The most correct $\rho^0(770)$ meson mass and width values

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Abstract. By application of the pion electromagnetic form factor and also the P-wave isovector $\pi\pi$ scattering phase shift parametrizations, following from the first principles, like unitarity and analyticity, for a description of the corresponding accurate data, it is clearly demonstrated that the $\rho^0(770)$ meson mass and width values obtained by the Gounaris-Sakurai model in a description of the same data can not be accepted as correct ones.

1 Introduction

The initial state radiation method gives very precise information on the pion electromagnetic (EM) form factor (FF) $F_\pi^{EM,I=1}(t)$ [1, 2], measuring total cross-section

$$\sigma_{tot}(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2(0)}{3t}\beta_\pi^3(t)\left|F_\pi^{EM,I=1}(t) + Re^{i\phi}\frac{m_\omega^2}{m_\omega^2 - t - im_\omega\Gamma_\omega}\right|^2 \tag{1}$$

with the pion velocity $\beta_\pi(t) = \sqrt{1 - \frac{4m_\pi^2}{t}}$, the $\rho - \omega$ interference phase ϕ to be expressed as $\phi = arctg\frac{m_\rho\Gamma_\rho}{m_\rho^2 - m_\omega^2}$ and the amplitude R as a free parameter.

On the other hand, the most accurate up to now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data at the elastic region with theoretical errors, to be generated from the existing inaccurate experimental information by the Garcia-Martin-Kamiński-Peláez-Yndurain (GKPY) Roy-like equations with an imposed crossing symmetry condition [3], appeared.

Further we demonstrated that the the mass and width values of the $\rho^0(770)$ meson following from the application of the Gounaris-Sakurai model to a description of these data are not trustworthy.

2 Analysis

The extensively quoted pion EM FF G.-S. model [4] has been constructed by assuming that for a wide energy of the elastic region up to $t = 1$ GeV^2 the P-wave isovector $\pi\pi$ scattering

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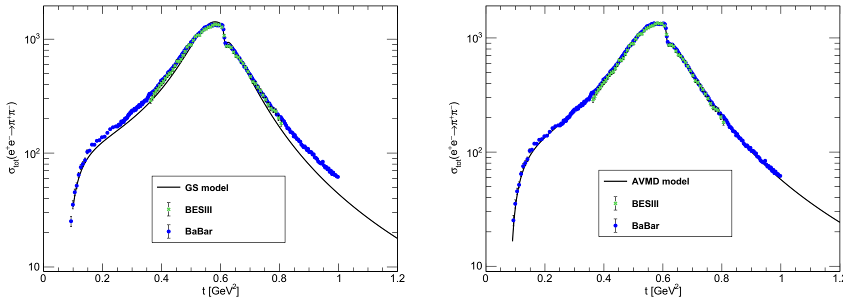


(a) (b)

Figure 1: Optimal description of the unified BESIII-BaBaR data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ at the elastic region: (a) by pion EM FF G.-S. model (b) by pion EM FF U&A model.

phase shift $\delta_1^1(t)$ satisfies a two parametric effective-range formula of the Chew-Mandelstam type [5]

$$\frac{q^3}{\sqrt{t}} cotg\delta_1^1(t) = a + bq^2 + q^2 h(t); \quad h(t) = \frac{2}{\pi}\frac{q}{\sqrt{t}} ln(\frac{\sqrt{t}+2q}{2m_\pi}); \quad q = [(t-4m_\pi^2)/4]^{1/2} \quad (2)$$

and takes the following form

$$F_\pi^{GS}(t) = \frac{m_\rho^2 + m_\rho\Gamma_\rho(\frac{3}{\pi}\frac{m_\pi^2}{q_\rho^2} ln(\frac{m_\rho+2q_\rho}{2m_\pi}) + \frac{m_\rho}{2\pi q_\rho} - \frac{m_\pi^2 m_\rho}{\pi q_\rho^3})}{(m_\rho^2 - t) + \Gamma_\rho(\frac{m_\rho^2}{q_\rho^3})(q^2(h(t) - h(m_\rho^2)) + q_\rho^2 h'(m_\rho^2)(m_\rho^2 - t)) - im_\rho\Gamma_\rho(\frac{q}{q_\rho})^3\frac{m_\rho}{\sqrt{t}}}. \quad (3)$$

Its application to an optimal description of the unified BESIII-BaBaR data [1, 2] at the elastic region (see Fig.1a) with χ^2/ndf=40.6341 gives parameters

$$\begin{aligned} m_\rho &= (775.73 \pm 0.10)MeV \\ \Gamma_\rho &= (126.51 \pm 0.13)MeV \end{aligned} \quad (4)$$

not in a coincidence with values presented by Review of Particle Physics [6].

