Effective Potential Formalism at Finite Temperature in Dual QCD and Deconfinement Phase Transition

Garima Punetha[1,a] and H. C. Chandola[1]

[1] *Center of Advanced Study, Department of Physics, Kumaun University Nainital-263001, India*

Abstract. We study the pure-gauge QCD phase transition at finite temperatures in the dual QCD theory, an effective theory of QCD based on the magnetic symmetry. We formulate the effective thermodynamical potential for finite temperatures using the path-integral formalism in order to investigate the properties of the pure-gauge QCD vacuum. Thermal effects bring a first-order deconfinement phase transition.

1 Introduction

One of the crucial areas of high energy physics research is to examine the study of Quantum Chromodynamics (QCD) [1, 2], contemplated as the fundamental theory of quarks and gluons. In the weak coupling regime of QCD, the perturbative calculations for deep inelastic scattering agree well with experimental data. Nevertheless, in the infrared regime, the description of QCD vacuum and non-perturbative processes yet remains as evident confront in the establishment of QCD as a local quantum field theory. One of its utmost unusual distinctive speculation is that at sufficiently high temperature (T) or chemical potential (μ_B), QCD is believed to be in Quark Gluon Plasma (QGP) phase by virtue of what color charges are screened preferably than confined [3–5]. Exploring the aforesaid new states of matter under extreme conditions is necessary quest and such an one concisely prevail the universe at about a few microseconds after the Big Bang. Nowadays the hottest matter has been reproduced recurrently in laboratory by heavy-ion collisions [6–10] of well as at the Large Hadron Collider (LHC) [11–16]. Moreover, one of the insight which have been put onward in the recent past is that QGP properties may be obtained by a magnetic component and such magnetic component has been related to thermal abelian monopoles disappearing from the magnetic condensate that are assumed to induce color confinement at low temperatures. In this paper, we formulate the effective thermodynamical potential for finite temperatures using the path-integral formalism in order to investigate the properties of the pure-gauge QCD vacuum at finite temperatures using dual QCD formulation based on magnetic symmetry.

2 SU(3) Dual QCD Formulation

The formulation involves imposing the magnetic symmetry as an internal isometry H admitting some additional Killing vector fields ($\hat{m}$) with the Killing condition $\mathcal{L}_{\xi_i} g_{AB} = 0$, which are internal such

[a] e-mail: garimapunetha@gmail.com

that H is a Cartan's subgroup of G and commutes with it satisfying the canonical commutation relation, $[\xi_i, \xi_j] = f_{ij}^{\ k}\,\xi_k$ [17, 18]. For the case of realistic $SU(3)$ color gauge group we consider G as the simple non-Abelian $SU(3)$ group and H as the little group $U(1) \otimes U'(1)$, then the homotomy $\Pi_2(G/H) \rightarrow \Pi_2(SU(3)/U(1) \otimes U'(1))$ is completely determined in terms of magnetic Killing vector $\hat{m}$ and the monopoles thus emerge as the topological charges. The killing vector $\hat{m}$ automatically selects another $\hat{m}'$ obtained by the symmetric product of $\hat{m}$,

$$D_\mu \hat{m}' = 0, \tag{1}$$

$$\hat{m}' = \sqrt{3}\hat{m} * \hat{m}. \tag{2}$$

The most general gauge potential in $SU(3)$ QCD which satisfies the above constraints may be written as,

$$\mathbf{W}_\mu = A_\mu \hat{m} + A'_\mu \hat{m}' - g^{-1}(\hat{m} \times \partial_\mu \hat{m}) - g^{-1}(\hat{m}' \times \partial_\mu \hat{m}'), \tag{3}$$

