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Double heavy tri-hadron bound state with strange flavor

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Abstract. Through the Born-Oppenheimer Approximation, we have performed a comprehensive investigation of the DD^*K, $D\bar{D}^*K$, $BB^*\bar{K}$ and $B\bar{B}^*\bar{K}$ molecular states. In the framework of One-Pion Exchange model as well as the treatments of the coupled-channel effects and S-D wave mixing, we find a loosely bound tri-meson molecular state these systems with the isospin configuration $|0, \frac{1}{2}, \pm\frac{1}{2}\rangle$ and quantum number $I(J^P) = 1/2(1^-)$, where the $\frac{1}{2}$ is the total isospin of the three-body system, the 0 is the isospin of the D^*K, $\bar{D}^*K$, $B^*\bar{K}$ and $\bar{B}^*\bar{K}$. With the estimated error, the mass of the DD^*K or $D\bar{D}^*K$ molecule is $4317.92^{+3.66}_{-4.32}$ MeV or $4317.92^{+6.13}_{-6.55}$ MeV. We also extend our calculations to the bottom sector and find tri-meson bound states for the $BB^*\bar{K}$ and $B\bar{B}^*\bar{K}$ with the mass $11013.65^{+8.49}_{-8.84}$ MeV and $11013.65^{+8.68}_{-9.02}$ MeV respectively.

1 Introduction

The investigations on the three body systems have been an important issue in recent years. The explorations on the few-body systems were first proposed to focus on the few nucleon systems or nucleons and hyperons [1–4]. Systematical calculations on the resonances containing three mesons were also studied [5–7]. The $X(2175)$ as a resonant state of the $\phi K\bar{K}$ system was stated in [5]. Theoretical study on the $KK\bar{K}$ system and dynamical generation of the $K(1460)$ resonance was performed in [6]. Theoretical support for the $\pi(1300)$ and the reported $f_0(1790)$ as a candidate of $f_0(980)\pi\pi$ molecular resonances were claimed in [7]. Theoretical interpretations on some low lying excited baryons could be reproduced by the resonances composed of two mesons and one baryon were also be proposed in [8–10]. Many theorists have extended the investigations on the three body systems to the charm sector. The description of the $Y(4260)$ as a resonant state of $J/\psi K\bar{K}$ system stated in [11]. The energy and width of a narrow $I = 1/2$ DNN quasibound state had been discussed in [12]. The studies on the NDK, $\bar{K}DN$ and $ND\bar{D}$ molecules publicised in [13]. The $N\bar{K}K$ system and the signature of a $N^*(1920)(1/2^+)$ state were studied and the Fixed Center Approximation to the Faddeev equations was used in [14]. A complete description of the DKK and $DK\bar{K}$ with the Fixed Center Approximation was also discussed in [15]. The systems were also studied in [16, 17]. Very recently, the theoretical calculations on the $BD\bar{D}$ and BDD systems were performed, and the hints of a bound state in the energy region 8935-8985 MeV with the quantum numbers $I(J^P) = 1/2(0^-)$ for the $BD\bar{D}$ system was claimed [18]. E. Wilbring *et al*. had calculated the molecule-meson scattering amplitudes for the $Z_b(10610)$ or $Z_b(10650)$ by employing the molecule assumptions and the dimmer tricks [19]. They had predicted the phase shifts for

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the scattering of B and B^* mesons off the exotic mesons $Z_b(10610)$ and $Z_b(10650)$, and the possibility for universal bound states of three B and B^* mesons form the Efimov effect had been ruled out. Moreover, the three-particle Efimov states in a finite volume via the dimmer formalism had been extensively explored in [20, 21].

The one pion exchange model (OPE) has been widely used to study the di-meson or di-baryon molecular systems. Since the pion has the lowest mass, longest effect range and the constituent hadrons should be well separated, the long range one-pion-exchange should play a dominant role among the possible exchange mesons. It is reasonable to extend the OPE model to a system containing three mesons, if we only focus on searching for its loosely bound states. The Born-Oppenheimer Approximation has been employed to study the few-body systems containing several heavy particles and several light particles [22]. For a three-particle system, it allows us to separate the system into two part. One part is the motion of the light particles relative to the heavy particles with fixed locations; The other part is the relative motion between the heavy particles. The Born-Oppenheimer Approximation has also been used to explore the properties of the di-meson molecules [23].

