Three-nucleon bound state calculations using the three dimensional formalism

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Abstract. The traditional method of carrying out few-nucleon calculations is based on the angular momentum decomposition of operators relevant to the calculation. Expressing operators using a finite-sized partial wave basis enables the calculations to be carried out using a small amount of numerical work. Unfortunately, certain calculations that involve higher energies or long range potentials, require including a large number of partial waves in order to get converged results. This is problematic because such an approach requires a numerical implementation of heavily oscillating functions. Modern computers made it possible to carry out few-nucleon calculations without using angular momentum decomposition and instead to work directly with the three dimensional degrees of freedom of the nucleons. In this paper we briefly describe the, so called *3D* approach and present preliminary results related to the ^{3}He bound state obtained within this formalism.

1 Introduction

The starting point of our bound state calculations is the Faddeev equation, for the Faddeev component of the three-nucleon (3N) wave function $|\,\psi\rangle$:

$$|\,\psi\rangle = \check{G}_0(E)\left(\check{V}_1^{2\mathrm{N}} + \check{V}_1^{3\mathrm{N}}\right)\left(1 + \check{P}\right)|\,\psi\rangle \tag{1}$$

where E is the bound state energy, $\check{G}_0(E)$ is the free propagator, $\check{V}_i^{2\mathrm{N}}$ is the two nucleon (2N) potential acting between particles j and k ($i \neq j \neq k \neq i$) and $\check{V}_i^{3\mathrm{N}}$ is a part of the 3N potential that is symmetric with respect to the exchange of particles j and k ($i \neq j \neq k \neq i$). Finally $\check{P}$ is an operator composed from operators $\check{P}_{ij}$ that exchange particles i and j:

$$\check{P} = \check{1} + \check{P}_{12}\check{P}_{23} + \check{P}_{13}\check{P}_{23}.$$

Using this operator, the full 3N bound state $|\,\Psi\rangle$ can be reconstructed from the Faddeev component using:

$$|\,\Psi\rangle = (\check{1} + \check{P})\,|\,\psi\rangle. \tag{2}$$

Eq. (1) is solved using the iterative Arnoldi algorithm (see e.g. [1]) and the same operator form of the Faddeev component as was used in [2]. In this form $|\,\psi\rangle$ and $|\,\Psi\rangle$ are defined by scalar functions $\phi^i_{tT}(p, q, \hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{q}})$ of the Jacobi momenta $\boldsymbol{p}$ and $\boldsymbol{q}$. The complete set of these

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scalar functions $\{\phi^i_{tT}(p, q, \hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{q}})\}$ for each two-body subsystem isospin t, total isospin T and $i = 1 \ldots 8$, spans a linear space and we will refer to vectors from this space using Greek letters $\alpha, \beta, \phi, \ldots$. In practical numerical calculations these vectors are represented as multidimensional arrays and the implementation of the Arnoldi algorithm contains parallelized subroutines that add vectors, apply linear operators and compute scalar products. In our current calculations we use two values of the total isospin $T = \frac{1}{2}, \frac{3}{2}$ as opposed to [2] where we used only $T = \frac{1}{2}$.

The possibility to represent the 3N Faddeev component $|\ \psi\rangle$ as a vector ϕ in a space spanned by the scalar functions from the operator form allowed us to rewrite (1) as:

$$\phi = \check{A}(E)\phi, \tag{3}$$

where $\check{A}$ is an energy dependent linear operator acting in the space spanned by functions $\phi^i_{tT}(p, q, \hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{q}})$. Eq. (3) is solved by trying various values of the energy $E_1, E_2, \ldots$, solving the eigenequation:

$$v_j \beta_j = \check{A}(E_j)\beta_j \tag{4}$$

and searching for an energy E_j such that $v_j = 1$. Once such an energy is found for the scalar function $\beta = \beta_j$ then $E = E_j$, $\phi = \beta$ and the scalar functions γ for the full bound state $|\ \Psi\rangle$ can be calculated using another operator $\check{B}$ that corresponds to the permutations $\check{1} + \check{P}$:

$$\gamma = \check{B}\beta.$$

2 Results for ^{3}He

In this section we present preliminary results for the ^{3}He bound state obtained using the 3D approach. In our calculations we used first generation NNLO 2N [6] and 3N [7] potentials (for more information see e.g. [3–6], [7–9]). For the Coulomb interaction we chose a screened potential from [10].

In Fig. 1 we show the convergence of the calculated bound state energy with increasing screening radius. At screening radii greater then $R \approx 10$ [fm] the bound state energy converges to -7.989 [MeV]. In Fig. 2 we plot a few scalar functions for the bound state of ^{3}He.

It should be noted that the general operator form of the 3N force has already been developed in [11], however the numerical performance of our bound state calculations would not directly benefit from using this form. Instead we use here our newly developed method of performing multidimensional integrations [12] related to the 3N force. This new method greatly reduces the numerical load of the bound state calculations by allowing Eq. (4) to be solved efficiently for different values of the energy.

3 Summary

Our preliminary results for the ^{3}He bound state are promising and prove the validity of the 3D approach in calculations with the Coulomb interaction. A very similar approach has been used to calculate the ^{3}H bound state making it possible to calculate observables related to the beta decay of the triton. Current work is concentrated on investigating the convergence of the calculations with an increasing number of grid points used in multidimensional integrations and aims at testing various models of 2N and 3N interactions.

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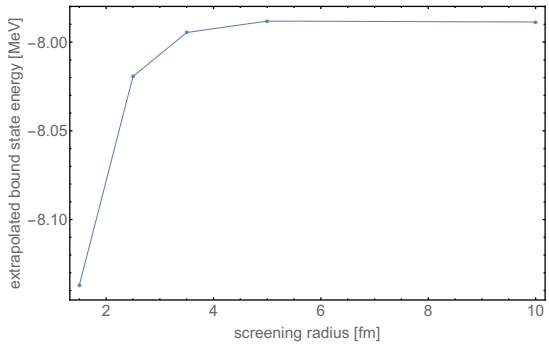

Figure 1. Convergence of the ^{3}He bound state energy with the screening radius R. The bound state energy converges to $E \approx -7.989$ [MeV] above $R = 10$ [fm]. For the screened Coulomb potential used in the calculation [10], the potential goes to 0 at $3R = 30$ [fm]. We used first generation NNLO 2N [6] and 3N [7] potentials and simplified the preliminary calculations by using only the 2N neutron-proton version of the interaction (also for the proton-proton pair).

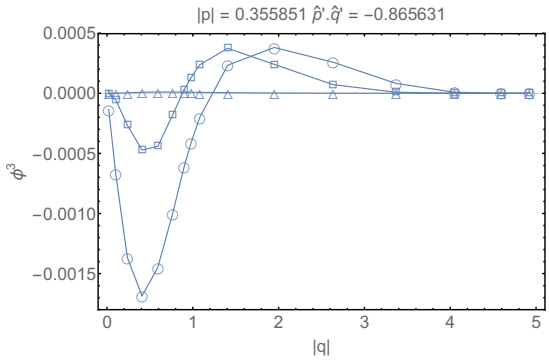

Figure 2. Cross section through selected scalar functions that define the bound state of ^{3}He. Circles, squares and triangles correspond to $T = \frac{1}{2}, t = 0$; $T = \frac{1}{2}, t = 1$ and $T = \frac{3}{2}, t = 1$ respectively.

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