XYZ **states as hadronic molecules**

A.V. Nefediev[1,2,∗]

[1]P.N. Lebedev Physical Institute of the Russian Academy of Sciences, 119991, Leninskiy Prospect 53, Moscow, Russia
[2]National Research Nuclear University MEPhI, 115409, Kashirskoe highway 31, Moscow, Russia

Abstract. In the past decade, a lot of new hadrons containing heavy quarks were discovered which do not fit into the scheme provided by the traditional quark models. Such states are known as the *XYZ* states and they are conventionally referred to as exotic ones. At present, there is no consensus on their nature, and different models and approaches have been suggested to explain their unusual properties. The talk is devoted to a brief overview of the molecule model for such exotic states.

1 Introduction

Before 2003, when viewed from the position of both theory and experiment, spectroscopy of heavy quark flavours did not look as an exciting field of research. Indeed, by that time, all $\bar{c}c$ excitations below the open-charm threshold and many $\bar{b}b$ states below the open-bottom threshold were already discovered, together with few higher vector states directly accessible in the e^+e^- annihilation. All observed states were predictably well described by the quark model in its simplest nonrelativistic form with the Cornell-like potential supplied with spin-dependent interactions [1]. No surprises were anticipated until the first unusual charmonium-like state, $X(3872)$, was discovered by the Belle Collaboration in the reaction $B^+ \rightarrow K^+(\pi^+\pi^- J/\psi)$ [2]. Later, this state was also found in another hidden-flavour mode, $\pi^+\pi^-\pi^0 J/\psi$ [3], and in the open-charm final state $D\bar{D}^*$ (this mode comes as a sum of two three-body final states, $D\bar{D}\pi$ and $D\bar{D}\gamma$) [4]. Further measurements confirmed quite unexpected properties of this state which lies about 100 MeV lower than "prescribed" by the quark model for the radially excited axial charmonium $\chi_{c1}(2P)$ (the quantum numbers of the $X(3872)$ are $J^{PC} = 1^{++}$ [5]) and, what is even more important, incredibly close (within less than 1 MeV) to the neutral $D\bar{D}^*$ threshold and demonstrates a strong isospin violation through the nearly equal decay rates of its di- and tri-pion decays to the vector charmonium J/ψ. The unusual nature of this states was reflected in the name it was awarded — the X. The discovery of the $X(3872)$ started a new era in the spectroscopy of heavy quarks for which a very important role was played by the B-factories at e^+e^- colliders, such as Belle at KEK and BABAR at SLAC. This also started a new naming scheme for the states which could not be described by the quark model and, for this reason, were regarded as exotic states — since 2003 such states have been named using the letters from the end of the latin alphabet: X, Y, Z. The total number of the reported exotic states in the spectrum of charmonium exceeds 20, and about a half of them are believed to be well established and confirmed. Two exotic states are found in
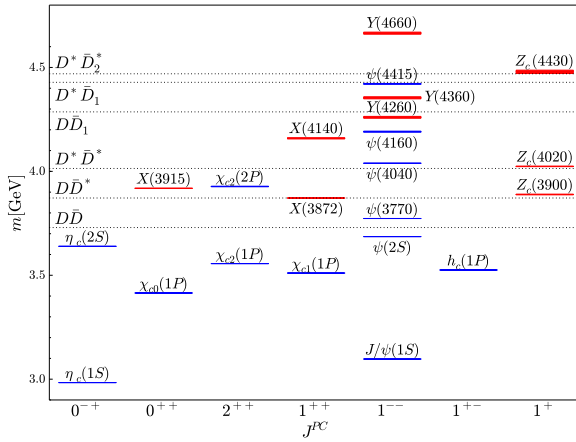

∗e-mail: nefediev@lebedev.ru

Figure 1. The measured hadronic states in the spectrum of charmonium. States which can(not) be described by the quark model are marked with blue(red). Here the *X*(3915) is shown as a scalar state, however, the 2^{++} option is not yet excluded — see the discussion in chapter 4.2.
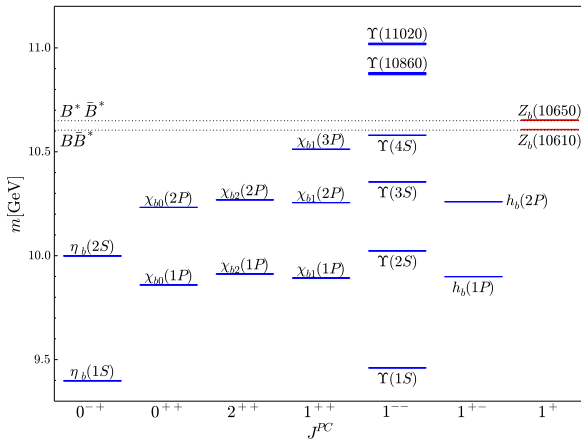

