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# $a_0(980)$-$f_0(980)$ mixing in $\chi_{c1}$ decay

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**Abstract.**
We investigate the isospin breaking in the $\chi_{c1} \to \pi^0\pi^+\pi^-$ and $\chi_{c1} \to \pi^0\pi^0\eta$ decays and its relation to the $a_0(980) - f_0(980)$ mixing, which was measured by the BESIII Collaboration. The isospin violation is introduced through the use of different masses for the charged and neutral kaons, either in the propagators of pairs of mesons created in the $\chi_{c1}$ decay, or in the propagators inside the $T$ matrix. We find that the most important effect in the total amplitude is the isospin breaking inside the $T$ matrix which is essential to get a good agreement with the experimental measurement of the mixing.

## 1 Introduction

Since the discoveries of $a_0(980)$ and $f_0(980)$ mesons several decades ago, explanations of their nature have been a topic of much discussion. Both $a_0(980)$ and $f_0(980)$ have a mass around the $K\bar{K}$ threshold, and couple to $K\bar{K}$. Their nature is still discussed, either as $q\bar{q}$, hybrids, tetraquarks and meson-meson molecules [1–7]. Possible mixing between $a_0(980)$ (isospin 1) and $f_0(980)$ (isospin 0) could help to constrain models and parameters.

The $a_0(980) - f_0(980)$ mixing was examined from $J/\psi \to \phi a_0(980)$ reaction at the upgraded Beijing electron positron collider with the BESIII detector in Ref. [8]. In Ref. [9], the authors investigated the isospin-violating mixing of the light scalar mesons $a_0(980)$ and $f_0(980)$ within the unitarized chiral approach. They improved the theoretical understanding of the phenomenon of $a_0 - f_0$ mixing and confirmed that the dominant mixing effect comes from the kaon mass difference. Investigation of the $a_0(980) - f_0(980)$ mixing from $a_0(980) \to f_0(980)$ transition was done in Ref. [10]. They found that the $a_0(980) \to f_0(980)$ transition can provide additional constraints to the parameters of $a_0(980)$ and $f_0(980)$ mesons. Experimental measurements of both reactions were studied by BESIII Collaboration in Ref. [11].

## 2 Formalism

We follow a similar approach to the Refs. [12, 13] in order to study the $\chi_{c1} \to \pi^0 f_0(980) \to \pi^0\pi^+\pi^-$ and $\chi_{c1} \to \pi^0 a_0(980) \to \pi^0\pi^0\eta$ decays (For detail analysis see Ref. [14]). There are three independent SU(3) scalars from $\phi\phi\phi$; $Trace(\phi\phi\phi)$, $Trace(\phi)Trace(\phi\phi)$ and

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$[Trace(\phi)]^3$. The authors discuss these three $SU(3)$ scalars and conclude that only the structure $Trace(\phi\phi\phi)$ yields results in good agrement with the recent experiment of BESIII [15] on the $\chi_{c1} \to \eta\pi^+\pi^-$ decay. Hence, in the present work we adopt $Trace(\phi\phi\phi)$ structure. Thus we obtain the combinations $\pi^0\pi^0\eta$ and $\pi^0 K\bar{K}$, as follows

$$Trace(\phi\phi\phi) = \sqrt{3}\,\pi^0\pi^0\eta + \frac{\pi^0}{\sqrt{2}}(3\,K^+K^- - 3\,K^0\bar{K}^0), \tag{1}$$

where we have neglected the $\eta'$ components because of its large mass and small couplings to these scalar mesons. Then the full amplitude for the isospin-allowed $a_0(980)$ production (with final state $\pi^0\eta$) is obtained considering also the rescattering of the pairs of mesons as indicated in Fig. 1,

$$t = \vec{\epsilon}_{\chi_{c1}} \cdot \vec{p}_{\pi^0}\ \tilde{t}_{\pi^0\eta}, \tag{2}$$

with

$$\begin{aligned}\tilde{t}_{\pi^0\eta} &= V_p\,(h_{\pi^0\eta} + h_{\pi^0\eta}\,G_{\pi^0\eta}\,t_{\pi^0\eta\to\pi^0\eta}\\ &+ h_{K^+K^-}\,G_{K^+K^-}\,t_{K^+K^-\to\pi^0\eta}\\ &+ h_{K^0\bar{K}^0}\,G_{K^0\bar{K}^0}\,t_{K^0\bar{K}^0\to\pi^0\eta})\,,\end{aligned} \tag{3}$$

where the weights $h_i$ are obtained from Eq. (1): $h_{\pi^0\eta} = 2\sqrt{3}$, $h_{K^+K^-} = 3/\sqrt{2}$ and $h_{K^0\bar{K}^0} = -3/\sqrt{2}$.

