

Scalar isoscalar form factors for energies above 1 GeV

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Abstract. We present a new parametrization for scattering amplitudes and form factors, which is consistent with high-accuracy dispersive representations at low energies but at the same time allows for a data description of higher mass resonances such as the $f_0(1500)$ and $f_0(2020)$. The formalism is general and thus can be applied to many decay processes. As an example we discuss the decay of $\bar{B}_s^0 \rightarrow J/\psi\pi\pi(KK)$. From the amplitude fixed in a fit to the experimental data pole positions and residues are extracted via Padé approximants.

1 Introduction

Dispersion theory is a powerful tool in order to rigorously describe final-state interactions of the lightest mesons, where scattering phase shifts are known with high precision. Since it is model independent it can even be applied to the decays of heavy mesons into pions and kaons [1–3]. Not only does it ensure analyticity and unitarity, but it is also consistent by construction with the high-precision analyses using Roy- and Roy-like equations [4–9].

However due to additional channels as well as resonances above 1 GeV its application is limited to a energy regime below said threshold. In order to extend the description to a larger energy range we need to leave the safe grounds of dispersion theory to find an effective parametrization. This model should not spoil unitarity and analyticity, but it should also be able to describe effects coming from additional channels and resonances. Furthermore, it should be consistent with the constraints given by dispersion theory. Such a parametrization was provided for the vector pion form factor by Ref. [10] and generalized for the scalar pion form factors in this work [11].

2 Formalism

In this paragraph we present the formalism as it was derived in Refs. [10, 11]. Although it is general, to be concrete we here proceed along the example of an isoscalar scalar partial wave and include the channels $\pi\pi$ (channel 1), $K\bar{K}$ (channel 2), and 4π (channel 3), which we model by either $\rho\rho$ or $\sigma\sigma$.

In order to describe the unitary partial wave-amplitude T_{if} from the initial state i to the final state f we use a Bethe-Salpeter equation of the form

$$T_{if} = V_{if} + \sum_n V_{in} G_{nn} T_{nf}. \quad (1)$$

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Here V_{in} denotes the scattering potential from channel i into the intermediate state n . The loop function G_{nm} describes the propagation of the intermediate state.

Similar to the two-potential formalism [12] we proceed by splitting the scattering kernel into two parts $V = V_0 + V_R$. As seen in experiment inelasticities at higher energies are typically accompanied by resonances. Thus we define the resonance exchange potential V_R such that it primarily contributes at higher energies, is suppressed at lower ones, and provides the coupling to all considered channels. This separation also allows us to rewrite the full scattering amplitude into $T = T_0 + T_R$. We assume that T_0 fulfills a Bethe-Salpeter equation with V_0 by itself and incorporates the most relevant crossed channel interactions. Thus we can parametrize it with a unitary representation only using phases and inelasticities. As in our case the dipion system interacts strongly in the S -wave with the dikaon system at a cms energy of $\sqrt{s} = 1$ GeV, we assume a coupled-channel description of them via V_0 . By construction T_0 contains the physics of $f_0(500)$ (σ) as well as $f_0(980)$. The corresponding scattering matrix is given by

$$T_0 = \begin{pmatrix} \frac{\eta e^{2i\delta} - 1}{2i\sigma_1} & g e^{i\psi} & 0 \\ g e^{i\psi} & \frac{\eta e^{2i(\psi-\delta)} - 1}{2i\sigma_2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{with} \quad \sigma_i = \sqrt{1 - \frac{4m_i^2}{s}}. \quad (2)$$

The $\pi\pi$ scattering phase shift δ and inelasticity η as well as the $\pi\pi \rightarrow K\bar{K}$ scattering phase ψ and modulus g were subject to many rigorous dispersive studies [4–9] and thus serve as possible input for our parametrization. In this work we explicitly used the coupled channel results of Ref. [13].

As T_0 is unitary by itself the resonance exchange amplitude T_R is not independent of T_0 . The full solution can be written as

$$T = T_0 + \Omega [1 - V_R \Sigma]^{-1} V_R \Omega^t, \quad (3)$$

where Ω denotes the vertex function, which can be calculated numerically as an Omnès solution [14, 15] of the integral equation

$$\Omega_{ij}(s) = \begin{cases} \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} dz \frac{(T_0)_m^*(z) \sigma_n(z) \Omega_{nj}(z)}{z - s - i\epsilon} & i, j \in \{1, 2\} \\ 1 & i = j \neq 1, 2 \\ 0 & \text{else.} \end{cases} \quad (4)$$

Similarly we also need to introduce the self energy Σ , which we write as a once-subtracted dispersion integral

$$\Sigma_{ij}(s) = \frac{s}{\pi} \int_{4m_\pi^2}^{\infty} dz \frac{\Omega_{ni}^*(z) \sigma_n(z) \Omega_{nj}(z)}{z - s - i\epsilon}. \quad (5)$$

The subtraction constant can be absorbed in the not yet defined resonance potential V_R . Note that for Σ_{33} only the four-pion phase space contributes, which we model through the decay of two ρ or σ mesons. See Refs. [11, 16] for details.