Figure 2. The measured hadronic states in the spectrum of bottomonium. States which can(not) be described by the quark model are marked with blue(red).

the spectrum of bottomonium — $Z_b(10610)$ and $Z_b(10650)$. Although there is no full consensus among phenomenologists concerning particular criteria used to assign this or that name to a newly discovered exotic state, it is more or less commonly accepted that the name *Z* is used for charged (that is, isovector) resonances (for example, $Z_c(3900)$, $Z_c(4020)$, $Z_c(4430)$, $Z_b(10610)$, $Z_b(10650)$) while vector states are called *Y*'s (for example, $Y(4230)$, $Y(4260)$, $Y(4360)$, $Y(4660)$). The name *X*, following the tradition started with the *X*(3872), is assigned to newly observed exotic states, sometimes temporarily, until their quantum numbers are unambiguously determined or their nature is clarified (for example, $X(3915)$, $X(4140)$). In Figs. 1 and 2 the *XYZ* exotic states are shown (in red) together with ordinary quarkonia (given in blue) and relevant open-flavour thresholds (see the horizontal dashed lines).

2 Models

As was mentioned above, most of states in the spectrum of charmonium and bottomonium can be explained as plain $\bar{Q}Q$ quarkonia which are quite well described with the quark model
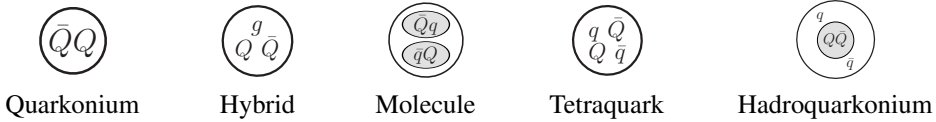
Figure 3. Models for the *XYZ* exotic states.

based on the nonrelativistic Hamiltonian

$$H = 2m_Q + \frac{p^2}{m_Q} + \sigma r - \frac{4\alpha_s}{3r} + C + V_{\text{spin-dep}}, \tag{1}$$

where m_Q is the quark mass, σ is known as the string tension parameter which sets the strength of the confining force between coloured objects, α_s is the strong coupling constant, and C stands for a constant energy shift. The factor 4/3 in the Coulomb part of the potential is the fundamental Casimir operator of the colour $SU(3)$ group.

The standard approach to the spin-dependent potential $V_{\text{spin-dep}}$ in Eq. (1) is to identify it order by order in the $1/m_Q$ expansion. This is well justified for heavy quarks and results in a well-known expression for the spin-dependent potential to the order $1/m_Q^2$ [6, 7]. A systematic approach to $\bar{Q}Q$ mesons based on the Hamiltonian (1) can be found in Ref. [8] and in many later works. Thus, the quark model provides a good description of the $\bar{Q}Q$ spectra below the open-flavour thresholds, as well as leptonic, radiative, and some hadronic widths of the $\bar{Q}Q$ quarkonia. In the meantime, quark models are known to fail for the exotic states. The reason for such a failure is rather obvious — the effects of gluons and light quarks, such as hadronic loops, strong thresholds, pions, and so on, cannot be captured by simple quantum mechanical approaches, like those based on the Hamiltonian (1), even after relativisation. This calls for building alternative approaches to the phenomenology of exotic states containing heavy quarks. The most popular models for the exotics are collected in Fig. 3. They include: (i) a plain $\bar{Q}Q$ assignment which, if applicable, is typically tested first for each newly observed state, (ii) a hybrid meson (or simply hybrid) in which a quark-antiquark pair is augmented by gluonic degrees of freedom (see, for example, citations contained in Ref. [9]), (iii) a hadronic molecule — an extended four-quark object which will be discussed in detail below (for a recent review see Ref. [10]), (iv) a tetraquark whose dominating wave function component is, similarly to the molecule, a four-quark one, however, tetraquarks are believed to be compact objects formed by the confining forces, similarly to ordinary mesons or hybrids (see Ref. [11] for a review and relevant references), and (v) hadroquarkonium which consists of a heavy-quark core surrounded by a cloud of light quarks [12].