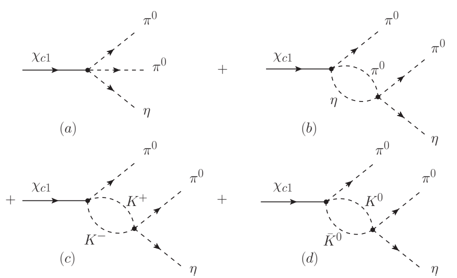


**Figure 1.** Diagrams involved in the $a_0(980)$ production in the $\chi_{c1} \to \pi^0 a_0(980) \to \pi^0\pi^0\eta$ reaction: (a) tree-level; and rescattering of (b) $\pi^0\eta$, (c) $K^+K^-$, (d) $K^0\bar{K}^0$.

For the isospin-forbidden $f_0(980)$ production (with final state $\pi^+\pi^-$) there is no tree-level contribution, only the rescattering diagrams taken into account, as shown in Fig. 2,

$$t = \vec{\epsilon}_{\chi_{c1}} \cdot \vec{p}_{\pi^0}\ \tilde{t}_{\pi^+\pi^-}, \tag{4}$$

where

$$\begin{aligned}\tilde{t}_{\pi^+\pi^-} &= V_p\,(h_{\pi^0\eta}\,G_{\pi^0\eta}\,t_{\pi^0\eta\to\pi^+\pi^-}\\ &+ h_{K^+K^-}\,G_{K^+K^-}\,t_{K^+K^-\to\pi^+\pi^-}\\ &+ h_{K^0\bar{K}^0}\,G_{K^0\bar{K}^0}\,t_{K^0\bar{K}^0\to\pi^+\pi^-})\,.\end{aligned} \tag{5}$$

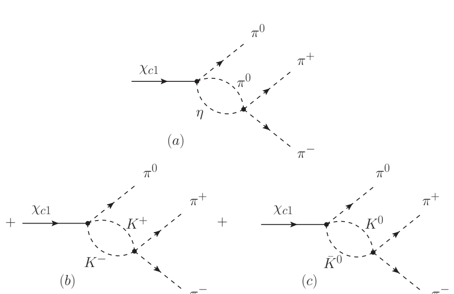


**Figure 2.** Diagrams involved in the $f_0(980)$ production in the $\chi_{c1} \to \pi^0 f_0(980) \to \pi^0\pi^+\pi^-$ reaction: rescattering of (a) $\pi^0\eta$, (b) $K^+K^-$, (c) $K^0\bar{K}^0$.

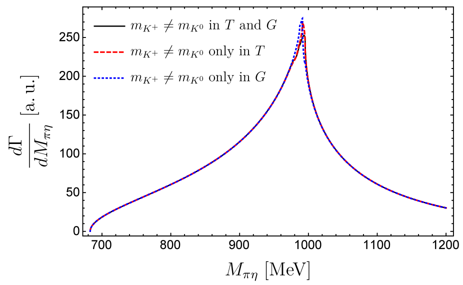


**Figure 3.** Invariant mass distribution of $\pi^0\eta$ in the $\chi_{c1} \to \pi^0 a_0(980) \to \pi^0\pi^0\eta$ reaction. (See text for explanations).

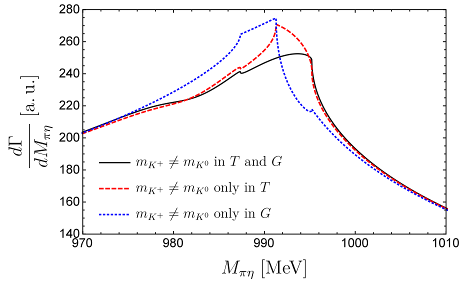


**Figure 4.** Zoom around the $a_0(980)$ peak in the invariant mass distribution of $\pi^0\eta$ in the $\chi_{c1} \to \pi^0 a_0(980) \to \pi^0\pi^0\eta$ reaction. (See text for explanations).

For the case of $a_0(980)$ production, the invariant mass distribution is written as

$$\frac{d\Gamma}{dM_{\rm inv}(\pi^0\eta)} = \frac{1}{(2\pi)^3}\frac{1}{4M^2_{\chi_{c1}}}\frac{1}{3}p^2_{\pi^0}\; p_{\pi^0}\; \tilde{p}_\eta |\tilde{t}_{\pi^0\eta}|^2, \tag{6}$$

where

$$p_{\pi^0} = \frac{\lambda^{1/2}(M^2_{\chi_{c1}}, m^2_{\pi^0}, M^2_{\rm inv}(\pi^0\eta))}{2M_{\chi_{c1}}}, \quad \tilde{p}_\eta = \frac{\lambda^{1/2}(M^2_{\rm inv}(\pi^0\eta), m^2_{\pi^0}, m^2_\eta)}{2M_{\rm inv}(\pi^0\eta)}. \tag{7}$$

