Instabilities of Collective Neutrino Oscillations Induced by Non-standard Neutrino Interactions

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Abstract. We study the effect of non-standard neutrino interactions (NSIs) on the growth of instabilities in neutrino energy spectra of a core-collapse supernova for different neutrino intensities and/or types of NSIs, notably including the exotic neutrino magnetic moment. Although it is usually attested that instabilities virtually smear out all potentially observable signatures, we show that, instead, there are regimes in which they act as a magnifying glass, bringing tiny effects to the eye of the observer.

1 Introduction

Compact astrophysical objects, in particular, protoneutron stars born in supernova explosions, provide a scene to test physics at its extreme, in particular, physics beyond the Standard Model (BSM) [1]. Luckily, core-collapse supernovae typically deposit the most of the explosion energy into neutrinos, which lets one probe exotic properties of this elusive particle. Moreover, after thirty years since the first observation of supernova neutrinos from SN1987A, experimental techniques have been substantially improved. For instance, the JUNO detector that should start operating in a couple of years will be able to collect over 5000 neutrino+antineutrino events for an explosion in the Milky Way, also offering an unprecedented neutrino energy resolution $3\% / \sqrt{E/\text{MeV}}$ and a low energy threshold of 1.8 MeV in the dominant, inverse beta decay channel [2]. In this connection, it is very interesting to study the effect of BSM neutrino physics near a protoneutron star on the neutrino energy spectra observed at infinity.

In the present paper, we focus on the effect of a tiny neutrino magnetic moment $\mu$ on the neutrino collective oscillations taking place near the neutrinosphere of a protoneutron star. Our study is inspired by the fact that the (yet unknown) Dirac or Majorana nature of neutrino affects the properties of the magnetic moment matrix, e.g., in the Majorana case, only transition (flavor-changing) magnetic moments are allowed. Moreover, in two recent papers [3], an ultrahigh sensitivity of the neutrino spectra to $\mu$ in the Majorana case was claimed, at least at the level of $10^{-20} \mu_B$ (cf. the present experimental constraints $\mu \lesssim 10^{-15} \mu_B$ or even weaker [4]), which could probably provide a solid opportunity to probe the neutrino nature. Such an ultrahigh sensitivity virtually suggests that interaction with the magnetic moment triggers some instabilities (which are, in general, known to be present in collective oscillations, see, e.g., [5–7]). We carry out a reanalysis of these results, quantifying the...
development of the magnetic moment signatures in neutrino flavor-energy spectra with the distance from the center of the star. It turns out that, despite Dirac and Majorana neutrinos lead to similar but different instability growth patterns, the overall effect of the magnetic moment at the “spatial infinity” remains modest in both cases, orders of magnitude lower than the one reported in Ref. [3]. In fact, the difference stems from a difference between the Hamiltonian from Ref. [3] and the one derived by us, the former virtually expressing a specific type of neutrino non-standard interaction that triggers a huge instability leading to quite observable spectral splits [3].

2 The Model

To study the effect of a nonzero magnetic moment on the evolution of the neutrino flavor spectra, we resort to the single-angle scheme and a two-flavor model outside the neutrinosphere, i.e., the last scattering surface for neutrinos, whose radius is set to $r_{\text{NS}} = 50$ km. We focus primarily on the growth of instabilities due to the nonlinear neutrino self-interaction, thus, wave packet separation effects and the effect of nonzero trajectory curvature are left beyond our setup. Majorana neutrinos with momentum $p$ are described by a density matrix $\rho_{\text{M}}(p; r)$, whose 4 rows/columns correspond to two neutrino and two antineutrino flavors $e, x, \bar{e}, \bar{x}$; in the Dirac case, neutrino-antineutrino transitions are forbidden and we use two matrices $\rho_D(p; r), \bar{\rho}_D(p; r)$, describing neutrino flavors/helicities $e+, x+, e-, x-$ and their antineutrino counterparts $\bar{e}-, \bar{x}-, \bar{e}+, \bar{x}+$, respectively. At the neutrinosphere, the density matrices are fixed to the Fermi–Dirac distributions $s_f(E)$ with different temperatures for different flavors $f$, with $E \approx p$ being the neutrino energy (see the upper pane of Fig. 1 or Ref. [3])

$$\rho_{\text{M}}(p; r_{\text{NS}}) = \text{diag}(s_e(p), s_x(p), s_{\bar{e}}(p), s_{\bar{x}}(p)), \quad \rho_D(p; r_{\text{NS}}) = \text{diag}(s_e(p), s_x(p), 0, 0), \quad \bar{\rho}_D(p; r_{\text{NS}}) = \text{diag}(s_{\bar{e}}(p), s_{\bar{x}}(p), 0, 0). \quad (1)$$