3 Introduction to hadronic molecules

As a hadronic molecular state, or simply a hadronic molecule, we understand a large probability to observe a resonance in a given hadron-hadron channel. Since the proximity of an open-flavour threshold implies naturally a large admixture of a meson-meson component in the wave function of the resonance, then near-threshold exotic states are strong candidates for hadronic molecules. The nature of such resonances can be different — one needs to solve a dynamical problem to find out whether this is a bound or virtual state or an above-threshold resonance. It should be noticed, however, that because of the presence of various elastic and inelastic decay channels, the poles in the complex energy plane describing near-threshold

states are always located on unphysical Riemann Sheets, away from the real axis. Thus, such poles cannot be nominated as bound or virtual states in the proper sense of the word. However, it is still convenient to stick to this terminology bearing in mind the position of the pole relative to the nearest relevant hadronic threshold. The binding forces in the molecule can also be quite different — for example, this can be a *s*- or *t*-channel exchange.

Remarkably, there exists a nice commonly accepted and known for many years example of a molecular state in hadronic physics — the deuteron. Indeed, a $^3S_1$ two-nucleon system is known to possess a pole in the isosinglet channel which is located on the physical Riemann Sheet and describes the deuteron as a bound state. Meanwhile, a $^1S_0$ two-nucleon system also has a near-threshold pole, however, in the isovector channel and located on the second (unphysical) Riemann Sheet which describes a virtual state just below the threshold.

It was suggested by Weinberg many years ago that, for a near-threshold resonance, it is possible to define the admixture of the hadronic molecule in its wave function in a model-independent way. To this end, one can define the probability to observe a compact state ($0 \leqslant Z \leqslant 1$) in the bound state wave function and to extract its value from the data on the scattering length and effective range. Applied to deuteron, this approach indeed demonstrates that the latter is a molecule state with $Z \to 0$ [13–15]. The Weinberg's method can be formulated in terms of the pole counting rules [16] which state that an elementary (provided by the confining forces of QCD) state is described by a large and negative effective range, a small scattering length and, as a result, corresponds to two nearly symmetric near-threshold poles in the momentum complex plane. On the contrary, a composite (molecular) state has a small effective range (its sign is not determined since range corrections may change it), a large scattering length and is described by a single near-threshold pole. Generalisation of the Weinberg's method to the case when there is no bound state in the system and to composite objects formed by unstable constituents can be found in Refs. [17, 18].

The experimental information on near-threshold states comes predominantly from their production reactions in which the line shapes are measured in the open-flavour and hidden-flavour channels. Then bound states reveal themselves as narrow below-threshold peaks (which, however, are very difficult to identify experimentally) and broad above-threshold humps in the elastic channels. Virtual states produce cusps in the inelastic channels and broad humps in the elastic ones. Thus, a combined analysis of all measured production and decay channels for a given resonance is necessary to reveal its nature.

From the point of view of theory, it is important to notice that the exotic *XYZ* states contain heavy quarks. Then, since for $m_Q \gg \Lambda_{QCD}$ the spin of the heavy quark decouples, the exotic systems at hand are subject to Heavy Quark Spin Symmetry (HQSS) which allows one to relate various properties of exotic states with different orientations of the heavy-quark spin — the so-called spin partners. Although being approximate, this symmetry appears to be a rather accurate symmetry of QCD, especially in the *b*-quark sector, and provides an important and useful tool for the phenomenology of exotic states — see, for example, Refs. [19–28].

Below we consider few examples of exotic *XYZ* states and discuss their possible molecule interpretation.

4 The *X*-family

4.1 *X*(3872)

As was already mentioned above, the first and the most well-studied representative of the *XYZ* family of exotic near-threshold states is the *X*(3872). It has the quantum numbers $J^{PC} = 1^{++}$ and its mass and width are $M_X = 3871.68 \pm 0.17$ MeV and $\Gamma_X < 1.2$ MeV [5]. The main

observation modes for the $X(3872)$ are $\pi^+\pi^- J/\psi$, $\pi^+\pi^-\pi^0 J/\psi$, $D\bar{D}^*$, $\gamma J/\psi$, $\gamma\psi'(3686)$ [5]. It is also seen in the reaction $e^+e^- \to \gamma X(3872)$.

