Decay $D_s^+ \to \phi\ell^+\nu_\ell$ in Covariant Quark Model

N. R. Soni[1,*] and *J. N.* Pandya[1,**]

[1]Applied Physics Department, Faculty of Technology and Engineering,
The Maharaja Sayajirao University of Baroda, Vadodara 390001, Gujarat, *INDIA*.

Abstract. We present the branching fractions for the leptonic and semileptonic decays $D_s^+ \to \ell^+\nu_\ell$ for $\ell = e, \mu, \tau$ and $D_s^+ \to \phi\ell^+\nu_\ell$ for $\ell = e, \mu$. We use the covariant confined quark model with built-in infrared confinement for computation of required transition form factors in the whole physical range of momentum transfer. Our results on the leptonic decays are found to agree well with the available experimental data. We obtain the following results for the semileptonic branching fractions of D_s^+ mesons: $\mathcal{B}(D_s^+ \to \phi e^+\nu_e) = 2.85$ % and $\mathcal{B}(D_s^+ \to \phi\mu^+\nu_\mu) = 2.70$ %. The ratios of branching fractions are found to be $\mathcal{B}(D_s^+ \to \phi\mu^+\nu_\mu)/\mathcal{B}(D_s^+ \to \phi e^+\nu_e) = 0.95$ which is consistent with the recent BESIII data.

1 Introduction

With the advancement of detector and accelerator technology updated and accurate results are available. BESIII in Beijing China is providing lot of results on the D and D_s meson decays. Out of which, the leptonic and semileptonic decays of $D_{(s)}$ provide very important tool for understanding dynamics of weak and strong interaction for the charm sector [1]. The branching fractions of these decays are proportional to the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, therefore these channels provide the direct determinations of $c \to s$ matrix element.

The branching fraction for leptonic decays of D_s meson ($D_s^+ \to \ell^+\nu_\ell$) was reported by CLEO [2–4], *BABAR* [5], Belle [6] and BESIII [7] collaborations. Very recently, BESIII collaboration have also reported the most precise results on leptonic decay constant from the leptonic branching fraction for the channel $D_s \to \mu^+\nu_\mu$ [8].

The branching fractions of $D_s \to \phi\ell^+\nu_\ell$ for electron channel were reported by BESIII [9], *BABAR* [10] and CLEO [11] collaborations but only the BESIII collaboration [9] has reported the branching fractions for muon channel. The form factors for $D_s \to \phi\ell^+\nu_\ell$ are also computed for the first time using lattice quantum chromodynamics (LQCD) by HPQCD collaboration [12]. The semileptonic branching fractions of D and D_s mesons are also computed using other theoretical approaches such as heavy meson chiral theory [13, 14], chiral unitary approach [15, 16], constituent quark model [17], light front quark model [18].

We have been able to compute the form factors and semileptonic decay widths for $D \to (K, \pi)\ell^+\nu_\ell$ channel [19] and the same was found consistent with recent BESIII data [20]. Further, in this paper, we compute the transition form factors to determine leptonic and

*e-mail: nrsoni-apphy@msubaroda.ac.in
**e-mail: jnpandya-apphy@msubaroda.ac.in

semileptonic branching fractions of the channel $D_s \to \phi \ell^+ \nu_\ell$ for $\ell = e$ and μ using the covariant confined quark model (CCQM) with built-in infrared confinement developed by G. V. Efimov and M. A. Ivanov [21–24]. One of the key feature of the CCQM is the computation of form factors in the entire physical range of momentum transfer. This method is applicable to hadronic system with any number of quark. Recently, this approach has been successfully employed in studing the decay properties of $B_{(s)}$ mesons [25–28], baryons [29–31] and exotic states [32–34].