As V_0 as described above alone fails to describe effects above 1 GeV, we include them via the resonance exchange potential V_R . It includes s -channel particle exchanges of N_R resonances, which are not covered by V_0 . In order to reduce its impact at low energies we subtract it at $s = 0$, which leaves us with

$$(V_R)_{ij}(s) = \sum_{r=1}^{N_R} g_i^r \frac{s}{m_r^2} \frac{1}{s - m_r^2} g_j^r. \quad (6)$$

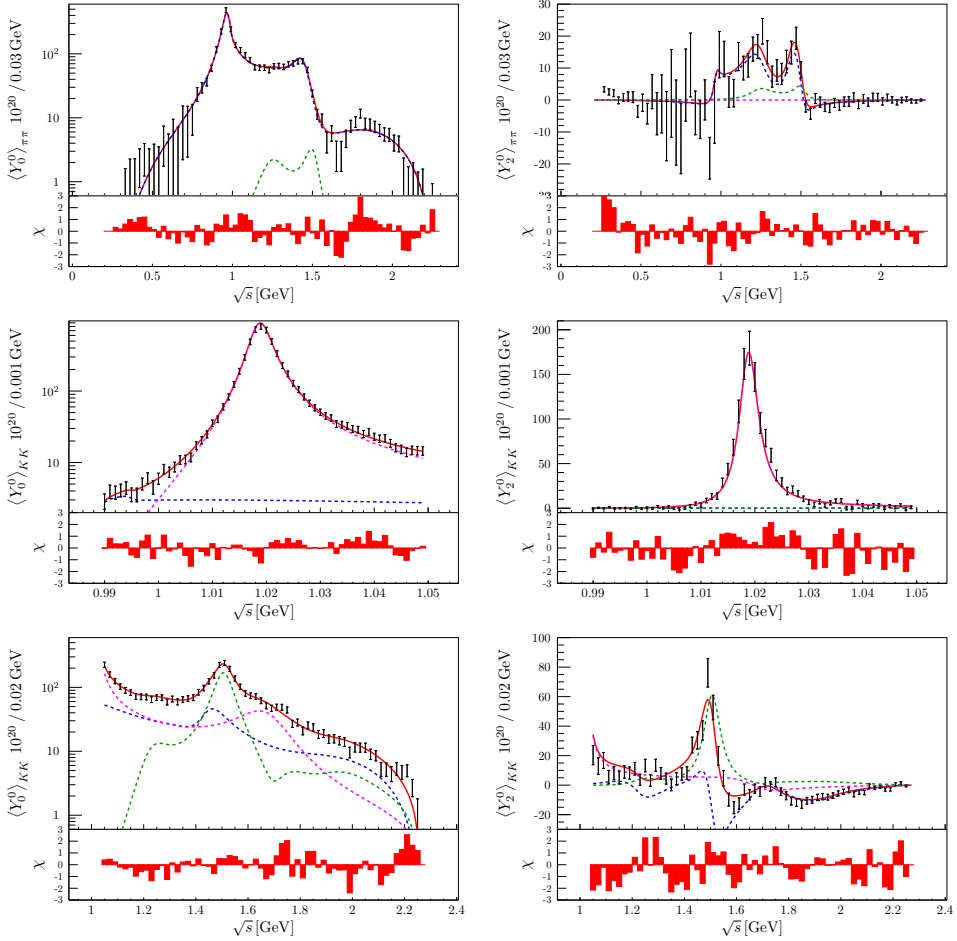


Figure 1. Example fits to the angular moments $\langle Y_0^0 \rangle$ (left) and $\langle Y_2^0 \rangle$ (right) of $\bar{B}_s^0 \rightarrow J/\psi \pi^+ \pi^-$ [18] (top two) and $\bar{B}_s^0 \rightarrow J/\psi K^+ K^-$ [19] (bottom four) using $N_R = 2$ with a constant polynomial and an additional $\rho\rho$ channel. The overall fit result is depicted by the solid red line. Additionally we show P -wave (dashed purple), D -wave (dashed green) and S -wave contributions or SD -wave interference (dashed blue). On the lower axis we depict the fit residuals given by $\chi = (\langle Y_L^0 \rangle_{\text{measured}} - \langle Y_L^0 \rangle_{\text{fit}}) / \sigma_{\text{measured}}$.

Here we introduced the bare mass m_r as well as the bare resonance-channel coupling constant g'_i , which need to be determined by a fit.