In the molecular model, the $X(3872)$ can be viewed as a shallow bound state ($E_B \ll 1$ MeV) with a large admixture of the $D\bar{D}^*$ component in its wave function. The short-range component of the wave function is responsible for the $X(3872)$ production at high energies, for its radiative decays and decays to light hadrons while the hadronic (long-range) component is responsible for open-charm decays. It is worthwhile mentioning that the three-body ($D\bar{D}\pi$) dynamics is important in the $X(3872)$ because of a very specific ordering of thresholds: $m_D + m_{D^*} > 2m_D + m_\pi$ and $m_{D^*} - m_D - m_\pi \ll m_\pi$.

For a recent review and relevant references on the subject see Ref. [29].

4.2 *X(3915)*

Another exotic state, $X(3915)$, is seen in the reactions $B \to KX \to K(\omega J/\psi)$ and $\gamma\gamma \to X \to \omega J/\psi$ [30, 31], however, quite surprisingly for a charmonium, not seen in the $D\bar{D}$ final state. Only two options for the quantum numbers of the $X(3915)$ are compatible with the data: 0^{++} and 2^{++}, however, the angular distributions for the final-state leptons and pions emerging from the decays of the J/ψ and ω favour the 0^{++} option [31] (this analysis was criticised in Ref. [32]). In the meantime, there exists a good in all respects candidate for the ordinary $\chi_{c0}(2P)$ charmonium — the state $X^*(3860)$ reported recently by the Belle Collaboration [33].

From the theory point of view, this states is also problematic. Its $\chi_{c0}(2P)$ identification [34] is quite questionable [35, 36]. On the other hand, the proposal of Ref. [32] to associate the $X(3915)$ with a tensor exotic state and, this way, to override the veto on the 2^{++} quantum numbers imposed in Ref. [31] does not work for the $X(3915)$ as a $D^*\bar{D}^*$ tensor molecule [37].

Thus, the most plausible assumption left for the $X(3915)$ seems to be its identification with a scalar $D_s\bar{D}_s$ molecule, as suggested in Ref. [38]. However, to verify this conjecture one needs to go beyond the simple order-of-magnitude estimates contained in the cited work.

5 The *Y*-family

5.1 *Y(4260)*

Among the exotic vector states, the $Y(4260)$ found by the BABAR Collaboration in 2005 [39] is one of the most well studied. This state is not seen in the R-ratio scan that was considered as an evidence of its possible hybrid nature [40]. Indeed, the quark-antiquark pair in the hybrid appears to be in the colour octet state, so it cannot annihilate to photons. The hybrid model predicts quite a peculiar pattern of open-flavour decays for such states [41–46] which can be employed to tell hybrids from conventional quarkonia. A competing model for the $Y(4260)$, as a compact object, is its $[cs]$-$[\bar{c}\bar{s}]$ diquark-antidiquark assignment [47].

In the meantime, proximity of the $D_1\bar{D}$ threshold implies a possible molecule interpretation of the $Y(4260)$ [48] which gives a good description of the data in $J/\psi\pi\pi$, $h_c\pi\pi$ and $D\bar{D}^*\pi$ final states and, in particular, provides a natural explanation for the $Z_c(3900)$ (yet another molecule candidate — see below) appearance in the $Y(4260)$ decays [49]. It is also worth mentioning that the prediction of the molecular model for the behaviour of the cross section $e^+e^- \to \gamma X(3872)$ in the energy region near $Y(4260)$ [50] was confirmed experimentally shortly after its publication [51].

Finally, similarly to the case of the $X(3872)$, the wave function of the $Y(4260)$ may be a mixture of a short-range (for example, associated with the hybrid or tetraquark) part and a long-range one given by the $D_1\bar{D}$ molecule.