where

$$A_\mu = \hat{m}.\mathbf{W}_\mu, \;\; A'_\mu = \hat{m}'.\mathbf{W}_\mu,$$

A_μ and A'_μ are the Abelian (color electric) component of $\mathbf{W}_\mu$ along $\hat{m}$ and $\hat{m}'$ respectively while the second part, determined completely by the magnetic symmetry are of topological in origin and are of dual nature [17–24]. Rotating the magnetic vector $\hat{m}$ to a fix time independent direction using the following parametrization in such a way as to exhibit the full homotopy class of the mapping,

$$U = exp\Big[-\beta'(-\frac{1}{2}t_3 + \frac{1}{2}\sqrt{3}t_8)\Big] \times e^{-\alpha t_n} exp\Big[-(\beta - \frac{1}{2}\beta')t_3 e^{-\alpha t_2}\Big], (\beta = n\varphi, \beta' = n'\varphi), \tag{4}$$

where t_i (i= 1, 2,3,......8) are the adjoint representations of the $SU(3)$ generators. This leads to the value of gauge potential with proper choices of A_μ and A'_μ, in the following form,

$$\mathbf{W}_\mu \xrightarrow{U} g^{-1}\Big[\Big((\partial_\mu\beta - \frac{1}{2}\partial_\mu\beta')cos\alpha\Big)\hat{\xi}_3 + \frac{1}{2}\sqrt{3}(\partial_\mu\beta' cos\alpha)\hat{\xi}_8\Big], \tag{5}$$

where

$$A_\mu = -\frac{1}{2g}sin^2\alpha\partial_\mu\beta', \;\; A'_\mu = 0. \tag{6}$$

The non-trivial dual structure of the QCD vacuum, become more transparent in absence of quarks and may be reduced in the following form,

$$\pounds = -\frac{1}{4}B_{\mu\nu}^2 - \frac{1}{4}B'^{\,2}_{\mu\nu} + |(\partial_\mu + i\frac{4\pi}{g}B_\mu^{(d)})\phi|^2 + |(\partial_\mu + i\frac{4\pi\sqrt{(3)}}{g}B'^{(d)}_\mu)\phi'|^2 - V(\phi^*\phi). \tag{7}$$

where

$$V = \frac{48\pi^2}{g^4}\lambda(\phi^*\phi - \phi_0^2)^2 + \frac{432\pi^2}{g^4}\lambda'(\phi^*\phi' - \phi'^{\,2}_0)^2, \tag{8}$$

and ϕ_0 and ϕ'_0 are the non-zero vacuum expectation values of the fields ϕ and ϕ'. Further, the field equations associated with the Lagrangian for the λ_3 and λ_8 components using cylindrical symmetry and the above effective potential are derived in the following form,

$$\frac{d}{d\rho}\Big[\rho^{-1}\frac{d}{d\rho}\Big(\rho B(\rho)\Big)\Big] - (16\pi\alpha_s^{-1})^{1/2}\Big(\frac{n}{\rho} + (4\pi\alpha_s^{-1})^{1/2}B(\rho)\Big)\chi^2(\rho) = 0, \tag{9}$$

$$\frac{1}{\rho}\frac{d}{d\rho}\Big(\rho\frac{d\chi(\rho)}{d\rho}\Big)-\Big[\Big(\frac{n}{\rho}+(4\pi\alpha_s^{-1})^{1/2}B(\rho)\Big)^2+6\lambda\alpha_s^{-2}\Big(\chi^2-\phi_0^2\Big)\Big]\chi(\rho)=0. \tag{10}$$

$$\frac{d}{d\rho}\Big[\rho^{-1}\frac{d}{d\rho}\Big(\rho B^{'}(\rho)\Big)\Big]-(48\pi\alpha_s^{-1})^{1/2}\Big(\frac{n^{'}}{\rho}+(12\pi\alpha_s^{-1})^{1/2}B^{'}(\rho)\Big)\chi^{'2}(\rho)=0, \tag{11}$$

$$\frac{1}{\rho}\frac{d}{d\rho}\Big(\rho\frac{d\chi^{'}}{d\rho}\Big)-\Big[\Big(\frac{n^{'}}{\rho}+(12\pi\alpha_s^{-1})^{1/2}B^{'}(\rho^{'})\Big)^2+54\lambda^{'}\alpha_s^{-2}\Big(\chi^{'2}-\phi_0^{'2}\Big)\Big]\chi^{'}(\rho)=0. \tag{12}$$