The di-meson and di-baryon molecules are formed by the effective potential produced by the pion exchange. To a very good extent, it is a pion shared by the two constitutes providing an attraction which makes them bound. It can be regarded as a σ *bond* similar to the σ *bond* in hydrogen molecules. There is another kind of bond called π *bond* universally existing in benzene molecules, which is a pair of electrons shared by the six carbon atoms. Therefore, it is fascinating that whether there exists a hadronic π *bond* in a system containing more than two hadrons. Specifically, whether there exist hadronic π *bond* in the tri-meson systems like DD^*K, $D\bar{D}^*K$, $BB^*\bar{K}$ and $B\bar{B}^*\bar{K}$.

2 One-pion-exchange model and the two-body sector

The OPE interaction means there should be only one pion exchanged by any two constituents. The DD^*K or $D\bar{D}^*K$ sharing one virtual pion seems like a benzene molecule with an electron pair shared by its six carbon atoms. We call the interaction produced by one pion shared by more than two hadrons as a hadronic π *bond*. However, the conservations of the angular momentum and C parity don't allow the direct channel of the DK, D^*K and $\bar{D}^*K$. Thus, the interactions in the DD^*K or $D\bar{D}^*K$ system are dominant by the coupled-channel effects. Specifically, if we use a, b and c to label the locations of the three distinct mesons in the original channel, which is $D_aD_b^*K_c$ ($D_a\bar{D}_b^*K_c$). It can change into $D_aD_bK_c^*$ ($D_a\bar{D}_bK_c^*$) via one-pion-exchange between b and c. While the channel $D_aD_bK_c^*$ ($D_a\bar{D}_bK_c^*$) can also change into $D_a^*D_bK_c$ ($D_a^*\bar{D}_bK_c$) through a pion exchanged between a and c. When a pion arises between a and b, the channel $D_a^*D_bK_c$ ($D_a^*\bar{D}_bK_c$) returns back into the original channel $D_aD_b^*K_c$ ($D_a\bar{D}_b^*K_c$). Within this scenario, one might expect that the three channels are coupled together to form a loosely bound state. The same analysis can also be extended to the $BB^*\bar{K}$ and $B\bar{B}^*\bar{K}$ systems.

The Lagrangians with the SU(2) chiral symmetry and C parity conservation read

$$\mathcal{L}_P = -i\frac{2g}{f_\pi}\bar{M}P_b^{*\mu}\partial_\mu\phi_{ba}P_a^\dagger + i\frac{2g}{f_\pi}\bar{M}P_b\partial_\mu\phi_{ba}P_a^{*\mu\dagger}, \tag{1}$$

where the heavy flavor meson fields $P^{(*)}$ represent $P^{(*)} = (D^{(*)0}, D^{(*)+})$, $(B^{(*)-}, \bar{B}^{(*)0})$ or $(K^{(*)-}, \bar{K}^{(*)0})$. The ϕ in the above represents the exchanged pion matrix:

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 & \pi^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 \end{pmatrix}. \tag{2}$$

In the OPE model, there are three coupling coefficients need to be determined. One is the pion decay constant $f_\pi = 132$ MeV, we got from chiral symmetry. And one is the pionic coupling constant $g = 0.59$ extracted from the width of D^{*+}[24]. The other one is the coupling constant g'. Here, we take the experimental result $g' = 0.90$ which was obtained from the full widths of K^* in the PDG [24].