Within the single-angle scheme, evolution of the density matrices with respect to the only radial parameter $r$ is given by the Heisenberg equation(s)

$$\partial_r \rho_{\text{M},D}(p; r) = i[H_{\text{M},D}(p; r), \rho_{\text{M},D}(p; r)], \quad \partial_r \bar{\rho}_D(p; r) = i[\bar{H}_D(p; r), \bar{\rho}_D(p; r)]; \quad (3)$$

$$H_{\text{M}} = \frac{\Delta m^2}{4p} \begin{pmatrix} \bar{\mu} & 0 \\ 0 & \bar{\mu} \end{pmatrix} + G_F \sqrt{2} \begin{pmatrix} \sqrt{\gamma} & 0 \\ 0 & -\sqrt{\gamma} \end{pmatrix} - B_{\text{M}} \begin{pmatrix} 0 & m_{\text{M}} \\ m_{\text{M}} & 0 \end{pmatrix} + H_{\text{M}}^\text{self}; \quad (4)$$

$$H_D = \frac{\Delta m^2}{4p} \begin{pmatrix} \bar{\mu} & 0 \\ 0 & \bar{\mu} \end{pmatrix} \pm \frac{\Delta m^2}{2p} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + B_{\text{D}} \begin{pmatrix} 0 & m_{\text{D}} \\ m_{\text{D}} & 0 \end{pmatrix} + \begin{pmatrix} H_{\text{D}}^\text{self} \\ -H_{\text{D}}^\text{self} \end{pmatrix}; \quad (5)$$

$$H_{\text{M}}^\text{self}(p; r) = G_F \sqrt{2} n_e(r) \int \text{d}p' \left[ \text{tr}(\rho_{\text{M}}(p'; r)G)G + \left(\rho_{\text{M}}(p'; r) - \rho_{\text{M}}^T(p'; r)\right)^T \right]. \quad (6)$$

$$H_{\text{D}}^\text{self}(p; r) = G_F \sqrt{2} n_e(r) \int \text{d}p' \left[ \text{tr}(\rho_{\text{D}} - \rho_{\text{D}}^T)p'p + \rho_{\text{D}}(p'; r) - \rho_{\text{D}}^T(p'; r) \right] \mathcal{P}. \quad (7)$$

$$\bar{\mu} \equiv \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}, \quad \sqrt{\gamma} \equiv \begin{pmatrix} m_e - m_\mu/2 & 0 \\ 0 & m_\mu/2 \end{pmatrix}, \quad m_{\text{M}} = \begin{pmatrix} 0 & i\mu \\ -i\mu & 0 \end{pmatrix}, \quad m_{\text{D}} = \begin{pmatrix} \mu_{ee} & \mu_{ex} \\ \mu_{xe} & \mu_{xx} \end{pmatrix}. \quad (8)$$

In the above equations, the notation is as follows. A transformation $\Xi \rightarrow \Xi^c = C\Xi C$, with $C \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, swaps neutrino and antineutrino flavors. Another matrix transformation used

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1It is worth mentioning that, in principle, decoherence effects due to wave packet separation can also contribute to the observable neutrino spectra [8]. Regarding the Thomas precession of the neutrino spin, an estimation shows that in the conditions discussed, it is much smaller than the Larmor precession included in our consideration.
above is $\Xi \rightarrow \Xi' \equiv \frac{\Xi + \gamma G}{2}$, where $G \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The helicity projector $P \equiv \frac{I \pm G}{2}$. The external fields affecting the oscillations are the number densities $n_{e,n}(r)$ of electrons and neutrons (the proton number density $n_p = n_e$ due to the assumed electroneutrality of the medium) and the component $B_z(r)$ of the magnetic field transversal to the neutrino trajectory. Finally, the neutrino mass/mixing parameters are $\Delta m^2 = \pm 2.5 \times 10^{-3} \text{eV}^2$ (normal/inverted hierarchy) and $\theta = 9^\circ$. The above effective Hamiltonians have been rederived by us using the standard approach treating the neutrino-neutrino interaction in the mean-field fashion; notably, $H_M$ does not coincide in the self-interaction term with the one given in Ref. [3] (a detailed derivation of effective Hamiltonians (4), (5) will be given elsewhere [9]). It is also worth noting that in the absence of the magnetic moment(s), both the Dirac and the Majorana Hamiltonians become block-diagonal and so do the density matrices $\rho_M$, $\rho_D$, $\bar{\rho}_D$, meaning that oscillations do not mix different neutrino helicities. As a result, if $\mu = 0$, the Dirac and the Majorana scenarios lead to the same evolution of neutrino spectra.

In our analysis, we took the diagonal Dirac magnetic moments $\mu_{ee} = \mu_{xx} = 0$ and the transition magnetic moment $\mu_{ex} = \mu$ up to $10^{-15} \mu_B$; the magnetic field was assumed to be a dipole field with the strength of $10^{12} \text{G}$ at the neutrinosphere. The effective neutrino number density $n_e(r)$ depends on the total neutrino luminosity $I$ and contains a geometric factor accounting for the non-pointlike neutrino source [3, 10]. Finally, it is the luminosity that controls the nonlinear term in the evolution equation, and thus the growth of instabilities; we made our simulations for $I = 10^{52} - 10^{56} \text{sec}^{-1}$.