2 Theoretical Framework

The CCQM is an effective quantum field theoretical approach for hadronic interactions [21–24]. Here, the interaction Lagrangian for meson is written in terms of constituent quarks given by,

$$L_{int} = g_M M(x) \int dx_1 \int dx_2 F_M(x; x_1, x_2) \cdot \bar{q}^a_{f_1}(x_1) \Gamma_M q^a_{f_2}(x_2) + H.c. \tag{1}$$

Similarly, the interaction lagrangian for the hadronic state with any number of quarks can be written. Here Γ_M is the Dirac matrix in accordance to the relevant mesonic field $M(x)$ and F_M is the vertex function that characterizes the effective size of the meson. For simplicity, we choose the Gaussian function for the vertex factor. It is important to note that any form of the vertex function can be taken but is should have appropriate falloff behavior in order to remove the ultraviolet divergences in the loop integral. g_M is the coupling constant computed using the Compositeness conditions [35, 36] that effectively guarantees the confinement of quarks within the hadrons. For detailed computation technique, we suggest to follow the Ref. [24]. The independent model parameters such as quark mass and size parameter are determined by fitting calculated decay constants Fig. 1 to the experimental or lattice results. We take the quark masses m_c and m_s to be 1.672 and 0.428 GeV respectively and the size parameters for D_s^+ and ϕ mesons to be 1.784 and 0.883 GeV. All other parameters such as meson masses, life time of D_s meson, CKM matrix elements are taken from the PDG [37].

The branching fraction for leptonic decay can be written as

$$\mathcal{B}(D_s^+ \to \ell^+ \nu_\ell) = \frac{G_F^2}{8\pi} m_{D_s} m_\ell^2 \left(1 - \frac{m_\ell^2}{m_{D_s}}\right)^2 f_{D_s}^2 |V_{cs}|^2 \tau_{D_s} \tag{2}$$

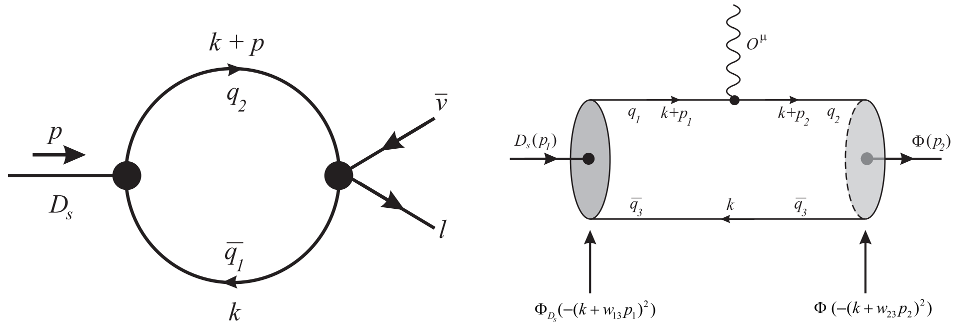
and the results for the leptonic branching fractions for $\ell = e, \mu$ and τ are tabulated in Tab. 1.

Table 1: Leptonic D_s^+-decay branching fraction ($\tau_{D_s^+} = 5.04 \times 10^{-13}$ s)

Channel	Present	PDG 2018 [37]
$D_s^+ \to e^+ \nu_e$	1.33×10^{-7}	$< 8.3 \times 10^{-5}$
$D_s^+ \to \mu^+ \nu_\mu$	5.64×10^{-3}	$(5.50 \pm 0.23) \times 10^{-3}$
$D_s^+ \to \tau^+ \nu_\tau$	5.49%	$(5.48 \pm 0.23)\%$

Next, we compute the semileptonic branching fractions of D_s meson. The matrix element for the feynman diagram Fig. 1 in terms of form factors can be written as

$$\langle \phi(p_2, \epsilon_\nu) | \bar{s} O^\mu c | D_s \rangle \quad = \quad N_c g_{D_s} g_\phi \int \frac{d^4k}{(2\pi)^4 i} \Phi_{D_s}(-(k + w_{13} p_1)^2) \Phi_\phi(-(k + w_{23} p_2)^2)$$