6 The Z-family

6.1 $Z_c(3900)$

The Z-states in the spectrum of charmonium attract special attention due to their undoubtedly exotic nature. Indeed, being charged (isovector) states they definitely contain a $c\bar{c}$ pair, so that their minimal quark content is four-quark. The charmonium-like state $Z_c(3900)$ observed by the BESIII Collaboration in 2013 [52] has the quantum numbers $J^{PC} = 1^{+-}$, the mass $M = 3886.6 \pm 2.4$ MeV and the width $\Gamma = 28.1 \pm 2.6$ MeV [5]. It, therefore, resides just above the $D\bar{D}^*$ threshold — see Fig.1 — that hints its molecular interpretation as an isovector cousin of the $X(3872)$ (with the opposite C-parity). Its main observation modes are the decays to the $J/\psi\pi$ and $D\bar{D}^*$ final states. The nature of this state is still obscure — for example, as argued in Ref. [53], the data are compatible with both virtual state and resonance. If the $Z_c(3900)$ pole in the energy complex plane resides above the $D\bar{D}^*$ threshold, a nontrivial interplay of different dynamics is needed to explain it.

6.2 $Z_b(10610)$ **and** $Z_b(10650)$

Charged 1^{+-} states $Z_b(10610)$ and $Z_b(10650)$ in the spectrum of bottomonium were observed by the Belle Collaboration in 2011 [54]. Their parameters, $M_{Z_b} = 10607.2 \pm 2.0$ MeV, $\Gamma_{Z_b} = 18.4 \pm 2.4$ MeV, $M_{Z'_b} = 10652.2 \pm 1.5$ MeV, $\Gamma_{Z'_b} = 11.5 \pm 2.2$ MeV [5], demonstrate that they reside very close to the $B\bar{B}^*$ and $B^*\bar{B}^*$ threshold, respectively, — see Fig. 2. The seven observation modes for this states are

$$\Upsilon(10860) \to \pi Z_b^{(\prime)} \to \pi B\bar{B}^*, \ \pi B^*\bar{B}^*, \ \pi\pi\Upsilon(1S), \ \pi\pi\Upsilon(2S), \ \pi\pi\Upsilon(3S), \ \pi\pi h_b(1P), \ \pi\pi h_b(2P), \tag{2}$$

and the molecular interpretation for the Z_b's, suggested in the pioneering work [20] and further developed in many later works, allows one to explain the nearly equal rates of the two-pion decays $\Upsilon(10860) \to \pi\pi h_b$ and $\Upsilon(10860) \to \pi\pi\Upsilon$ which proceed with and without the heavy quark spin flip, that, naively, would imply a strong HQSS violation, quite unexpected and unnatural for bottomonia.

Furthermore, the line shapes of the Z_b's in the reactions quoted in Eq. (2) can be well described in the molecular model framework [55–58].

7 Spin partners

A specific prediction from the HQSS for molecular states is the existence of the spin partners with a different heavy quark spin orientation. In particular, if the $Y(4260)$ is indeed a $D_1\bar{D}$ molecule, then another vector state, $Y(4360)$ [59, 60], may be identified as a $D_1\bar{D}^*/D_2\bar{D}^*$ molecule, too [61]. Similarly, if the $Z_c(3900)$ is identified as a $D\bar{D}^*$ molecule, its $D^*\bar{D}^*$ counter part should be nothing more than the $Z_c(4020)$ [62]. The existence of the $X(3872)$ as a 1^{++} bound state in the $D\bar{D}^*$ system was predicted to result in the existence of a tensor 2^{++} molecule X_{c2} at the $D^*\bar{D}^*$ threshold [23, 26, 27]. Finally, for the $Z_b(10610)$ and $Z_b(10650)$, the existence of the spin partner states W_{bJ} with the quantum numbers J^{++} ($J = 0, 1, 2$) was predicted in Refs. [20, 21]. Their line shapes in various elastic and inelastic channels and the corresponding pole positions are predicted from the data on the Z_b states in Ref. [63]. It is important to mention that, because of the positive C-parity, the W_{bJ} molecules can be produced in the radiative decays from the $\Upsilon(10860)$ to lower lying bottomonia, so that the probability of such decays is additionally suppressed by the electromagnetic fine structure constant $\alpha = \frac{1}{137}$. However, a large statistics expected at Belle-II should allow to override this suppression, so that searches for the Z_bs' partner states should be regarded as an important part of the Belle-II physical programme [9].

8 Conclusions

The molecule model provides a commonly accepted and phenomenologically successful approach to exotic states with heavy quarks which allows one to describe specific line shapes of exotic near-threshold resonances, predict spin partner states and explain the mass splittings, such as $M_{Y(4260)} - M_{X(3872)} \approx M_{D_1(2420)} - M_{D^*}$ and $M_{Z_b(10650)} - M_{Z_b(10610)} \approx M_{B^*} - M_B$.

