

Radiative pion capture in ${}^2\text{H}$, ${}^3\text{He}$ and ${}^3\text{H}$

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Abstract. We investigate the $\pi^- + {}^2\text{H} \rightarrow \gamma + n + n$, $\pi^- + {}^3\text{He} \rightarrow \gamma + {}^3\text{H}$, $\pi^- + {}^3\text{He} \rightarrow \gamma + n + d$, $\pi^- + {}^3\text{He} \rightarrow \gamma + n + n + p$ and $\pi^- + {}^3\text{H} \rightarrow \gamma + n + n + n$ capture reactions using realistic two-nucleon and three-nucleon potentials and the single nucleon Kroll-Ruderman-type transition operator. We obtain predictions for the total capture rates for all these processes, calculating rigorously the initial and final nuclear states.

1 Introduction

Experiments on pion capture were started in 1951 by Panofsky, Aamodt and Hadley [1], who measured the ratio of pionic ($\pi^- + p \rightarrow n + \pi^0$) to radiative ($\pi^- + p \rightarrow n + \gamma$) capture of stopped negative pions in hydrogen. Consecutive measurements provided information on the pion properties and delivered photon spectra from radiative pion capture on different nuclei. Many such experiments were performed at the Lawrence Radiation Laboratory in Berkeley as well as at the Swiss Institute for Nuclear Research (later the Paul Scherrer Institute) and their results were analyzed with the theoretical models available at that time. All the early experimental and theoretical work prior to January 1976 was described in Ref. [2].

In this contribution we restrict ourselves to reactions with two and three nucleons. The $\pi^- + {}^2\text{H} \rightarrow \gamma + n + n$ reaction has attracted a lot of attention. In 1951 Watson and Stuart [3] showed that the corresponding photon spectrum is very sensitive to the low-energy properties of the neutron-neutron interaction. Many further theoretical efforts and experiments have led to our present day knowledge about the neutron-neutron scattering length a_{nn} . This work is summarized in the reviews by Ślaus, Akaishi, and Tanaka [4] and by Gårdestig [5].

Studies of the (not only radiative) negative pion capture in the three-nucleon bound states were pioneered in 1952 by Messiah [6]. Further calculations [7–15] often concentrated on various branching ratios, because the measurements [16–20] yielded no absolute capture rates but relative probabilities of various reactions and, in some cases, photon spectra.

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Phillips and Roig [13, 14] were the first to calculate radiative breakup rates for the $\pi^- + {}^3\text{He} \rightarrow \gamma + n + d$, $\pi^- + {}^3\text{He} \rightarrow \gamma + n + n + p$ and $\pi^- + {}^3\text{H} \rightarrow \gamma + n + n + n$ processes. They treated final state interactions among the three nucleons within the Amado model [21, 22], by solving the Faddeev equations with a simple separable s -wave nucleon-nucleon potential. Despite very simplistic dynamical input, Phillips and Roig could describe the shapes of the experimental photon spectra from Refs. [18–20] and the branching ratios given in Ref. [18]. Although many reactions with the few-nucleon systems were later treated within much more advanced theoretical frameworks, actually no calculations for radiative pion capture in trinucleons have been performed with modern realistic nuclear forces.

Recently we have established and cross-checked a theoretical framework for muon capture on the $A \leq 3$ nuclei [23, 24]. We have now adapted our momentum space scheme for corresponding radiative capture reactions and obtain, for the first time, predictions with a consistent treatment of the initial and final nuclear states calculated from realistic two-nucleon and three-nucleon forces.

In Sect. 2 we introduce our single nucleon transition operator and show results for the total capture rates of the $\pi^- + {}^2\text{H} \rightarrow \gamma + n + n$, $\pi^- + {}^3\text{He} \rightarrow \gamma + {}^3\text{H}$, $\pi^- + {}^3\text{He} \rightarrow \gamma + n + d$, $\pi^- + {}^3\text{He} \rightarrow \gamma + n + n + p$, and $\pi^- + {}^3\text{H} \rightarrow \gamma + n + n + n$ reactions. We summarize in Sect. 3.

2 Results

The radiative pion capture process is treated in the same way as muon capture from the lowest atomic orbit. In particular, the kinematics of muon capture and radiative capture is basically the same; only the muon mass has to be replaced by the negative pion mass.