On the other hand an application of the pion EM FF model (see Fig.1b)

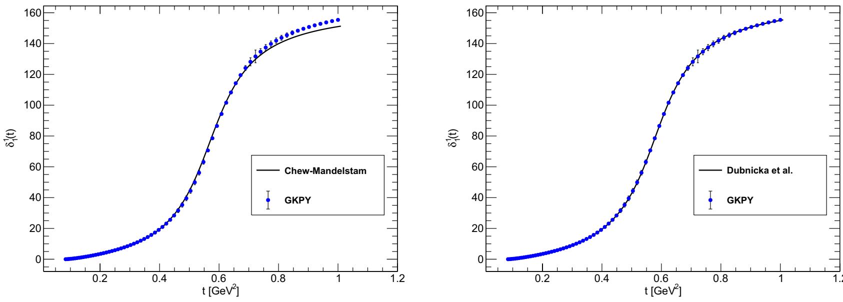
$$F_\pi^{EM,I=1}(q) = \frac{(q-q_Z)(q_N-q_P)}{(q-q_P)(q_N-q_Z)}\frac{(q_N-q_\rho)(q_N-q_{\bar\rho})}{(q-q_\rho)(q-q_{\bar\rho})}(f_{\rho\pi\pi}/f_\rho). \quad (5)$$

following from the first principles [7] to a description of the same data gives slightly lower values of parameters

$$\begin{aligned} m_\rho &= (763.026 \pm 0.10)MeV \\ \Gamma_\rho &= (144.233 \pm 0.13)MeV \end{aligned} \quad (6)$$

than in [6].

Which of these three sets of ρ^0 meson parameters can be considered to be correct ones? For a solution of this problem the most accurate up to now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data [3] have been exploited.



(a) (b)

Figure 2: Description of accurate $\delta_1^1(t)$ data: (a) by effective-range formula of Chew-Mandelstam type with χ^2/ndf=2.4499 (b) by the model independent parametrization following from the first principles with $\chi^2/ndf = 0.0244$.

If the effective-range formula of the Chew-Mandelstam type (2) is used to a description of $\delta_1^1(t)$ data (see Fig.2a) the following ρ^0 meson parameter values

$$\begin{aligned} m_\rho &= (772.42 \pm 0.03) MeV \\ \Gamma_\rho &= (153.85 \pm 0.11) MeV. \end{aligned} \tag{7}$$

are found [8], which do not coincide, neither with the values obtained by description of the BESIII-BaBaR data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ by G.-S. charged pion EM FF model, nor with values of PDG.

On the other hand, if fully solvable mathematical scheme through the phase representation of the pion EM FF

$$F_\pi^{EM,I=1}(t) = P_n(t) exp\Big[\frac{t}{\pi}\int_4^\infty \frac{\delta_1^1(t')}{t'(t'-t)}dt'\Big], \tag{8}$$

elaborated in [9, 10], is applied, a perfect description of $\delta_1^1(t)$ data (see Fig.2b), by a model independent phase shift $\delta_1^1(t)$ representation [8]

$$\delta_1^1(q) = arctg\frac{A_3q^3 + A_5q^5 + ...}{1 + A_2q^2 + A_4q^4 + ...} \tag{9}$$

with $\chi^2/ndf = 0.0244$ is achieved and the ρ^0 meson mass and width

$$\begin{aligned} m_\rho &= (763.56 \pm 0.51) MeV \\ \Gamma_\rho &= (143.09 \pm 0.82) MeV. \end{aligned} \tag{10}$$

are found to be consistent with (6).

The same values of ρ^0 meson parameters are found also by a generalization of formula (5) to the region of the excited states of the ρ^0 meson [8], what can not be said about results obtained by a generalization of the pion EM FF G.-S. model [4] to the same region.

This all convinces us that the $\rho^0(770)$ meson mass and width values obtained by the Gounaris-Sakurai model can not be accepted as correct ones.

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References

[1] M. Ablikin *et al.*, Phys. Lett. B **753**, 629 (2016)
[2] J. P. Lees *et al.*, Phys. Rev. D **86**, 032013 (2012)
[3] R. Garcia-Martin *et al.*, Phys. Rev. D **83**, 074004 (2011)
[4] G. J. Gounaris and J. J. Sakurai, Phys. Rev. Lett. **21**, 244 (1968)
[5] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960)
[6] C. Patrignani, *et al.*, Chin. Phys. C **40** 100001 (2016)
[7] S. Dubnicka, A. Z. Dubnickova, Acta Phys. Slovaca **60**, 1 (2010)
[8] E. Bartos, S. Dubnicka, A. Z. Dubnickova, R. Kamiński, A. Liptaj, Phys. Rev. D **96**, 113004 (2017)
[9] S. Dubnicka, A. Z. Dubnickova, A. Liptaj, Phys. Rev. D **90**, 114003 (2014)
[10] S. Dubnicka, A. Z. Dubnickova, R. Kamiński, A. Liptaj, Phys. Rev. D **94**, 054036 (2016)