Study of the possible role of triangle singularities in $B^- \rightarrow D^{*0} \pi^- \pi^0 \eta$ and $B^- \rightarrow D^{*0} \pi^- \pi^+ \pi^-$

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Abstract. Studying the effects of triangle singularities in hadronic processes is of the utmost importance since they can originate peaks that may wrongfully be associated with resonances. In this work, the role of the triangle mechanism in the decays $B^- \rightarrow D^{*0} \pi^- \pi^0 \eta$ and $B^- \rightarrow D^{*0} \pi^- \pi^+ \pi^-$ is explored. Here, the singularity appears when B^- decays into $D^* K^{*0} K^-$, then K^{*0} decays into K^+ through pion emission, and $K^- K^+$ fuse together forming either the $a_0(980)$ or $f_0(980)$ which then decays into $\pi^0 \eta$ or $\pi^+ \pi^-$, respectively. As a result, the $K^* K^+ K^-$ loop generates a peak in the invariant mass of $\pi^- a_0$ or $\pi^- f_0$ around 1420 MeV. The branching ratios that come from this peak are $Br(B^- \rightarrow D^{*0} \pi^- a_0; a_0 \rightarrow \pi^0 \eta) = (1.66 \pm 0.45) \times 10^{-6}$ and $Br(B^- \rightarrow D^{*0} \pi^- f_0; f_0 \rightarrow \pi^+ \pi^-) = (2.82 \pm 0.75) \times 10^{-6}$, which are well within the measurable range. Thus, this work makes interesting predictions about an effect of a triangle singularity in an experiment that is feasible in present experimental facilities.

1 Introduction

Non conventional hadron states, called exotics, have been the target of intense study. A concept very relevant to these studies are the triangle singularities, which produce peaks that appear from simple kinematical effects. They were first described by Landau in Ref. [4] (see also Ref. [5]). The peaks produced by the singularities can be misidentified with resonances, such as the " $a_1(1420)$ " meson measured by the COMPASS collaboration [6] which was shown to be an effect of the triangle mechanism [7–9].

In this work we intend to study the $B^- \rightarrow D^{*0} \pi^- \pi^0 \eta$ ($\pi^+ \pi^-$) decays via a_0 (f_0) formation¹. These reactions start with $B^- \rightarrow D^{*0} K^- K^{*0}$ where $K^{*0} \rightarrow \pi^- K^+$ and then $K^+ K^- \rightarrow a_0$ (f_0) which generates a triangle singularity around $M_{\text{inv}}(\pi^- a_0$ (f_0)) = 1418 MeV (see Eq. (18) of Ref. [5]). This peak is very visible in the mass distributions. Finally, the branching fractions are then calculated and found to be within measurable range.

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¹The mechanism studied here was suggested in Ref. [7], without any proposal as to how the $K^* \bar{K}$ could be formed, and no qualitative evaluation of the process. Here we give a proposal as to how the mechanism can occur and do a quantitative calculation of its amplitudes

2 Formalism

In order to analyse the following decays, $B^- \rightarrow D^{*0} \pi^- \pi^0 \eta$ ($\pi^+ \pi^-$) we will first evaluate the $B^- \rightarrow D^* \pi R$ ($R = a_0, f_0$). The total t matrix for this decay will be,

$$-it_{B \rightarrow D^* \pi R} = i \sum_{\text{pol. of } K^*} \int \frac{d^4 q}{(2\pi)^4} \frac{i t_{B^- \rightarrow D^{*0} K^{*0} K^-}}{q^2 - m_K^2 + i\epsilon} \frac{i t_{K^* K^+ \pi^-}}{(P - q)^2 - m_{K^*}^2 + i\epsilon} \frac{i t_{K^+ K^- R}}{(P - q - k)^2 - m_K^2 + i\epsilon}. \quad (1)$$

Now we need to calculate the three vertices, $t_{B^- \rightarrow D^{*0} K^{*0} K^-}$, $t_{K^* K^+ \pi^-}$ and $t_{K^+ K^- R}$. We first look at the $B^- \rightarrow D^{*0} K^- K^{*0}$ vertex which can be in s -wave, since both D^{*0} and K^{*0} have $J^P = 1^-$. Thus we take the amplitude of the interaction to be,

$$t_{B^- \rightarrow D^{*0} K^{*0} K^-} = C \epsilon_\mu(K^*) \epsilon^\mu(D^*). \quad (2)$$

We can calculate C since we know $\text{Br}(B^- \rightarrow D^{*0} K^{*0} K^-) = (1.5 \pm 0.4) \times 10^{-3}$ [13].

For the decay $K^{*0} \rightarrow \pi^- K^+$ we can use the chiral invariant lagrangian with local hidden symmetry [14, 15], which gives

$$-it_{K^* K^+ \pi^-} \simeq -ig \vec{\epsilon}_{K^*} \cdot (\vec{p}_\pi - \vec{p}_{K^+}), \quad (3)$$

with $\vec{p}_{K^+}$ and $\vec{p}_\pi$ in the CM frame of πR , and $g \simeq 4.3$.

