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Chiral anomaly and strange-nonstrange mixing

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Abstract. As a first step, a simple and pedagogical recall of the η-η' system is presented, in which the role of the axial anomaly, related to the heterochiral nature of the multiplet of (pseudo)scalar states, is underlined. As a consequence, η is close to the octet and η' to the singlet configuration. On the contrary, for vector and tensor states, which belong to homochiral multiplets, no anomalous contribution to masses and mixing is present. Then, the isoscalar physical states are to a very good approximation nonstrange and strange, respectively. Finally, for pseudotensor states, which are part of an heterochiral multiplet (just as pseudoscalar ones), a sizable anomalous term is expected: $\eta_2(1645)$ roughly corresponds to the octet and $\eta_2(1870)$ to the singlet.

1 Introduction

The meson $\eta' \equiv \eta'(958)$ is special: its large mass and its flavor content are strongly influenced by the so-called axial anomaly [1–3] (the classical $U(1)_A$ symmetry of QCD is broken by quantum fluctuations). Roughly speaking, η' corresponds to a flavor singlet, while $\eta \equiv \eta(547)$ to the octet. In Sec. 1, we recall some basic features of the η-η' and we connect them to the *heterochirality* [4, 5] of pseudoscalar states and their chiral partners, the scalar states.

A natural question is if the axial anomaly affects other mesons. Interestingly, it turns out that the axial anomaly does *not* affect the vector states $\omega(782)$ and $\phi(1020)$ and the tensor states $f_2(1270)$ and $f_2'(1525)$ (see Sec. 3): $\omega(782)$ and $f_2(1270)$ are (almost purely) nonstrange and $\phi(1020)$ and $f_2'(1525)$ strange. This fact can be nicely understood by the *homochirality* of the corresponding chiral multiplets, which involve left- and right-handed currents. For homochiral multiplets, no anomalous mixing is realized [4].

Are there other mesons for which the anomaly plays a role? This seems to be the case of pseudotensor mesons (Sec. 4). As shown in the phenomenological study of Ref. [6], the mesons $\eta_2(1645)$ and $\eta_2(1870)$ roughly correspond to octet and singlet states (the mixing angle is similar to the one of η and η'). The pseudotensor mesons belong to a heterochiral multiplet (just as pseudoscalar states), hence one can understand why the axial anomaly is relevant.

2 Pseudoscalar sector

First, we review some features of the pseudoscalar sector. We consider the strange-nonstrange basis $\eta_N = \sqrt{1/2}(\bar{u}u + \bar{d}d)$, $\eta_S = \bar{s}s$ and the octet-singlet basis $\eta_8 = \sqrt{1/6}(\bar{u}u + \bar{d}d - 2\bar{s}s)$, $\eta_0 = \sqrt{1/3}(\bar{u}u +$

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$\bar{d}d + \bar{s}s$). The physical fields $\eta \equiv \eta(547)$ and $\eta' \equiv \eta'(958)$ are a mixture of η_N and η_S (and, similarly, of η_0 and η_8), according to:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_P & \sin\theta_P \\ -\sin\theta_P & \cos\theta_P \end{pmatrix} \begin{pmatrix} \eta_N \\ \eta_S \end{pmatrix} , \begin{pmatrix} \eta_0 \\ \eta_8 \end{pmatrix} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} \\ \sqrt{1/3} & -\sqrt{2/3} \end{pmatrix} \begin{pmatrix} \eta_N \\ \eta_S \end{pmatrix} . \tag{1}$$

The determination of θ_P is one important aspect of the problem.

We now introduce the Lagrangian terms for masses and mixing. The flavour invariant term is simply given by:

$$\mathcal{L}_{P,U(3)} = -\frac{m_P^2}{2}\left(\eta_N^2 + \eta_S^2\right) = -\frac{m_P^2}{2}\left(\eta_0^2 + \eta_8^2\right) . \tag{2}$$

After spontaneous symmetry breaking $m_P^2 \propto (m_u + m_d)/2$, see e.g. Ref. [7]. If only $\mathcal{L}_{P,U(3)}$ is taken into account, one could use η_0-η_8 or η_N-η_S (the octet-singlet choice is mathematically preferable). Next, the fact that the s-quark is more massive than the quarks u and d is taken into account by the Lagrangian

$$\mathcal{L}_{P,S} = -\frac{\delta_{P,S}}{2}\eta_S^2 , \tag{3}$$

with $\delta_{P,S} = 2(m_K^2 - m_\pi^2)$ (m_K and m_π are the kaon and pion masses). If $\mathcal{L}_{U(3)} + \mathcal{L}_S$ is considered, the physical states are η_N (with squared mass m_P^2) and η_S (with squared mass $m_P^2 + \delta_S$). Last, the octet-singlet splitting is parametrized by