Using the Lagrangian (7), the string tension of the associated flux tube configuration governed by the field equations (9), (10), (11) and (12) may be obtained in the following form,

$$k\,(B,\chi,B^{'},\chi^{'})=2\pi\int_0^\infty\rho\,d\rho\Big[\frac{1}{2\rho^2}\Big(\frac{d}{d\rho}(\rho\,B(\rho))\Big)^2+\Big(\frac{d}{d\rho}\chi(\rho)\Big)^2+\Big(\frac{4\pi}{g}B(\rho)+\frac{n}{\rho}\Big)^2\chi^2(\rho)$$
$$+3\lambda\alpha_s^{-2}(\chi^2-\phi_0^2)^2\Big]+2\pi\int_0^\infty\rho\,d\rho\Big[\frac{1}{2\rho^2}\Big(\frac{d}{d\rho}(\rho\,B^{'}(\rho))\Big)^2$$
$$+\Big(\frac{d}{d\rho}\chi^{'}(\rho)\Big)^2+\Big(\frac{4\pi\sqrt{3}}{g}B^{'}(\rho)+\frac{n^{'}}{\rho}\Big)^2\chi^{'2}(\rho)+27\lambda^{'}\alpha_s^{-2}(\chi^{'2}-\phi_0^{'2})^2\Big]. \tag{13}$$

Imposing color reflection and the asymptotic boundary condition to the λ_3 and λ_8 components appropriate for the large-scale behavior of QCD $B(\rho)\stackrel{\rho\to\infty}{\longrightarrow}-\frac{ng}{4\pi\rho}$ and $\phi\stackrel{\rho\to\infty}{\longrightarrow}\phi_0$, $B^{'}(\rho)\stackrel{\rho\to\infty}{\longrightarrow}-\frac{n^{'}g}{4\sqrt{3}\pi\rho}$ and $\phi^{'}\stackrel{\rho\to\infty}{\longrightarrow}\phi_0^{'}$ leads to the asymptotic solution for $B(\rho)$ and $B^{'}(\rho)$ as $B(\rho)=-\frac{ng}{4\pi\rho}[1+F(\rho)]$ and $B^{'}(\rho)=-\frac{n^{'}g}{4\sqrt{3}\pi\rho}[1+G(\rho)]$, where the function $F(\rho)$ and $G(\rho)$, in asymptotic limit for the system (13) are obtained as,

$$F(\rho)\stackrel{\rho\to\infty}{\longrightarrow}-n+C\sqrt{\rho}\,exp\,(-m_B\,\rho), \tag{14}$$

$$G(\rho)\stackrel{\rho\to\infty}{\longrightarrow}-n^{'}+C\sqrt{\rho}\,exp\,(-m_B^{'}\,\rho). \tag{15}$$

In view of the relationship of k with Regge slope parameter and $\alpha^{'}=0.9GeV^{-2}$ and using the numerical computation of equation (13), we obtain the vector and scalar glueball masses for some typical values of strong coupling in full infrared sector of QCD presented in table 1 [25].

Table 1. The masses of vector and scalar glueball as functions of α_s.