The resonance R couples to $\pi^0 \eta$ or $\pi^+ \pi^-$ in s -wave which means that the t matrix of the vertex will be a constant,

$$t_{K^+ K^- R} = g_{K^+ K^- R}. \quad (4)$$

Then, one needs only to substitute Eqs. (2), (3) and (4) in Eq. (1), to obtain the amplitude $t_{B^- \rightarrow D^{*0} \pi R}$.

For the three body decay of $B^- \rightarrow D^{*0} \pi^- R$, the mass distribution is

$$\frac{d\Gamma}{dM_{\text{inv}}(\pi R)} = \frac{1}{(2\pi)^3} \frac{|\vec{p}_{D^*}| |\vec{p}_\pi|}{4M_B^2} \sum_{\text{pol.}} |t_{B^- \rightarrow D^{*0} \pi R}|^2, \quad (5)$$

but since a_0 and f_0 are resonances that will decay to $\pi^0 \eta$ and $\pi^+ \pi^-$, respectively, one will end up with four particles in the final state. The solution is to consider the resonance as a normal particle and then adding a mass distribution to the decay width in Eq. (5). Then, for the $a_0(980)$ case, Eq. (5) becomes

$$\frac{d^2\Gamma}{dM_{\text{inv}}(\pi a_0) dM_{\text{inv}}(\pi^0 \eta)} = \frac{1}{(2\pi)^5} \frac{|\vec{p}_{D^*}| |\vec{k}| |\vec{q}_\eta|}{4M_B^2} \sum \sum |\tilde{t}_{B^-, D^* \pi R} \times t_{K^+ K^- \rightarrow \pi^0 \eta}|^2. \quad (6)$$

For the case of f_0 we just need to substitute, in Eq. (6), $t_{K^+ K^- \rightarrow \pi^0 \eta} \rightarrow t_{K^+ K^- \rightarrow \pi^+ \pi^-}$, $M_{\text{inv}}(\pi a_0) \rightarrow M_{\text{inv}}(\pi f_0)$, $M_{\text{inv}}(\pi^0 \eta) \rightarrow M_{\text{inv}}(\pi^+ \pi^-)$, and $|\vec{q}_\eta| \rightarrow |\vec{q}_\pi|$, with

$$|\vec{q}_\pi| = \frac{\lambda^{1/2}(M_{\text{inv}}^2(\pi^+ \pi^-), m_{\pi^+}^2, m_{\pi^-}^2)}{2M_{\text{inv}}(\pi^+ \pi^-)}. \quad (7)$$

The amplitudes $t_{K^+ K^- \rightarrow \pi^0 \eta}$ and $t_{K^+ K^- \rightarrow \pi^+ \pi^-}$ are calculated using the chiral unitary approach as done in Refs. [10, 11].

Finally, we can substitute everything we have calculated into Eq. (6) and obtain,

$$\frac{1}{\Gamma_{B^-}} \frac{d^2\Gamma}{dM_{\text{inv}}(\pi R) dM_{\text{inv}}(R)} = \frac{g^2}{(2\pi)^5} \frac{|\vec{p}_{D^*}| |\vec{q}_\eta| |\vec{k}|^3}{4M_B^2} |t_T \times t_{K^+ K^- \rightarrow \pi^0 \eta(\pi^+ \pi^-)}|^2 \frac{C^2}{\Gamma_{B^-}}, \quad (8)$$

Figure 1: The derivative of the mass distribution with regards to $M_{\text{inv}}(R)$.

with (see Refs. [5] and [16])²

$$t_T = \int \frac{d^3 q}{(2\pi)^3} \left(2 + \frac{\vec{q} \cdot \vec{k}}{|\vec{k}|^2} \right) \frac{1}{8\omega^* \omega \omega'} \frac{1}{k^0 - \omega' - \omega^*} \frac{1}{P^0 + \omega + \omega' - k^0} \frac{1}{P^0 - \omega - \omega' - k^0 + i\epsilon} \times \\ \times \frac{\{2P^0 \omega + 2k^0 \omega' - 2[\omega + \omega'][\omega + \omega' + \omega^*]\}}{P^0 - \omega^* - \omega + i\epsilon}, \quad (9)$$

where $\omega^*(\vec{q}) = \sqrt{m_{K^*}^2 + |\vec{q}|^2}$, $\omega'(\vec{q}) = \sqrt{m_K^2 + |\vec{q} + \vec{k}|^2}$ and $\omega(\vec{q}) = \sqrt{m_K^2 + |\vec{q}|^2}$.

