ρ meson GPDs and IPDs with a light-front constituent quark model

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Abstract. We investigate the ρ meson unpolarized Generalized Parton Distributions based on a Light-Front Constituent Quark Model. Moreover, by employing a Gaussian form wave packet, we apply the obtained generalized parton distributions to get the Impact Parameter dependent parton Distributions.

1 Introduction

It is well-known that the usual parton distributions (PDFs) can give the longitudinal information of hadron targets in the deep inelastic scattering (DIS) processes. To investigate the transverse information, the generalized parton distributions (GPDs) could be a powerful tool [1]. In our recent work, a Light-Front Constituent Quark Model (LFCQM) is employed to study the GPDs [2] of the ρ meson (spin-1) and its Impact Parameter dependent parton Distributions (IPDs) [3].

2 GPD and our model

The unpolarized GPDs of a spin-1 particle H_i , $i = 1, 5$ are defined as [4]

$$V_{\lambda'\lambda} = \frac{1}{2} \int \frac{d\omega}{2\pi} e^{ix(Pz)} \langle p', \lambda' | \bar{q}(-\frac{1}{2}z) \not{n} q(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z=\omega n} = \sum_i \epsilon'^{* \nu} V_{\nu\mu}^{(i)} \epsilon^\mu H_i^q(x, \xi, t) \quad (1)$$

where ϵ (or ϵ') is the polarization vector of the incoming (or outgoing) spin-1 particle, and $V_{\nu\mu}^{(i)}$ are Lorentz structures. By taking the forward limit, one can obtain the ρ meson PDF through GPD H_1 as $q(x) = [q^1(x) + q^{-1}(x) + q^0(x)]/3 = H_1^q(x, 0, 0)$. The ρ meson form factors (FFs) are related to its GPDs by the following sum-rules,

$$\int_{-1}^1 dx H_i^q(x, \xi, t) = G_i^q(t) \ (i = 1, 2, 3), \quad \int_{-1}^1 dx H_i^q(x, \xi, t) = 0 \ (i = 4, 5), \quad (2)$$

and the charge, magnetic and quadrupole form factors are given by [5]

$$G_C(t) = G_1(t) + \frac{2}{3}\eta G_Q(t), \quad G_M(t) = G_2(t), \quad G_Q(t) = G_1(t) - G_2(t) + (1 + \eta)G_3(t). \quad (3)$$

The normalization condition is $G_C(0) = 1$.

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In analogy to the chiral interaction Lagrangian for the $\pi \rightarrow q\bar{q}$ vertex [6], the effective Lagrangian for the $\rho \rightarrow \bar{q}q$ is taken as

$$\mathcal{L}_I = -\frac{iM}{f_\rho} \bar{q} \Gamma^\mu \tau_q \cdot \rho_\mu = -\frac{i\sqrt{2}M}{f_\rho} \left[\frac{\bar{u} \Gamma^\mu u - \bar{d} \Gamma^\mu d}{\sqrt{2}} \rho_\mu^0 + \bar{u} \Gamma^\mu d \rho_\mu^+ + \bar{d} \Gamma^\mu u \rho_\mu^- \right], \quad (4)$$

where ρ_μ is the massive spin one field, f_ρ is the ρ decay constant, and Γ^μ is a Bethe-Salpeter amplitude (BSA),

$$\Gamma^\mu = N \frac{\gamma^\mu - (k_q - k_{\bar{q}})^\mu / (M_{\text{inv}} + 2m_q)}{[k_q^2 - m_R^2 + i\epsilon][k_{\bar{q}}^2 - m_R^2 + i\epsilon]},$$

where N is the normalization constant, m_q and m_R are the constituent quark and the regulator masses, respectively. The bound state invariant mass M_{inv} is defined differently in the valence and nonvalence regions [2]. In our approach, we take $m_q = 0.403$ GeV and $m_R = 1.61$ GeV. Our ρ meson mean square charge radius $\langle r^2 \rangle = 0.52$ fm², magnetic moment $\mu = 2.06/(2m_\rho)$ and quadrupole moment $Q_\rho/m_\rho^2 = -0.323$. All the calculated observables agree with experiment data and most of previous calculations [2].

The longitudinal information can be investigated when the skewness variable ξ is nonzero. Our results for ρ FFs and GPDs with $\xi = -0.4$ are show in Fig. 1. After leading order DGLAP evolution to Lattice scale, we find the Mellin moments of the obtained GPDs, see in Table 1, match well between our model and the Lattice QCD calculation [8].

Table 1. Mellin moments of GPDs.

	$\int dx x H_1$	$\int dx x^2 H_1$	$\int dx x^3 H_1$	$\int dx x H_5$	$\int dx x^2 H_5$	$\int dx x^3 H_5$
Lattice QCD	0.33(2)	0.17(5)	0.06(4)	0.29_{-23}^{+22}	-0.001(15)	-0.01(6)
Our result	0.34(2)	0.15(1)	0.08(1)	0.044(3)	0.048(5)	0.039(5)

3 IPDs

To obtain the information how PDF are distributed in position space, one can take Fourier transform of $V_{\mathcal{N}}$ w.r.t. the transverse momenta $\mathbf{p}_\perp$ and $\mathbf{p}'_\perp$, which gives the so-called IPDs [3]. Experimentally there is a largest measured value $|t|_{\text{max}}$ and thus the measurement accuracy is $\sigma \sim (|t|_{\text{max}})^{-1/2}$. To account this limit, we introduce a Gaussian form wave packet in initial and final states,

$$G(\mathbf{p}_\perp, 1/\sigma^2) = \exp(-\mathbf{p}_\perp^2 \sigma^2 / 2). \quad (5)$$

The IPD for H_1^q (which relates to DIS structure function F_1) is

$$q_\sigma(x, b) = \int_0^\infty \frac{\Delta_\perp d\Delta_\perp}{2\pi} J_0(b\Delta_\perp) e^{-\Delta_\perp^2 \sigma^2 / 4} H_1(x, 0, -\Delta_\perp^2), \quad (6)$$

with $q_\sigma(b) = \int_0^1 dx q_\sigma(x, b)$.

Our results for IPDs are shown in Fig. 2. We see that as σ becomes smaller, the wave functions of the initial and final states get closer to a plane wave, and the parton distribution becomes more transversely localized in the position space. Following the same way, IPDs for $G_{C,M,Q}$ are also introduced in our calculation.

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Figure 1. ρ form factors and GPDs with $\xi = -0.4$.

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Figure 2. Contour plots of the IPD $q(b)$ with a wave packet with $\sigma = 1 \text{GeV}^{-1}$.