In the calculations, each vertex in the Feynman diagram needs a form factor to suppress the high momentum contribution. We take the conventional form for the form factor as in the Bonn potential model, which is $F(q) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - q^2} = \frac{\Lambda^2 - m_\pi^2}{\tilde{\Lambda}^2 + \vec{q}^2}$, where m_π is the mass of the exchanged meson and $\tilde{\Lambda}^2 = \Lambda^2 - (\Delta M^{(\prime)})^2$, $\Delta M = \{(M_D^{*2} + M_K^{*2}) - (M_D^2 + M_K^2)\}/\{2(M_D + M_K^*)\}$ and $\Delta M' = \{(M_B^{*2} + M_K^{*2}) - (M_B^2 + M_K^2)\}/\{2(M_B + M_K^*)\}$. The role of the form factor is to remove or suppress the contribution from the ultraviolet region of the exchanged momentum.

After carefully solving the couple-channel Schrödinger equation with the treatment of the S-D wave mixing, we find that for the D^*K system, the isospin $I = 1$ case gives a repulsive potential, obviously, it has no bound solution. While the isospin $I = 0$ case corresponds to an attractive potential then has a bound state. For the $B^*\bar{K}$ system, we find out a loosely bound state for the $B^*\bar{K}$ with its isospin $I = 0$. We assume the $D_{s1}(2460)$ reported in the experiment is dominant by a loosely bound D^*K molecule [25]. The cutoff parameter could be fixed definitely at 803.20 MeV. Then the binding energy and the root-mean-square radius of the bound state obtained are 42.29 MeV and 1.14 fm, respectively. The dominant component is the S wave with a probability 98.92 %. While the corresponding component from D wave is 1.08 %. Numerically, the probability of the component D^*K is 75.83 % while that of the component DK^* is 24.17 %. We also adopt the speculations proposed in [26], that the $D_{s1}(2460)$ has a counter partner state in bottom meson sector. It might be a good candidate of the $B^*\bar{K}$ molecular state with mass 5778 MeV. Thus its binding energy might be 42.19 MeV. Similar to the treatment for the D^*K, we fix the cutoff parameter at 1451.00 MeV. Then the binding energy and the root-mean-square radius of the bound state obtained are 42.19 MeV and 0.96 fm, respectively. The probability for the S wave and D wave component are 99.32 % and 0.68 %. The probability of the component $B^*\bar{K}$ is 78.66 % while that of the component $B\bar{K}^*$ is 21.34 %.

3 Three-body sector and Born-Oppenheimer Approximation

Based on the main idea of the BOA, there are two kinds of degree of freedoms in the DD^*K system as an example, the motion of the K related to the D and D^*, and the relative motion between D and D^*. In the Born-Oppenheimer Approximation, both of the two degrees of freedom are independent, and they can be treated separately. Therefore, the total wave functions of the three-body system DD^*K can be expressed as

$$\begin{aligned} |\Psi_{Total}\rangle &= C_0\{\varphi(\vec{R})\psi(\vec{r_b})\,|D_a D_b^* K_c\rangle + C\varphi'(\vec{R})[\psi'(\vec{r_a}) + \psi'(\vec{r_b})]\,|D_a D_b K_c^*\rangle \\ &+ \varphi(\vec{R})\psi(\vec{r_a})\,|D_a^* D_b K_c\rangle\}, \end{aligned} \tag{3}$$

where the $\varphi(\vec{R})$ and $\varphi'(\vec{R})$ are normalized, which satisfy $\int \varphi^*(\vec{R})\varphi(\vec{R})d\vec{R} = 1$, $\int \varphi'^*(\vec{R})\varphi'(\vec{R})d\vec{R} = 1$. The C_0 is the normalization coefficient. The wave functions $\psi(\vec{r_a})$ and $\psi'(\vec{r_a})$ can be determined in the last section. The C is a variation parameter.

The total Hamiltonian of the three-body system DD^*K is

$$H_T = T_h + V_h + T_K + V_K,$$

where the T_h and V_h denote the relative kinetic energy and the effective potential between the D and D^* respectively. The T_K and V_K is the kinetic energy and effective potential of the K in the DD^*K system. The Schrödinger equation for the three-body system can be simplified as

$$E_3|\Phi(\vec{R})\rangle = H'_T|\Phi(\vec{R})\rangle = [T_h(\vec{R}) + V_h(\vec{R}) + V_{BO}(\vec{R})]|\Phi(\vec{R})\rangle,$$

where $H'_T = H_T - E_b$. The reduced energy $E_3 = E_T - E_b$.