### 3 The Simulation

As mentioned above, we numerically evolved the neutrino flavor-energy spectra $s_f(E; r)$ from the neutrinosphere $r = r_{NS}$ (see the upper pane in Fig. 1) to the distance $r = 400$ km where the neutrino self-interaction gives way to other effects not leading to instabilities (e.g., the MSW resonance occurring at larger distances as a result of competition between the matter and the vacuum terms in the Hamiltonian). For distances smaller than 400 km, generally speaking, it is the interplay between the matter and the self-interaction terms that shapes the effects taking place.

In order to quantify the effect of a small magnetic moment on the neutrino spectra, we introduce a spectral residual $\epsilon(r) = \frac{1}{2} \left( \sum_f \int \left| s^{(0)}_f(E; r) - s^{(0)}_f(E; r) \right| dE \right) / \left( \sum_f \int \left| s^{(0)}_f(E; r) \right| dE \right)$, $f = e, x, \bar{e}, \bar{x}$, where $s^{(0)}_f(E; r)$ is the neutrino spectral density calculated for the magnetic moment $\mu$. The above residual is nothing but a relative deviation of the neutrino spectra from the zero magnetic moment case.

Typical flavor-energy spectra we found are depicted in the lower pane of Fig. 1 (the upper pane shows the initial spectra at $r = r_{NS}$). As usual (see, e.g., [10]), the two hierarchies lead to essentially different spectral swaps/splits. For small neutrino luminosities $I \leq 10^{52} \text{sec}^{-1}$, the oscillations in the region in question are strongly suppressed by the matter potential. The interaction with the magnetic moment always remains feeble, but it still has a chance to show up if its effect is magnified by an exponentially growing instability induced by the nonlinear self-interaction term. Indeed, a typical growth pattern example of such an instability is presented in Fig. 2, which clearly features a region of exponential growth. However, this growth ends in a ’saturation’ not resulting in any significant changes in the neutrino spectra. The latter, saturated values are the main goal of our study.

Dependence of the spectral residual $\epsilon(r)$ at $r = 400$ km on the luminosity $I$ of the star and the neutrino magnetic moment $\mu$ is shown in Figs. 3, 4 for Dirac and Majorana neutrinos, respectively. In these figures, one observes that at luminosities below a certain critical value
Figure 1. Initial neutrino flavor-energy spectra at the neutrinosphere (above) and the final ones at \( r = 400 \) km (left/right below for the normal/inverted hierarchy). The evolution is calculated for \( I = 10^{54} \text{sec}^{-1} \) in the absence of the magnetic moment.

Figure 2. A typical example of the development of an instability triggered by the neutrino magnetic moment, in terms of the spectral residual \( \varepsilon(r) \). The figure shows the Dirac neutrino/normal hierarchy case for the luminosity \( I = 10^{55} \text{sec}^{-1} \).

\( \sim 3 \times 10^{54} \text{sec}^{-1} \), the residual depends only on the magnetic moment, and is roughly proportional to \( \mu^2 \). However, for supercritical luminosities, a significant enhancement of the residual takes place, apparently due to the nonlinear effects introduced by the neutrino self-action. Nevertheless, the value of the residual hardly reaches several thousandths, which means that experimental observation of the signatures of the magnetic moment below \( 10^{-15} \mu_B \) is barely feasible. It is worth mentioning here that the Hamiltonian derived in Ref. [3] for Majorana neutrinos leads to residuals up to \( \varepsilon \sim 0.1 \) [11], which seems, in contrast, quite observable given the typical expected neutrino event numbers for a SN explosion [2].

Also, it should be noted that the effect of the nonzero magnetic moment is somewhat larger for the inverse mass hierarchy for both types of neutrinos. The neutrino type, however, has little effect on the spectra observed far from the neutrinosphere, even though, for the Dirac neutrinos, the effect of the magnetic moment is slightly more pronounced than in the Majorana case, especially for the normal mass hierarchy. A deeper search and study of other types of non-standard neutrino interactions that could lead to observable saturation values of the spectral residual (beyond the one reported in Ref. [3]) is in progress now.
Figure 3. Spectral residual $\varepsilon(r)$ at $r = 400$ km for different neutrino luminosities $I$ and magnetic moments $\mu$. Dirac neutrinos, normal/inverted hierarchy (left/right).

Figure 4. Spectral residual $\varepsilon(r)$ at $r = 400$ km for different neutrino luminosities $I$ and magnetic moments $\mu$. Majorana neutrinos, normal/inverted hierarchy (left/right).

Acknowledgments

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References