonian of the nuclear core, $\xi_{N(\eta)} = m_{N(\eta)}/(m_N + m_\eta)$, and $\xi_A = Am_N/(Am_N + m_\eta)$.

The imaginary part of $V_{\eta N}$ is significantly smaller than its real part. This allows to calculate the width Γ_η perturbatively [1]. The SVM η-nuclear calculations are thus performed only for the real part of the ηN potential and Γ_η is evaluated using the expression

$$\Gamma_\eta = -2 \left\langle \Psi_{g.s.} \right| \mathrm{Im} V_{\eta N} \left| \Psi_{g.s.} \right\rangle, \tag{4}$$

where $|\Psi_{g.s.}\rangle$ is the SVM solution for the η-nuclear ground state corresponding to Re$V_{\eta N}$. Another possible way how to calculate Γ_η is to solve a generalized eigenvalue problem for complex Hamiltonian (including Im$V_{\eta N}$) using variationally determined SVM basis states for Re$V_{\eta N}$. This approach, already used in SVM calculations of kaonic nuclei [10], yields complex eigenenergy of the ground state $E = \mathrm{Re}(E) + \mathrm{iIm}(E)$ and consequently the width as $\Gamma_\eta = -2\mathrm{Im}(E)$. This method takes into account the effect of the non-zero imaginary part of $V_{\eta N}$ on the η binding energy. Namely, Im$V_{\eta N}$ acts as repulsion and thus makes the η meson less bound in the nucleus.

3 Results

Results of our SVM calculations of the η binding energies B_η and widths Γ_η in η^3He, η^4He, and η^6Li are summarized in Fig. 1. The calculations were performed using the GW and CS models and the parameter $\Lambda = 2$ and 4 fm^{-1}. In the GW model, η^6Li is rather comfortably bound for both values of Λ. On the other hand, the CS model yields η-nuclear quasi-bound state only for $\Lambda = 4$ fm^{-1}, with $B_\eta = 0.68$ MeV.

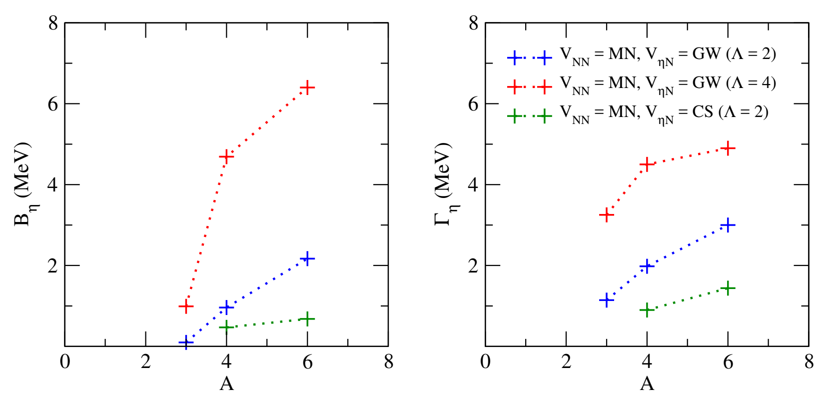

Figure 1. SVM calculations of the binding energy B_η and width Γ_η in η^3He, η^4He, and η^6Li using the Minnesota *NN* potential with Coulomb force included and two ηN interaction models - GW and CS.

Table 1. Comparison of self-consistent SVM calculations using two different width Γ_η evaluations - the mean-value approximation (Eq. 4) and the complex eigenvalue problem (cmplx) approach. Calculations were performed for the Minnesota *NN* potential with Coulomb force and the GW model. The η binding energy B_η and self-consistent energy $\delta\sqrt{s_{sc}}$ are shown as well.

η^3He		B_η [MeV]	Γ_η [MeV]	$\delta\sqrt{s_{sc}}$ [MeV]
$\Lambda = 2$ fm^{-1}	(Eq. 4)	0.11	1.37	−9.23
	(cmplx)	−0.25	1.32	−8.87
$\Lambda = 4$ fm^{-1}	(Eq. 4)	1.01	3.32	−13.18
	(cmplx)	0.36	3.44	−12.72

η^4He		B_η [MeV]	Γ_η [MeV]	$\delta\sqrt{s_{sc}}$ [MeV]
$\Lambda = 2$ fm^{-1}	(Eq. 4)	0.97	2.17	−19.64
	(cmplx)	0.77	2.22	−19.50
$\Lambda = 4$ fm^{-1}	(Eq. 4)	4.62	4.38	−29.73
	(cmplx)	4.40	4.41	−29.60

In Table 1, we compare two approaches to evaluation of the width Γ_η introduced in the previous section: the mean-value approach (Eq. 4) and the complex eigenvalue problem (cmplx) approach. Calculations of η^3He and η^4He were performed within the GW model with $\Lambda = 2$ and 4 fm^{-1}. It is apparent that the effect of the imaginary part of $V_{\eta N}$ on B_η, which is included in the cmplx approach, is quite significant in η^3He (with $\delta\sqrt{s}$ close to threshold) and decreases in η^4He with larger energy shift with respect to threshold. For the CS model (not shown in the table) the η^3He is not bound while in η^4He the effect of Im$V_{\eta N}$ is smaller (few tens of keV) due to the lower value of Im$V_{\eta N}$ than in the GW model. Table 1 illustrates that the size of the changes of B_η caused by Im$V_{\eta N}$ decreases with the magnitude of the subthreshold energy shift $\delta\sqrt{s}$. Namely, the strength of Im$V_{\eta N}$ has for both CS and GW interaction models maximum close to threshold and decreases with $\sqrt{s}$, as shown in Figure 2 of Ref. [3]. Moreover, the cmplx method confirms the estimate of Γ_η within the mean-value approach (Eq. 4), giving practically the same widths in all considered cases.

4 Summary

We performed few-body calculations of η-nuclear quasi-bound states in s-shell nuclei as well as in the p-shell nucleus ^{6}Li within our newly developed high-performance SVM code. We considered the Minnesota *NN* potential and two ηN interaction models - GW and CS. Calculations of η^6Li within the GW model yield the binding energy B_η and corresponding width consistent with previous RMF calculations [11]. The CS model gives quasi-bound state only for $\Lambda = 4$ fm^{-1}. This suggests that to bind η^6Li, the real part of the ηN scattering length should be greater than Re$a_{\eta N} = 0.67$ fm, predicted by the CS model.

Next, we repeated our previous study of the onset of η-nuclear binding in He isotopes taking into account the effect of Im$V_{\eta N}$ on the binding energy B_η. We observed considerable decrease of B_η in $^3_\eta$He and rather negligible effects in $^4_\eta$He as well as in $^6_\eta$Li. The η meson is barely bound in $^3_\eta$He even for the larger value of the cut-off parameter $\Lambda = 4$ fm^{-1}. This indicates that in order to study the η^3He system, one has to explore the resonance region as well, e.g., using the complex rotation method [12].

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