$$\mathcal{L}_{P,0} = -\alpha_P \eta_0^2 = -\alpha_P\left(\sqrt{2}\eta_N + \eta_S\right)^2 , \tag{4}$$

where $\alpha_P = \alpha_{P,gg} + \alpha_{P,A}$. Here, $\alpha_{P,gg}$ describes processes with two intermediate transverse gluons ($\bar{n}n \to \bar{n}n$, $\bar{n}n \to \bar{s}s$, etc.). This is a small perturbation. The parameter $\alpha_{P,A}$ represents an effective contribution of the axial anomaly; Eq. (4) with $\alpha_P \simeq \alpha_{P,A}$ was also obtained in e.g. Refs. [8, 9]. If $\mathcal{L}_{P,U(3)} + \mathcal{L}_{P,0}$ is considered, the physical states are η_8 (with squared mass m_P^2) and η_0 (with squared mass $m_P^2 + 2\alpha_P$). Thus, $\mathcal{L}_{P,S}$ and $\mathcal{L}_{P,0}$ lead to different basis, and the question is which one is dominant.

In the full case, one considers $\mathcal{L}_{U(3)} + \mathcal{L}_S + \mathcal{L}_{P,0}$. The pseudoscalar mixing angle θ_P can be calculated by the previous expressions: $\theta_P = -\frac{1}{2}\arctan\left[\frac{4\sqrt{2}\alpha_P}{2(m_K^2 - m_\pi^2 - \alpha_P)}\right]$. Numerically, θ_P varies between $-40°$ and $-45°$ [2, 3, 7, 10]. The mixing is rather large and the states are closer to octet and singlet ones, but the effect of the s-quark is also important. Note, in the limit $\alpha_P = 0$ one gets $\theta_P = 0$ (purely strange and nonstrange states). On the contrary, in the limit $m_K^2 - m_\pi^2 = 0$ ($\delta_{P,S} = 0$) one has $\theta_P = \frac{1}{2}\arctan\left[2\sqrt{2}\right] = 35.3°$, i.e. octet and singlet states, see Eq. (1).

In the recent work of Ref. [4], it was shown that the (pseudo)scalar multiplet is *heterochiral*. Namely, it is described by a matrix Φ (see [7]) which under chiral transformation changes as $\Phi \to e^{-i\alpha}U_L \Phi U_R^\dagger$ (the parameter α refers to $U(1)_A$). The Lagrangian $\mathcal{L}_\Phi^{\text{anomaly}} = -a_A^{(3)}[\det(\Phi) - \det(\Phi^\dagger)]^2$ preserves chiral symmetry but breaks $U(1)_A$ (this is a consequence of the determinant, see also Ref. [5]). This Lagrangian term reduces to Eq. (4) when condensation is considered and quadratic mass terms are isolated. In conclusion, the heterochiral (pseudo)scalar nonet can easily explain the emergence of an anomalous term affecting η and η'.

3 Vector (and tensor) mesons

Next, we consider the isoscalar vector states $\omega(782)$ and $\phi(1020)$. Just as before, one introduces the nonstrange-strange basis $\omega_N = \sqrt{1/2}(\bar{u}u + \bar{d}d)$, $\omega_S = \bar{s}s$ and the octet-singlet basis $\omega_8 = \sqrt{1/6}(\bar{u}u +$

$\bar{d}d - 2\bar{s}s)$, $\omega_0 = \sqrt{1/3}(\bar{u}u + \bar{d}d + \bar{s}s)$, for which Eq. (1) holds (upon, of course, renaming the fields). Also here, we consider three Lagrangians:

$$\mathcal{L}_{V,U(3)} = -\frac{m_V^2}{2}\left(\omega_N^{\mu 2} + \omega_S^{\mu 2}\right) , \mathcal{L}_{V,S} = -\frac{\delta_{V,S}}{2}\omega_S^{\mu 2} , \mathcal{L}_{V,0} = -\alpha_V \omega_0^2 . \tag{5}$$

There is an important difference in the last term. For vector states, the constant $\alpha_V = \alpha_{V,ggg}$ (three-gluon mixing processes, typically small): there is *no* contribution from the axial anomaly, $\alpha_{V,A} = 0$. As a consequence, $\omega(782)$ is basically nonstrange and $\phi(1020)$ strange (mixing angle $\theta_V = -3°$, equation analogous to Eq. (1) [11]). Similarly, for their axial-vector chiral partners it holds that: $f_1(1285)$ is almost purely nonstrange and $f_1(1420)$ purely strange [12].