In order to describe a transition form factor Γ_f of some source to the final state f we use the P -vector formalism [17], which gives us after some algebraic manipulation

$$\Gamma_f = \Omega_{fm} [1 - V_R \Sigma]_{mn}^{-1} M_n. \quad (7)$$

The direct transition between the source and the intermediate channel n is given by M_n . We parametrize it by

$$M_n(s) = P_n(s) - \sum_{r=1}^{N_R} g_n^r \frac{s}{s - m_r^2} \alpha^r. \quad (8)$$

The first term is a polynomial in s with free coefficients, which need to be determined by a fit to data. The normalization $\Gamma_f(0)$ corresponds to $P_f(0)$, which can be determined by using the Feynman-Hellmann theorem with an $\bar{s}s$ source. Thus the $\pi\pi$ channel has a normalization of $\Gamma_\pi(0) = 0$ and the $K\bar{K}$ $\Gamma_K(0) = 2/\sqrt{3}$ [1]. Accordingly we also fix $\Gamma_{4\pi} = 0$, which is OZI suppressed. The second term describes the mixing of the resonance r with the source, which is described by the additional parameter α' . This term cancels potential zeros at the bare mass positions.

3 Fits

In this section we discuss for illustration a fit to the decays $\bar{B}_s^0 \rightarrow J/\psi\pi^+\pi^- (K^+K^-)$ measured at LHCb [18, 19]. For more details on the free parameters we refer to Ref. [11].

We consider the angular moment averages defined by the partial-wave-expanded helicity amplitudes \mathcal{H}_λ^L . Here $\lambda = 0, \parallel, \perp$ refers to the helicity of the J/ψ , and L denotes the angular momentum of the dipion or dikaon system. The two most relevant angular moments for us are given by

$$\begin{aligned} \langle Y_0^0 \rangle &= \frac{p_\psi p_m}{\sqrt{4\pi}} \left\{ |\mathcal{H}_0^0|^2 + \sum_{\lambda=0,\parallel,\perp} \left(|\mathcal{H}_\lambda^1|^2 + |\mathcal{H}_\lambda^2|^2 \right) \right\} \quad \text{and} \\ \langle Y_2^0 \rangle &= \frac{p_\psi p_m}{\sqrt{4\pi}} \left\{ 2\text{Re}[\mathcal{H}_0^0 (\mathcal{H}_0^2)^*] + \frac{1}{\sqrt{5}} [2|\mathcal{H}_0^1|^2 - \sum_{\lambda=\parallel,\perp} |H_\lambda^1|^2] + \frac{\sqrt{5}}{7} [2|\mathcal{H}_0^2|^2 + \sum_{\lambda=\parallel,\perp} |H_\lambda^2|^2] \right\}. \end{aligned} \quad (9)$$

Here p_ψ denotes the momentum of the J/ψ in the \bar{B}_s^0 rest frame and p_m the pion (kaon) momentum in the dipion (dikaon) rest frame. For more information on the angular moments we refer to Refs. [1, 20].

In the following we assume that the S -wave \mathcal{H}_0^0 is proportional to the scalar isoscalar pion (kaon) form factor and can be described by Eq. (7), while we parametrize P - and D -waves as Breit-Wigner functions according to the LHCb studies [18, 19].

An example fit with $\chi^2/\text{ndf} = 376.2/(384 - 30 - 1) \approx 1.07$ using $N_R = 2$ in Eqs. (6) and (8) and a $\rho\rho$ channel is shown in Fig. 1. The first thing we see is that the event distribution $\langle Y_0^0 \rangle$ as well as the interference term $\langle Y_2^0 \rangle$ are well described for both $\bar{B}_s^0 \rightarrow J/\psi\pi^+\pi^-$ as well as $\bar{B}_s^0 \rightarrow J/\psi K^+K^-$. Furthermore we notice that the decay into pions is indeed S -wave dominated, while the decay into kaons also shows strong contributions of P - and D -waves. Therefore the former decay indeed seems to be particularly suitable in order to extract information about pion and kaon dynamics at higher energies.

Interestingly we see that the peak structure at 1.45 GeV in the event distribution of $\bar{B}_s^0 \rightarrow J/\psi\pi\pi$, corresponding to the $f_0(1500)$ resonance, is in line with the analysis performed by the LHCb collaboration. However at higher energies our fit seems to prefer a broader resonance with a higher mass. Because of the interplay of many resonances with different angular momenta it would be interesting to reanalyze the energy region above 1.5 GeV with higher statistics data sets and additional final channels such as an exclusive 4π channel, which is not yet available in partial-wave decomposed form [21].