(a) leptonic D_s decay (b) semileptonic D_s decay

Figure 1: Feynman diagrams

Table 2: Parameters of double pole parametrization Eq. (4) for the form factor

	A_0	A_+	A_-	V
$F(0)$	2.13	0.67	-0.95	0.91
a	0.59	1.06	1.21	1.20
b	-0.12	0.17	0.26	0.24

$$\begin{aligned} & \mathrm{tr}[O^{\mu} S_1(k+p_1)\gamma^5 S_3(k)\ \not\epsilon_{\nu}^{\dagger} S_2(k+p_2)] \\ = & \frac{\epsilon_{\nu}^{\dagger}}{m_1+m_2}\Big[-g^{\mu\nu} P\cdot q A_0(q^2) + P^{\mu}P^{\nu}A_+(q^2) + q^{\mu}P^{\nu}A_-(q^2) \\ & + i\varepsilon^{\mu\nu\alpha\beta}P_{\alpha}q_{\beta}V(q^2)\Big] \end{aligned} \tag{3}$$

with $P = p_1 + p_2$, $q = p_1 - p_2$, ϵ_ν is the polarization vector of the ϕ meson such that $\epsilon_\nu^\dagger \cdot p_2 = 0$. Also $p_1^2 = m_{D_s}^2$, $p_2^2 = m_\phi^2$ and $w_{ij} = m_{q_j}/(m_{q_i} + m_{q_j})$ which considers all the particles are on-shell.

In Fig. 2, we plot the form factors in the entire physical range of momentum transfer. The form factors are also written in terms of double pole approximation as

$$F(q^2) = \frac{F(0)}{1 - a\left(\frac{q^2}{m_{D_s}^2}\right) + b\left(\frac{q^2}{m_{D_s}^2}\right)^2} \tag{4}$$

The numerical results of form factors and associated double pole parameters are tabulated in Tab. 2. With the help of form factors, the semileptonic differential branching fraction can be written in terms of helicity amplitude as [38, 39],

$$\frac{d\Gamma(D_{(s)} \to \phi\ell^+\nu_\ell)}{dq^2} = \frac{G_F^2|V_{cq}|^2|\mathbf{p_2}|q^2}{12(2\pi)^3 m_{D_s}^2}\left(1 - \frac{m_\ell^2}{q^2}\right)^2\left[\left(1 + \frac{m_\ell^2}{2q^2}\right)\sum_{n=0,+,-}|H_n|^2 + \frac{3m_\ell^2}{2q^2}|H_t|^2\right] \tag{5}$$

Here $|\mathbf{p_2}| = \lambda^{1/2}(m_{D_s}^2, m_\phi^2, q^2)/2m_{D_s}$ is the momentum of ϕ meson in the rest frame of D_s meson and λ is the Källen function. The helicity amplitudes $H's$ in terms of invariant form