After solving the Schrödinger equation for the three-body system, we find out the loosely bound states for the DD^*K and $D\bar{D}^*K$ with the isospin configuration $|0, \frac{1}{2}, \pm\frac{1}{2}\rangle$. For the system DD^*K and $D\bar{D}^*K$, if we assume the $D_{s1}(2460)$ is molecular state of D^*K respectively, then the cutoff parameters Λ can be fixed at 803.20 MeV. The reduced three-body binding energy $|E_3|$ for the DD^*K ($D\bar{D}^*K$) is 8.29 MeV, and the root-mean-square radius for the D and D^* ($\bar{D}^*$) is 1.65 fm. For the DD^*K, the isospin configuration $|0, \frac{1}{2}, \frac{1}{2}\rangle$ contains the D^+D^{*+} and D^+D^{*0}, the $|0, \frac{1}{2}, -\frac{1}{2}\rangle$ contains the D^0D^{*+} and D^0D^{*0}. Taking into account the effective potentials and the channel-dependent coefficients, we find the effective potentials for the DD^* are cancelled out. Thus, only the BO potential provided by the K effect the three-body system. For the $D\bar{D}^*K$, the isospin configuration $|0, \frac{1}{2}, \frac{1}{2}\rangle$ contains the $D^+\bar{D}^{*0}$ and D^+D^{*-}, the $|0, \frac{1}{2}, -\frac{1}{2}\rangle$ contains the $D^0\bar{D}^{*0}$ and D^0D^{*-}. Similarily, without the BO potential, the four channel have no possibility to be bound states.

The $BB^*\bar{K}$ and $B\bar{B}^*\bar{K}$ also have a loosely bound states with the isospin configuration $|0, \frac{1}{2}, \pm\frac{1}{2}\rangle$. For the system $BB^*\bar{K}$ and $B\bar{B}^*\bar{K}$, if we assume there exists a $B^*\bar{K}$ molecule at 5778 MeV as a counter-partner of the D^*K, then the cutoff parameter Λ' can be fixed at 1451.00 MeV. The reduced three-body binding energy $|E_3|$ for the $BB^*\bar{K}$ ($B\bar{B}^*\bar{K}$) is 41.76 MeV, and the root-mean-square radius for the B and B^* ($\bar{B}^*$) is 0.65 fm. For the $BB^*\bar{K}$, the isospin configuration $|0, \frac{1}{2}, \frac{1}{2}\rangle$ contains the B^+B^{*+} and B^+B^{*0}, the $|0, \frac{1}{2}, -\frac{1}{2}\rangle$ contains the B^0B^{*+} and B^0B^{*0}. For the $B\bar{B}^*\bar{K}$, the isospin configuration $|0, \frac{1}{2}, \frac{1}{2}\rangle$ contains the $B^+\bar{B}^{*0}$ and B^+B^{*-}, the $|0, \frac{1}{2}, -\frac{1}{2}\rangle$ contains the $B^0\bar{B}^{*0}$ and B^0B^{*-}. Similar with the charm systems, the BO potential plays an important role to make them bound.