λ	α_s	$m_B(GeV)$	$m_\phi(GeV)$	$\kappa_{QCD}^{(d)}$
$\frac{1}{4}$	0.25	1.74	1.21	0.69
$\frac{1}{2}$	0.24	1.63	1.68	0.99
1	0.23	1.53	2.16	1.42
2	0.22	1.42	2.89	2.05

3 QCD Phase Transition

In order to find the stable vacuum in the field theory, the effective potential formalism indicates the vacuum energy at zero temperature and corresponds to the thermodynamical potential for finite temperature. The partition functional is written as,

$$Z[J]=\int D[\phi]D[B_\mu^{(d)}]D[\phi^{'}]D[B_\mu^{'(d)}]exp(i\int d^4x(\mathcal{L}_d^{(m)}-J|\phi|^2-J^{'}|\phi^{'}|^2)), \tag{16}$$

where, the quadratic source term has been introduced instead of the standard linear source term. Further, we separate the monopole field ϕ into its mean field ϕ and its fluctuation $\tilde{\phi}$ and formulate the effective thermodynamical potential as the function of QCD-monopole condensate expressed in the following form,

$$V_{eff}(\phi) = 3\lambda\alpha_s^{-2}(\phi^2 - \phi_0^2)^2 + 27\lambda^{'}\alpha_s^{-2}(\phi^{'2} - \phi^{'2}_0)^2 + 3\frac{T}{\pi^2}\int_0^\infty dkk^2 ln(1 - e^{-\sqrt{k^2+m_B^2}/T})$$
$$+\frac{T}{2\pi^2}\int_0^\infty dkk^2 ln(1 - e^{-\sqrt{k^2+m_\phi^2}/T}) + 3\frac{T}{\pi^2}\int_0^\infty dkk^2 ln(1 - e^{-\sqrt{k^2+m_B^{'2}}/T})$$
$$+\frac{T}{2\pi^2}\int_0^\infty dkk^2 ln(1 - e^{-\sqrt{k^2+m_\phi^{'2}}/T}), \quad (17)$$

where

$$m_B^2 = 8\pi\alpha_s^{-1}\phi^2, \quad m_\phi^2 = 12\lambda\alpha_s^{-2}\phi^2, \quad m_B^{'2} = 24\pi\alpha_s^{-1}\phi^{'2}, \quad m_\phi^{'2} = 108\lambda\alpha_s^{-2}\phi^{'2}. \quad (18)$$

Minimization of the thermodynamical potential leads to the thermal values of the VEV of the monopole field expressed in the following form,

$$<\phi>_0^{(T)} = 0 \text{ for } T \geq T_c, \quad <\phi>_0^{(T)} = \sqrt{\phi_0^2 - \left(\frac{4\pi\alpha_s + \lambda}{\lambda}\right)\frac{T^2}{8}} \text{ for } T < T_c. \quad (19)$$

and thus reveals the disappearance of the QCD monopole condensate at sufficiently high temperature which, in turn, indicates the restoration of the magnetic symmetry and the deconfinement of the quarks in such temperature region. The variation of $<\phi>_0^{(T)}$ for $\lambda = 2$ with temperature for $\alpha_s = 0.22$ has been shown in figure 1 and ultimately vanishes at the critical temperature of $0.241 GeV$. Similarly the variation of effective $V_{eff}(\phi, T)$ as a function of the QCD-monopole condensate ϕ, around the critical temperature value for the case of $\alpha_s = 0.22$ coupling has been depicted in figure 1. The minimum points of $V_{eff}(\phi, T)$ correspond to the meta-stable vacuum state and as the temperature increases, the broken gauge symmetry tends to be restored.

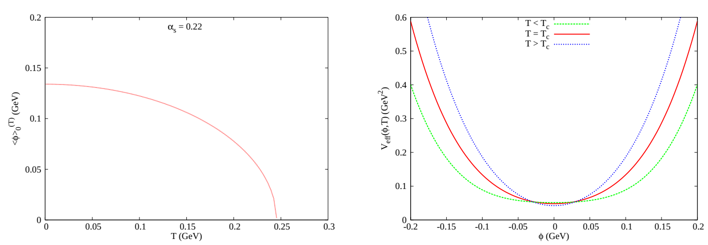

Figure 1. (Color online.) (a) The behavior of monopole condensate ($<\phi>_0^{(T)}$) with temperature (T) and (b) the finite temperature effective potential $V_{eff}(\phi, T)$ as a function of monopole condensate (ϕ) for the coupling $\alpha_s = 0.22$.