3 Results

In Fig. 1 we plot Eq. (8) for $B^- \rightarrow D^{*0} \pi^- \pi^0 \eta$ ($\pi^+ \pi^-$), where we fix $M_{\text{inv}}(\pi R) = 1418$ MeV which corresponds to the position of the triangle singularity. In both cases a strong peak can be seen around 980 MeV, corresponding to the a_0 and f_0 resonances, although in Fig. 1(b) the dispersion is bigger than in Fig. 1(a). Then, we integrate Eq. (8) over $M_{\text{inv}}(R)$ to obtain $\frac{d\Gamma}{dM_{\text{inv}}(\pi R)}$ which we show in Fig. 2. We see a peak of the distribution at around 1420 MeV and that the distribution also stretches up to large values of $M_{\text{inv}}(\pi R)$. The peak is a clear manifestation of the triangle singularity, and the stretching of the distribution to higher energies is a result of the $|\vec{k}|^3$ factor in Eq. (8) that contains a $|\vec{k}|$ factor from phase space and a $|\vec{k}|^2$ factor from the dynamics of the process.

Integrating the mass distributions over $M_{\text{inv}}(R)$ in Fig. 2, we obtain the branching fractions

$$\text{Br}(B^- \rightarrow D^{*0} \pi^- a_0; a_0 \rightarrow \pi^0 \eta) = (1.66 \pm 0.45) \times 10^{-6}, \quad (10a)$$

$$\text{Br}(B^- \rightarrow D^{*0} \pi^- f_0; f_0 \rightarrow \pi^+ \pi^-) = (2.82 \pm 0.75) \times 10^{-6}. \quad (10b)$$

4 Summary

We have studied the effects of the triangle mechanism for the reactions $B^- \rightarrow D^{*0} \pi^- a_0(980); a_0 \rightarrow \pi^0 \eta$ and $B^- \rightarrow D^{*0} \pi^- f_0(980); f_0 \rightarrow \pi^+ \pi^-$. We evaluated $\frac{d^2 \Gamma}{dM_{\text{inv}}(\pi^- R) dM_{\text{inv}}(R)}$ and have seen clear peaks in the $M_{\text{inv}}(R)$ distributions, corresponding to the $a_0(980)$ and $f_0(980)$. Integrating over $M_{\text{inv}}(R)$ we have obtained $\frac{d\Gamma}{dM_{\text{inv}}(\pi R)}$. In these distributions a clear

²To be correct in our analysis we have to use the width of K^{*0} , thus $\omega^* \rightarrow \omega^* - i\frac{\Gamma_{K^*}}{2}$ in Eq. (9).

Figure 2: The mass distribution of $B^- \rightarrow D^{*0}\pi^-\pi^0\eta$ (full line) and $B^- \rightarrow D^{*0}\pi^-\pi^+\pi^-$ (dashed curve).

peak appears for $M_{\text{inv}}(\pi a_0)$, $M_{\text{inv}}(\pi f_0)$ around 1420 MeV that is a consequence of the triangle singularity. Thus, this work serves as a warning not to identify this peak as a resonance. We have also calculated the branching ratios of these reactions and noted that they are within measurable range. Thus we present interesting triangle singularities effects that can be tested in present experimental facilities.

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References

- [1] R. H. Dalitz, T. C. Wong and G. Rajasekaran, Phys. Rev. **153** 1617 (1967)
- [2] T. Hyodo and D. Jido, Prog. Part. Nucl. Phys. **67** 55 (2012)
- [3] Y. Kamiya, K. Miyahara, S. Ohnishi, Y. Ikeda, T. Hyodo, E. Oset and W. Weise, Nucl. Phys. A **954** 41 (2016)
- [4] L. D. Landau, Nucl. Phys. **13**, 181 (1959)
- [5] M. Bayar, F. Aceti, F. K. Guo and E. Oset, Phys. Rev. D **94**, no. 7, 074039 (2016)
- [6] C. Adolph *et al.* [COMPASS Collaboration], Phys. Rev. Lett. **115**, no. 8, 082001 (2015)
- [7] X. H. Liu, M. Oka and Q. Zhao, Phys. Lett. B **753** 297 (2016)
- [8] M. Mikhasenko, B. Ketzer and A. Sarantsev, Phys. Rev. D **91**, no. 9, 094015 (2015)
- [9] F. Aceti, L. R. Dai and E. Oset, Phys. Rev. D **94**, no. 9, 096015 (2016)
- [10] J. A. Oller and E. Oset, Nucl. Phys. A **620** 438 (1997) Erratum: [Nucl. Phys. A **652** 407 (1999)]
- [11] J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. D **59** 074001 (1999) Erratum: [Phys. Rev. D **60** 099906 (1999)] Erratum: [Phys. Rev. D **75** 099903 (2007)]
- [12] L. L. Chau, Phys. Rept. **95** 1 (1983)
- [13] C. Patrignani *et al.* [Particle Data Group], Chin. Phys. C **40** no.10, 100001 (2016)
- [14] M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. **164** 217 (1988)
- [15] H. Nagahiro, L. Roca, A. Hosaka and E. Oset, Phys. Rev. D **79** 014015 (2009)
- [16] R. Pavao, S. Sakai and E. Oset, Eur. Phys. J. C **77** no.9, 599 (2017)