The discussion on the DD^*K and $D\bar{D}^*K$ molecules above are based on the assumption that the reported state $D_{s1}(2460)$ can fit into the D^*K molecule scenario, then the cutoff parameter Λ can be fixed at 803.20 MeV. It is interesting to investigate the sensitivity of the three-body eigen-energy to the variance of the cutoff parameter Λ. The numerical results on the $BB^*\bar{K}$ and $B\bar{B}^*\bar{K}$ molecules above are based on the hypothesis that there exists a $B^*\bar{K}$ molecule at 5778 MeV as a counter-partner of the D^*K, then the cutoff parameters Λ' can be fixed at 1451.00 MeV. We plot dependence of the three body binding energy $|E_3|$ on the two body binding energy $|E_b|$ for the DD^*K and $D\bar{D}^*K$ in Figure 1. That dependence for the $BB^*\bar{K}$ and $B\bar{B}^*\bar{K}$ also plotted in Figure 1. Where the red and green bands are from PDG2016 and PLB647,133, respectively. The three-body binding energies are $E^{DD^*K}_{I=1/2} = 8.29^{+4.32}_{-3.66}$ MeV, $E^{BB^*\bar{K}}_{I=1/2} = 41.76^{+8.49}_{-8.84}$ MeV. The error band is estimated as $m_K/(2m_u)$ with m_u the reduced mass of the two heavy particles. As the long-distance DD^* potential is related to that of the $D\bar{D}^*$ potential by G-parity [27], there could also exist a three-body $D\bar{D}^*K$ bound state, but with additional uncertainty $m_\pi^2/(2m_u)$ (which characterizes the natural energy scale of OPE [28]) from the unknown short-distance interaction. Thus, for the $D\bar{D}^*K$ and $B\bar{B}^*\bar{K}$ system, the three-body binding energies are $E^{D\bar{D}^*K}_{I=1/2} = 8.29^{+6.55}_{-6.13}$ MeV, $E^{B\bar{B}^*\bar{K}}_{I=1/2} = 41.76^{+8.68}_{-9.02}$ MeV. With the additional uncertainty from the missing short-distance interaction. Those correspond to two bound states with masses $4317.92^{+6.13}_{-6.55}$ MeV and $11013.65^{+8.68}_{-9.02}$ MeV.

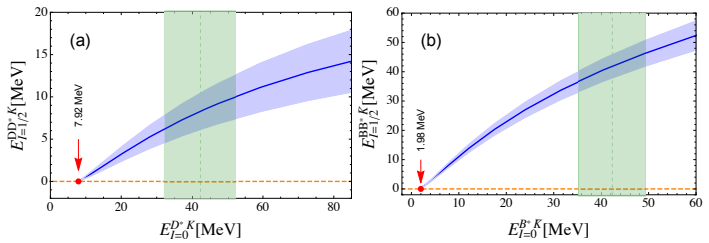

Figure 1. The dependence of the three-body binding energy $|E_3|$ on the two-body binding energy $|E_b|$. (a) is for the charm sector. (b) is for the bottom sector.

4 Summary and Discussion

In the present paper, we have performed an extensive study on the possibility of the DD^*K, $D\bar{D}^*K$, $BB^*\bar{K}$ and $B\bar{B}^*\bar{K}$ in forming the tri-meson molecules. With the hypothesis of the $D_{s1}(2460)$ as the D^*K molecule, the cutoff parameter Λ can be fixed at 803.20 MeV. After the calculation of the Schrödinger Equation within the Born-Oppenheimer Approximation as well as the OPE scheme, we find out a loosely bound tri-meson molecular state for the DD^*K and $D\bar{D}^*K$ system with the isospin configuration $|0, \frac{1}{2}, \pm\frac{1}{2}\rangle$. The reduced three-body binding energy of the DD^*K and $D\bar{D}^*K$ molecule are $8.29^{+4.32}_{-3.66}$ MeV and $8.29^{+6.55}_{-6.13}$ MeV. The root-mean-square between D and K as well as D^* ($\bar{D}^*$) and K are both 1.14 fm. The root-mean-square between D and D^* ($\bar{D}^*$) is 1.65 fm. The root-mean-square structure for the DD^*K and $D\bar{D}^*K$ seem like a triangle. As we have derived in Sect. 2, the two-body binding energy of the D^*K or $\bar{D}^*K$ is 42.29 MeV. Therefore, the total binding energy of the DD^*K and $D\bar{D}^*K$ molecules relative to the DD^*K threshold are both the $|E_T| = |E_3 + E_b|$ and equal to 50.58 MeV. With the estimated error, the mass of the DD^*K and $D\bar{D}^*K$ molecules are $4317.92^{+3.66}_{-4.32}$ MeV and $4317.92^{+6.13}_{-6.55}$ MeV.