On the other hand, for the case of $f_0(980)$ production, the invariant mass distribution reads

$$\frac{d\Gamma}{dM_{\rm inv}(\pi^+\pi^-)} = \frac{1}{(2\pi)^3}\frac{1}{4M^2_{\chi_{c1}}}\frac{1}{3}p^2_{\pi^0}\; p_{\pi^0}\; \tilde{p}_{\pi^+} |\tilde{t}_{\pi^+\pi^-}|^2, \tag{8}$$

with

$$p_{\pi^0} = \frac{\lambda^{1/2}(M^2_{\chi_{c1}}, m^2_{\pi^0}, M^2_{\rm inv}(\pi^+\pi^-))}{2M_{\chi_{c1}}}, \quad \tilde{p}_{\pi^+} = \frac{\lambda^{1/2}(M^2_{\rm inv}(\pi^+\pi^-), m^2_{\pi^+}, m^2_{\pi^-})}{2M_{\rm inv}(\pi^+\pi^-)}. \tag{9}$$

## 3 Results

We show in Figs. 3 and 4 the invariant mass distribution $d\Gamma/dM_{\pi\eta}$ from Eq. (6), where the shape of the $a_0(980)$ is clear. The solid line represents the case where different masses for the charged and neutral kaons are used in the propagators inside the $T$ matrix and also in the first rescattering loops $G_{K^+K^-}$ and $G_{K^0\bar{K}^0}$. The dashed line is the case where the different masses are used only inside the $T$ matrix and the dotted line only in the first rescattering loops $G_{K^+K^-}$ and $G_{K^0\bar{K}^0}$. As we can see, there is only a small difference in the curves around the $K\bar{K}$ threshold. By looking closer into this region, one can see in Fig. 4 that in the three curves there is a small cusp effect in $M_{\pi\eta}$ at $2\,m_{K^+}$ and $2\,m_{K^0}$; and in the dashed and dotted line, where the isospin-average kaon mass $\langle m_K\rangle$ is also used, the $a_0(980)$ peak appears at $2\,\langle m_K\rangle$.

For the isospin-breaking production of $f_0(980)$, we present in Fig. 5 the invariant mass distribution $d\Gamma/dM_{\pi\pi}$ from Eq. (8). We have a narrow peak around the threshold of $K\bar{K}$

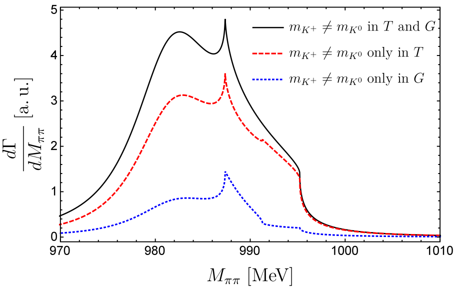


**Figure 5.** Invariant mass distribution of $\pi^+\pi^-$ in the $\chi_{c1} \to \pi^0 f_0(980) \to \pi^0\pi^+\pi^-$ reaction. (See text for explanations).

similar to the results of the literature [10]. We can see clearly the effect of the two different thresholds, at $M_{\pi\pi}$ equal to $2\,m_{K^+}$ and $2\,m_{K^0}$, and for the case of the dashed and dotted lines, we also see the cusp effect at $M_{\pi\pi}$ equal to $2\,\langle m_K \rangle$.

Finally, we show in Table 1 the results of the $a_0(980) - f_0(980)$ mixing in the $\chi_{c1} \to \pi^0\pi^+\pi^-$ and $\chi_{c1} \to \pi^0\pi^0\eta$ reactions. We calculate the ratio of the $\Gamma(\chi_{c1} \to \pi^0\pi^+\pi^-) / \Gamma(\chi_{c1} \to \pi^0\pi^0\eta)$. Here we integrate the whole mass distrubution of the $M_{\pi\eta}$ from $m_\pi + m_\eta$ up to 1200 MeV and also from 885 MeV to 1085 MeV.

**Table 1.** Comparison between experiment and theoretical results for the $a_0(980) - f_0(980)$ mixing in the $\chi_{c1} \to \pi^0\pi^+\pi^-$ and $\chi_{c1} \to \pi^0\pi^0\eta$ reactions.

| | $\Gamma(\chi_{c1} \to \pi^0\pi^+\pi^-) / \Gamma(\chi_{c1} \to \pi^0\pi^0\eta)$ |
|---|---|
| BESIII [11] | $(0.31 \pm 0.16(\text{stat}) \pm 0.14(\text{sys}) \pm 0.03(\text{para}))\%$ |
| $m_{K^+} \neq m_{K^0}$ | $M_{\pi\eta} \in [885, 1085]$ MeV |
| in $T$ and $G$ | 0.26 % |
| only in $T$ | 0.19 % |
| only in $G$ | 0.05 % |
| $m_{K^+} \neq m_{K^0}$ | $M_{\pi\eta} \in [m_\pi + m_\eta, 1200\ \text{MeV}]$ |
| in $T$ and $G$ | 0.17 % |
| only in $T$ | 0.12 % |
| only in $G$ | 0.03 % |

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