4 Results and Conclusions

To establish monopole condensation in QCD, we have studied the mechanism of quark confinement in the context of gauge theory of non-Abelian monopoles which has a built-in dual structure. The analysis of the dual QCD Lagrangian in the dynamically broken phase of magnetic symmetry has been shown to lead a precise flux tube structure to the QCD vacuum which appears as a dual version of Abrikosov vortices. Further we have investigated the behavior of the color-confinement at high temperature by studying the change of the properties in the QCD vacuum with temperature by formulating the effective potential for finite temperature using path-integral formalism. We have used the quadratic source term instead of the linear source term which is useful to obtain the effective potential for the negative-curvature region. Thermal effects reduce the QCD-monopole condensate and brings a first-order deconfinement phase transition. It demonstrates that for higher temperature ($T > T_c$), the magnetic symmetry tends to be restored and the system enters into the deconfinement phase. However, below T_c, the magnetic symmetry is dynamically broken pushing the system in confined phase where the local minima of effective potential correspond to the physical stable vacuum state.

Acknowledgments

Garima Punetha, is thankful to the organizers of AYSS 2018 and University Grant Commission, New Delhi, India, under the UGC-RFSMS fellowship for financial assistance.

References

[1] D. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973).
[2] H. D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973).
[3] E. Shuryak, Phys. Rep. **61**, 71 (1980).
[4] L. McLerran, Rev. Mod. Phys. **58**, 1021 (1986).
[5] K. Fukushima, J. Phys. **G 39**, 013101 (2012).
[6] J. D. Bjorken, Phys. Rev. **D 27**, 140 (1983).
[7] I. Arsene et al. (PHENIX Collaboration), Nucl. Phys. **A 757**, 1 (2005).
[8] B. B. Back et al. (PHOBOS Collaboration), Nucl. Phys. **A 757**, 28 (2005).
[9] J. Adams et al. (BRAHMS Collaboration), Nucl. Phys. **A 757**, 102 (2005).
[10] K. Adcox et al. (STAR Collaboration), Nucl. Phys. **A 757**, 184 (2005).
[11] G. Aad et al. (ATLAS Collaboration), Phys. Rev. Lett. **114**, 072302 (2015).
[12] K. Aamodt et al. (ALICE Collaboration), Phys. Lett. **B 734**, 31 (2014).
[13] V. Khachatryan et al. (CMS Collaboration), JHEP **09**, 091 (2010).
[14] A. Adare et al. (PHENIX Collaboration), Phys. Rev. Lett. **98**, 232301 (2007).
[15] X. Zhao and R. Rapp, Phys. Lett. **B 664**, 253 (2008).
[16] Y. Liu, Q. Zhen, N. Xu and P. Zhuang, Phys. Lett. **B 678**, 72 (2009).
[17] Y. M. Cho, Phys. Rev. **D 21**, 1080 (1980).
[18] Y. M. Cho, Phys. Rev. **D 23**, 2415 (1981).
[19] Y. M. Cho, Phys. Rev. Lett. **44**, 1115 (1980).
[20] H. C. Pandey and H. C. Chandola, Phys. Lett. **B 476**, 193 (2000).
[21] H. C. Chandola, Garima Punetha, H. Dehnen, Nucl. Phys. **A 945**, 226 (2016).
[22] Garima Punetha and H. C. Chandola, *Springer Proceedings in Physics edited by Bhuyan Bipul*, (Springer International Publishing, Switzerland, 2016) p. 653, Vol. 174.
[23] Garima Punetha and H. C. Chandola, EPL **116**, 11001 (2016).
[24] Garima Punetha and H. C. Chandola, EPJ Web of Conferences **177**, 09009 (2018).
[25] Y. M. Cho, Phys. Rev. Lett. **46**, 302 (1981).