Further developments of the model should include investigations of the relation between different heavy-quark sectors, a proper inclusion of the pions and compact components of the resonances' wave functions, generalisation to the $SU(3)$ flavour group for light quarks, tests of the accuracy of HQSS, especially in the charm sector.

References

[1] E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane, T.M. Yan, Phys. Rev. **D17**, 3090 (1978), [Erratum: Phys. Rev. **D21**, 313 (1980)]

[2] S.K. Choi et al. (Belle), Phys. Rev. Lett. **91**, 262001 (2003), `hep-ex/0309032`

[3] K. Abe et al. (Belle), `hep-ex/0505037`

[4] G. Gokhroo et al. (Belle), Phys. Rev. Lett. **97**, 162002 (2006), `hep-ex/0606055`

[5] M. Tanabashi et al. (Particle Data Group), Phys. Rev. **D98**, 030001 (2018)

[6] E. Eichten, F. Feinberg, Phys. Rev. **D23**, 2724 (1981)

[7] D. Gromes, Z. Phys. **C26**, 401 (1984)

[8] S. Godfrey, N. Isgur, Phys. Rev. **D32**, 189 (1985)

[9] E. Kou et al. (Belle II) (2018), `1808.10567`

[10] F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, B.S. Zou, Rev. Mod. Phys. **90**, 015004 (2018), `1705.00141`

[11] A. Esposito, A. Pilloni, A.D. Polosa, Phys. Rept. **668**, 1 (2016), `1611.07920`

[12] X. Li, M.B. Voloshin, Mod. Phys. Lett. **A29**, 1450060 (2014), `1309.1681`

[13] S. Weinberg, Phys. Rev. **130**, 776 (1963)

[14] S. Weinberg, Phys. Rev. **131**, 440 (1963)

[15] S. Weinberg, Phys. Rev. **137**, B672 (1965)

[16] D. Morgan, Nucl. Phys. **A543**, 632 (1992)

[17] L.N. Bogdanova, G.M. Hale, V.E. Markushin, Phys. Rev. **C44**, 1289 (1991)

[18] V. Baru, J. Haidenbauer, C. Hanhart, Yu. Kalashnikova, A.E. Kudryavtsev, Phys. Lett. **B586**, 53 (2004), `hep-ph/0308129`

[19] F.-K. Guo, C. Hanhart and U.-G. Meißner, Phys. Rev. Lett. **102**, 242004 (2009), `0904.3338`

[20] A.E. Bondar, A. Garmash, A.I. Milstein, R. Mizuk, M.B. Voloshin, Phys. Rev. **D84**, 054010 (2011), `1105.4473`

[21] M.B. Voloshin, Phys. Rev. **D84**, 031502 (2011), `1105.5829`

[22] T. Mehen, J.W. Powell, Phys. Rev. **D84**, 114013 (2011), `1109.3479`

[23] J. Nieves, M.P. Valderrama, Phys. Rev. **D86**, 056004 (2012), `1204.2790`

[24] F.-K. Guo, C. Hidalgo-Duque, J. Nieves, M.P. Valderrama, Phys. Rev. **D88**, 054007 (2013), `1303.6608`

[25] C. Hidalgo-Duque, J. Nieves, A. Ozpineci, V. Zamiralov, Phys. Lett. **B727**, 432 (2013), `1305.4487`

[26] M. Albaladejo, F.-K. Guo, C. Hidalgo-Duque, J. Nieves, M.P. Valderrama, Eur. Phys. J. **C75**, 547 (2015), `1504.00861`

[27] V. Baru, E. Epelbaum, A.A. Filin, C. Hanhart, U.-G. Meißner, A.V. Nefediev, Phys. Lett. **B763**, 20 (2016), `1605.09649`

[28] V. Baru, E. Epelbaum, A.A. Filin, C. Hanhart, A.V. Nefediev, JHEP **06**, 158 (2017), `1704.07332`

[29] Y.S. Kalashnikova, A.V. Nefediev (2018), `1811.01324`

[30] S. Uehara et al. (Belle), Phys. Rev. Lett. **104**, 092001 (2010), `0912.4451`