In Fig. 2 we show the extracted pion and kaon isoscalar scalar form factors using a $\rho\rho$ channel. We consider $N_R = 2$ with a constant polynomial in Eq. (8) (Fit 1), a linear polynomial (Fit 2), and $N_R = 3$ with a constant polynomial (Fit 3). We see that the three fits are consistent with each other below 1.5 GeV. Above this energy the form factors are not as strongly constrained and show more significant deviations. We can also compare our results to a two-channel Omnès solution and notice that below 1 GeV they seem to be consistent with each other.

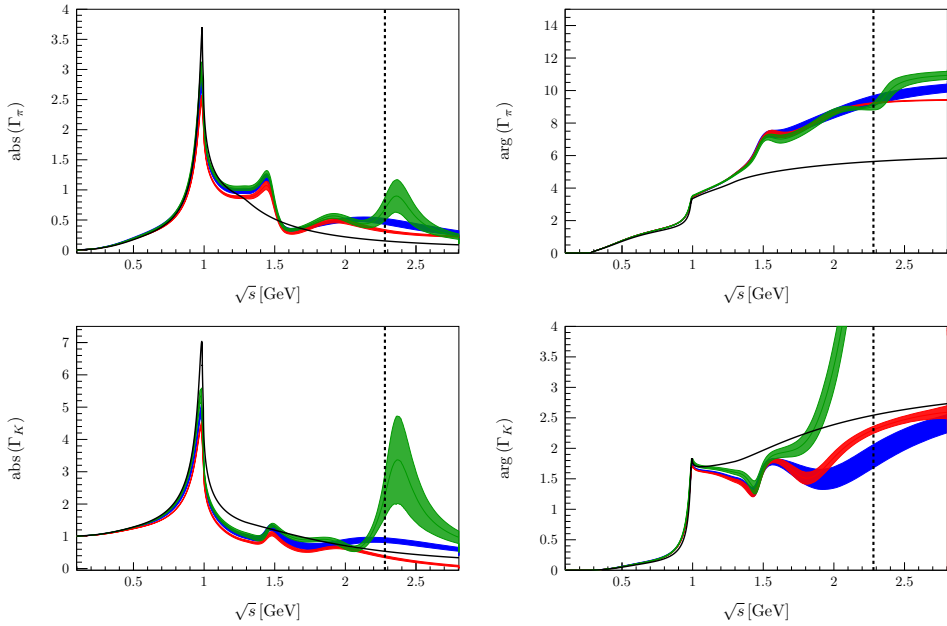


Figure 2. Modulus (left) and phase (right) of the extracted pion (top) and kaon form factor (bottom) with an additional $\rho\rho$ channel. We see Fit 1 in blue, Fit 2 in red, and Fit 3 in green. The dotted lines correspond to the kinematic energy range available in the \bar{B}_s^0 decay. Additionally we see a two-channel Omnès solution as a black solid line.

4 Pole positions

As we utilized a fully unitary and analytical amplitude to extract the S -wave it is possible to extract resonance poles. For this purpose we used Padé approximants [22–24]. With a series of different fits, which are further explained in Ref. [11], we extracted the resonance poles of four different S -wave resonances on the nearest unphysical sheet.

The first resonance corresponds to the $f_0(500)$. We obtain a pole with a mass of (442 ± 2) MeV and a width of (512 ± 10) MeV. By construction the pole position should be close to the one defined by the input scattering amplitude T_0 [13], which lies at $(441 - i 544/2)$ GeV. While the real parts are consistent with each other, we see some deviation in the imaginary parts. This shows that even at lower energies small shifts of our parametrization to the dispersive results are possible. These deviations might be improved upon by subtracting higher orders of the resonance exchange potential V_R .

A similar picture can be seen for the $f_0(980)$ resonance, which can still be covered by the dispersive solutions [8]. As a benchmark we again use the input [13], which prefers a pole position of $(998 - i 42/2)$ MeV. Compared to that we obtain a resonance with mass (996 ± 6) MeV and width (57 ± 11) MeV. As overall we could reproduce these two resonance poles we assume that our extraction with Padé approximants is sensible and also applicable for the two heavier resonances.

The first additional resonance that we obtain is the $f_0(1500)$ with a mass of (1465 ± 18) GeV and width (100 ± 19) MeV. This result is perfectly consistent with the extraction of LHCb given by a mass of (1465.9 ± 3.1) MeV and width (115 ± 7) MeV [18]. In contrast, our fit prefers to shift the second resonance to a higher mass and width. While LHCb found a resonance with a mass (1809 ± 22) MeV and width (263 ± 30) MeV [18], quoted as $f_0(1790)$,

we prefer a mass of (1910 ± 50) MeV and width (398 ± 79) MeV, which is more in line with the $f_0(2020)$. In order to fully establish this difference a data set with higher statistics or different exclusive channels would be necessary.

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