Our calculations have indicated that the heavy mesons in the three-body system can't be bound without the BO potential provided by the K, since the effective potential of some channels like $D^+D^{*+}(D^+\bar{D}^{*0})$ and $D^0D^{*0}(D^0D^{*-})$ are all repulsive. The virtual pion exchanged in the three body system plays a vital role in forming the tri-meson molecules. Its status seems like the π *bond* in the Benzene molecule where the six carbons share a pair of electrons. Obviously, the delocalized π *bond* also exists in the tri-meson molecular states.

We also extend our investigation to the bottom sector. With the assumption that there exists a $B^*\bar{K}$ molecule at 5778 MeV as a counter-partner of the D^*K, then the cutoff parameter Λ' can be fixed at 1451.00 MeV. We also find out a loosely bound tri-meson molecular state for the $BB^*\bar{K}$ and $B\bar{B}^*\bar{K}$ with the isospin configuration $|0, \frac{1}{2}, \pm\frac{1}{2}\rangle$. The reduced three-body binding energy $|E_3|$ for the $BB^*\bar{K}$ ($B\bar{B}^*\bar{K}$) is $41.76^{+8.49}_{-8.84}$ MeV. The two-body binding energy of the $B^*\bar{K}$ or $\bar{B}^*\bar{K}$ is 42.19 MeV. Thus, the total binding energy of the $BB^*\bar{K}$ and $B\bar{B}^*\bar{K}$ molecules relative to the BB^*K threshold is the $|E_T| = |E_3 + E_b|$ and equal to 83.95 MeV. With the estimated error, the mass of the $BB^*\bar{K}$ and $B\bar{B}^*\bar{K}$ molecule are $11013.65^{+8.49}_{-8.84}$ MeV. The root-mean-square radius for the B and B^* ($\bar{B}^*$) is 0.96 fm. The root-mean-square radius for the $\bar{K}$ and B^* ($\bar{B}^*$) is 0.65 fm. Thus, its root-mean-square structure for the $BB^*\bar{K}$ ($B\bar{B}^*\bar{K}$) also seems like a triangle.

The $D\bar{D}^*K$ molecule may be easily detected in experiment. As the experiments on the B meson decay have been well developed. It will provide us a good platform to probe the tri-meson molecule $D\bar{D}^*K$. The $D\bar{D}^*K$ molecule with the isospin $|0, \frac{1}{2}, \frac{1}{2}\rangle$ and $I(J^P) = 1/2(1^-)$ has one unit positive charge. It is possible to be probed in the process $B^+ \to T(4318)^+ \to$

$(J/\psi\pi^0K^+)\pi^0$, or to be detected in the process $B^0 \to T(4318)^+ \to (J/\psi\pi^0K^+)\pi^-$. Here we use $T(4318)^+$ denote the charge partner of the $D\bar{D}^*K$ molecular state. The $D\bar{D}^*K$ molecule with the isospin $|0, \frac{1}{2}, -\frac{1}{2}\rangle$ is neutral. Its main decay process could be $B^\pm \to T(4318)^0 \to (J/\psi\pi^0K^+)\pi^\pm$. The radiative decays $B^+ \to T(4318)^+ \to (J/\psi\pi^0K^+)\gamma$ and $B^\pm \to T(4318)^0 \to (J/\psi\pi^0K^0)\gamma$ might be another way to detect the tri-meson molecules. The LHCb, BABAR, Belle, BESIII and other collaborations have accumulated many decay data on the process $B^\pm \to (J/\psi\pi^+\pi^-)K^\pm$. These data mainly analyzed around 3872 MeV in the $J/\psi\pi^+\pi^-$ invariant mass. If we analyze these data with a higher mass at 4312 MeV in the $J/\psi\pi K$ invariant mass, It will unveil the information on the tri-meson system $D\bar{D}^*K$. Moreover, the DD^*K molecule has possibility to be probed in the forthcoming experiments on the pp collision.

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