[31] J.P. Lees et al. (BaBar), Phys. Rev. **D86**, 072002 (2012), `1207.2651`

[32] Z.Y. Zhou, Z. Xiao, H.Q. Zhou, Phys. Rev. Lett. **115**, 022001 (2015), `1501.00879`

[33] K. Chilikin et al. (Belle), Phys. Rev. **D95**, 112003 (2017), `1704.01872`

[34] X. Liu, Z.G. Luo, Z.F. Sun, Phys. Rev. Lett. **104**, 122001 (2010), `0911.3694`

[35] F.-K. Guo, U.-G. Meißner, Phys. Rev. **D86**, 091501 (2012), `1208.1134`

[36] S.L. Olsen, Phys. Rev. **D91**, 057501 (2015), `1410.6534`

[37] V. Baru, C. Hanhart, A.V. Nefediev, JHEP **06**, 010 (2017), `1703.01230`

[38] X. Li, M.B. Voloshin, Phys. Rev. **D91**, 114014 (2015), `1503.04431`

[39] B. Aubert et al. (BaBar), Phys. Rev. Lett. **95**, 142001 (2005), `hep-ex/0506081`

[40] S.L. Zhu, Phys. Lett. **B625**, 212 (2005), `hep-ph/0507025`

[41] A. Le Yaouanc, L. Oliver, O. Pene, J.C. Raynal, S. Ono, Z. Phys. **C28**, 309 (1985)

[42] N. Isgur, R. Kokoski, J. Paton, Phys. Rev. Lett. **54**, 869 (1985)

[43] F. Iddir, S. Safir, O. Pene, Phys. Lett. **B433**, 125 (1998), `hep-ph/9803470`

[44] Yu.S. Kalashnikova, Z. Phys. **C62**, 323 (1994)

[45] E. Kou, O. Pene, Phys. Lett. **B631**, 164 (2005), `hep-ph/0507119`

[46] Yu.S. Kalashnikova, A.V. Nefediev, Phys. Rev. **D77**, 054025 (2008), `0801.2036`

[47] L. Maiani, V. Riquer, F. Piccinini, A.D. Polosa, Phys. Rev. **D72**, 031502 (2005), `hep-ph/0507062`

[48] J.L. Rosner, Phys. Rev. **D74**, 076006 (2006), `hep-ph/0608102`

[49] Q. Wang, C. Hanhart, Q. Zhao, Phys. Rev. Lett. **111**, 132003 (2013), `1303.6355`

[50] F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, Phys. Lett. **B725**, 127 (2013), `1306.3096`

[51] M. Ablikim et al. (BESIII), Phys. Rev. Lett. **112**, 092001 (2014), `1310.4101`

[52] M. Ablikim et al. (BESIII), Phys. Rev. Lett. **110**, 252001 (2013), `1303.5949`

[53] M. Albaladejo, F.-K. Guo, C. Hidalgo-Duque and J. Nieves, Phys. Lett. **B755**, 337 (2016) `1512.03638`

[54] A. Bondar et al. (Belle), Phys. Rev. Lett. **108**, 122001 (2012), `1110.2251`

[55] T. Mehen, J. Powell, Phys. Rev. **D88**, 034017 (2013), `1306.5459`

[56] C. Hanhart, Yu.S. Kalashnikova, P. Matuschek, R.V. Mizuk, A.V. Nefediev, Q. Wang, Phys. Rev. Lett. **115**, 202001 (2015), `1507.00382`

[57] F.-K. Guo et al., Phys. Rev. **D93**, 074031 (2016), `1602.00940`

[58] Q. Wang, V. Baru, A.A. Filin, C. Hanhart, A.V. Nefediev, J.L. Wynen, Phys. Rev. **D98**, 074023 (2018), `1805.07453`

[59] B. Aubert et al. (BaBar), Phys. Rev. Lett. **98**, 212001 (2007), `hep-ex/0610057`

[60] X.L. Wang et al. (Belle), Phys. Rev. Lett. **99**, 142002 (2007), `0707.3699`

[61] Q. Wang et al., Phys. Rev. **D89**, 034001 (2014), `1309.4303`

[62] M.B. Voloshin, Phys. Rev. **D87**, 091501 (2013), `1304.0380`

[63] V. Baru, E. Epelbaum, A.A. Filin, C. Hanhart, A. Nefediev, Q. Wang, in preparation