The essential difference comes from the transition operator. We take it in the form of the one-body Kroll-Rudermann operator j_{KR} [25] and start with the nonrelativistic formula from Ref. [2]:

$$j_{KR} = -ie \frac{g_A}{g_V} \frac{1}{f_\pi} \boldsymbol{\epsilon} \cdot \boldsymbol{\sigma} \tau_-, \quad (1)$$

where e , g_A , g_V , f_π , $\boldsymbol{\epsilon}$, $\boldsymbol{\sigma}$ and τ_- are the elementary charge, axial-vector and vector coupling constants, the pion decay constant, the photon polarization vector, the nucleon spin operator and the isospin lowering operator, respectively.

In order to make a connection to our work on muon capture [23, 24] we modify Eq. (1) and obtain

$$j_{KR} = -i \frac{e}{f_\pi} \boldsymbol{\epsilon} \cdot \mathbf{j}_A, \quad (2)$$

where \mathbf{j}_A is the single nucleon axial current from Refs. [23, 26]. Presented here predictions are based on this single nucleon transition operator and concentrate on final state interactions in the nuclear sector. We calculate consistently two-nucleon and three-nucleon bound and scattering states using the AV18 NN potential [27] and the Urbana IX three-nucleon force [28]. To the best of our knowledge, we provide, for the first time, consistent predictions for the total capture rates of the $\pi^- + {}^2\text{H} \rightarrow \gamma + n + n$, $\pi^- + {}^3\text{He} \rightarrow \gamma + {}^3\text{H}$, $\pi^- + {}^3\text{He} \rightarrow \gamma + n + d$, $\pi^- + {}^3\text{He} \rightarrow \gamma + n + n + p$, and $\pi^- + {}^3\text{H} \rightarrow \gamma + n + n + n$ reactions, obtained with realistic nuclear forces. All the rates are listed in Table 1. A detailed discussion of the presented results, information about the corresponding differential capture rates as well as a comparison with earlier theoretical predictions and experimental data can be found in Ref. [29].

Table 1. Total rates Γ in 10^{15} 1/s for radiative pion capture in ${}^2\text{H}$, ${}^3\text{He}$ and ${}^3\text{H}$ obtained with the AV18 [27] nucleon-nucleon potential and the transition operator given in Eq. (2). Results calculated using the plane wave impulse approximation (PW 2NF), with a consistent treatment of the initial and final nuclear states based on two-nucleon forces only (Full 2NF) and, additionally, including the Urbana IX [28] three-nucleon force (Full 2NF+3NF) are presented.

$\pi^- + {}^2\text{H} \rightarrow \gamma + n + n$	Γ_{nn}
PW 2NF	0.318
Full 2NF	0.328
$\pi^- + {}^3\text{He} \rightarrow \gamma + {}^3\text{H}$	$\Gamma_{{}^3\text{H}}$
Full 2NF	2.059
Full 2NF+3NF	2.132
$\pi^- + {}^3\text{He} \rightarrow \gamma + n + d$	Γ_{nd}
PW 2NF	5.201
Full 2NF	2.013
Full 2NF+3NF	1.840
$\pi^- + {}^3\text{He} \rightarrow \gamma + n + n + p$	Γ_{nnp}
PW 2NF	3.816
Full 2NF	0.659
Full 2NF+3NF	0.615
$\pi^- + {}^3\text{H} \rightarrow \gamma + n + n + n$	Γ_{nnn}
PW 2NF	0.117
Full 2NF	0.141
Full 2NF+3NF	0.128

3 Summary and conclusions

Various electromagnetic and weak reactions on the single nucleon, deuteron, ${}^3\text{He}$, ${}^3\text{H}$ and other light nuclei should be ultimately studied within chiral effective field theory. Such a framework can provide consistent two-nucleon and many-nucleon forces as well as nuclear currents comprising one-body and many-body parts. Results of fully converged calculations should then be compared with precise experimental data, to reveal expected connections between these reactions.

In the present contribution, we studied radiative pion capture reactions using traditional nuclear forces (the AV18 nucleon-nucleon potential and the Urbana IX three-nucleon force) and a single-nucleon transition operator. Within this framework we provided the first realistic predictions for the total capture rates.

These calculations, as well as our studies of muon capture [23, 24] or neutrino induced reactions [30], can be extended to deal with more complete dynamical input. The formalism used in the present contribution will be applied to study other pion capture reactions in the future.

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