In Ref. [4] it was discussed why $\alpha_{V,A} = 0$. This is due to the fact that the corresponding chiral multiplets of vector (V_μ) and axial-vector (A_μ) states are *homochiral*. Namely, they enter into the right(left)-handed $R_\mu = V_\mu - A_\mu$ and $L_\mu = V_\mu + A_\mu$, which under chiral symmetry transforms as $L_\mu \longrightarrow U_{\mathrm{L}} R_\mu U_{\mathrm{L}}^\dagger$, $R_\mu \longrightarrow U_{\mathrm{R}} R_\mu U_{\mathrm{R}}^\dagger$ (in both cases, either only U_{L} or U_{R} appears, but no mixed terms). There is no term involving the determinant.

A similar analysis applies to the ground-state tensor mesons, which are also part of an heterochiral multiplet: $f_2(1270)$ is almost purely nonstrange and $f_2'(1525)$ strange, in agreement with the phenomenology [13].

4 Pseudotensor mesons

In the end, we consider the pseudotensor sector. We start from $\eta_{2,N} = \sqrt{1/2}(\bar{u}u + \bar{d}d)$, $\eta_S = \bar{s}s$ and $\eta_{2,8} = \sqrt{1/6}(\bar{u}u + \bar{d}d - 2\bar{s}s)$, $\eta_{2,0} = \sqrt{1/3}(\bar{u}u + \bar{d}d + \bar{s}s)$. The Lagrangian terms read

$$\mathcal{L}_{PT,U(3)} = -\frac{m_{PT}^2}{2}\left(\eta_{2,N}^{\mu\nu,2} + \eta_{2,S}^{\mu\nu,2}\right) , \mathcal{L}_{PT,S} = -\frac{\delta_S}{2}\eta_{2,S}^{\mu\nu 2} , \mathcal{L}_{PT,0} = -\alpha_{PT}\eta_{2,0}^{\mu\nu,2} \tag{6}$$

Here, $\alpha_{PT} = \alpha_{PT,gg} + \alpha_{PT,A}$, and the latter quantity is expected to be sizable, hence the anomaly is potentially large. This is due to the fact that the corresponding chiral multiplet $\Phi_{\mu\nu}$ is heterochiral, just as for pseudoscalar mesons. In fact, under chiral transformations it transforms as $\Phi_{\mu\nu} \to \mathrm{e}^{-\mathrm{i}\alpha} U_{\mathrm{L}} \Phi_{\mu\nu} U_{\mathrm{R}}^\dagger$ [4]. The corresponding Lagrangian term $\mathcal{L}_{\Phi_{\mu\nu}}^{\text{anomaly}} \propto (\varepsilon^{ijk}\varepsilon^{i'j'k'}\Phi^{ii'}\Phi^{jj'}\Phi_{\mu\nu}^{kk'} - h.c.)^2$ is chirally symmetric but breaks $U(1)_A$ (it is an extension of the determinant) and reduces to $\mathcal{L}_{PT,0}$ when the condensation of Φ is considered. The physical fields $\eta_2(1645)$ octet, $\eta_2(1870)$ are:

$$\begin{pmatrix}\eta_2(1645)\\ \eta_2(1870)\end{pmatrix} = \begin{pmatrix}\cos\theta_{PT} & \sin\theta_{PT}\\ -\sin\theta_{PT} & \cos\theta_{PT}\end{pmatrix}\begin{pmatrix}\eta_{2,N}\\ \eta_{2,S}\end{pmatrix} , \tag{7}$$

with $\theta_{PT} \simeq -\frac{1}{2}\arctan\left[\frac{4\sqrt{2}\alpha_{PT}}{2(m_{K_2(1770)}^2 - m_{\pi_2(1660)}^2 - \alpha_{PT})}\right]$. According to the phenomenological study of Ref. [6], $\theta_{PT} \simeq -42°$: a surprisingly large and negative mixing (similar to the pseudoscalar sector) is realized, a fact that can be nicely explained by the axial anomaly being important in this (heterochiral) sector.

5 Conclusions

We have studied the role of the axial anomaly for light mesons. For the so-called “heterochiral” multiplets [4] (pseudoscalar and pseudotensor states), a large strange-nonstrange mixing is expected (a known fact for η and η', some experimental evidence exists for pseudotensor mesons [6]). On the contrary, (axial-)vector and tensor mesons are “homochiral” and the anomaly does not affect the mixing: the isoscalar states are (almost) nonstrange and strange, respectively. Ongoing experimental activity at the JLab (e.g. Ref. [14]) can help to shed light on resonances between 1-2 GeV and hence on the role of the axial anomaly.

As recently shown, the axial anomaly can also be relevant in the baryonic sector. In particular, it can explain the large decay $N(1535) \to N\eta$ [15] and contribute to pion-nucleon scattering [16]. Moreover, the enigmatic pseudoscalar glueball [9] is also related to the axial anomaly and can be studied in the future.

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