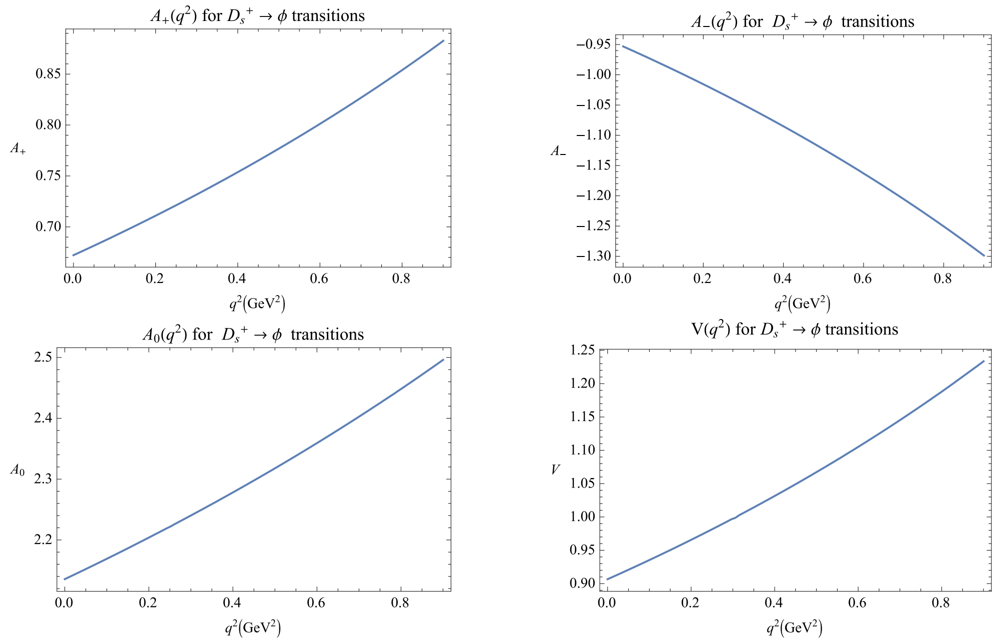


Figure 2: Form factors for $q^2 \geq 0$

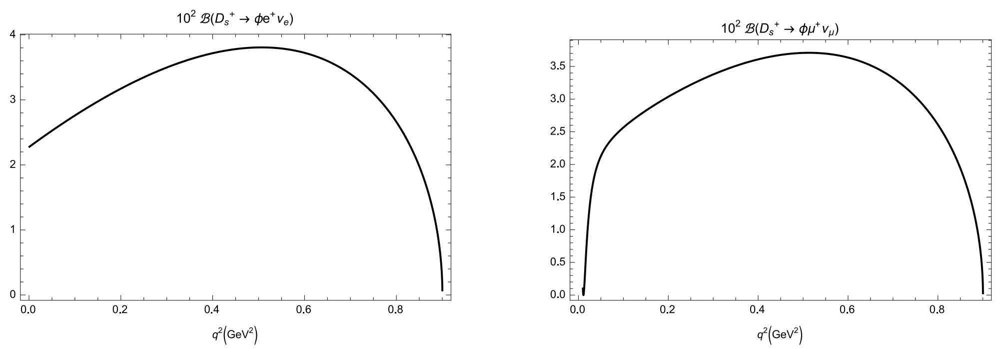


Figure 3: Differential branching fraction for the decays $D_s^+ \to \phi \ell^+ \nu_\ell$

factors can be written as

$$
\begin{aligned}
H_t &= \frac{1}{m_1 + m_2} \frac{m_1 |\mathbf{p_2}|}{m_2 \sqrt{q^2}} \left((m_1^2 - m_2^2)(A_+ - A_-) + q^2 A_- \right) \\
H_\pm &= \frac{1}{m_1 + m_2} (-(m_1^2 - m_2^2) A_0 \pm 2 m_1 |\mathbf{p_2}| V) \\
H_0 &= \frac{1}{m_1 + m_2} \frac{1}{2 m_2 \sqrt{q^2}} (-(m_1^2 - m_2^2)(m_1^2 - m_2^2 - q^2) A_0 + 4 m_1^2 |\mathbf{p_2}|^2 A_+). \qquad (6)
\end{aligned}
$$

Table 3: Semileptonic branching fraction for D_s meson (in %).

Channel	Present	Data	Reference
$\mathcal{B}(D_s^+ \to \phi e^+ \nu_e)$	2.85	$2.26 \pm 0.45 \pm 0.09$	BESIII [9]
		$2.61 \pm 0.03 \pm 0.08 \pm 0.15$	*BABAR* [10]
		$2.14 \pm 0.17 \pm 0.08$	CLEO [11]
$\mathcal{B}(D_s^+ \to \phi \mu^+ \nu_\mu)$	2.70	$1.94 \pm 0.53 \pm 0.09$	BESIII [9]
$\mathcal{R}_{\mu/e} = \frac{\mathcal{B}(D_s^+ \to \phi \mu^+ \nu_\mu)}{\mathcal{B}(D_s^+ \to \phi e^+ \nu_e)}$	0.95	0.86 ± 0.29	BESIII [9]

3 Result and Discussion

In this article, we have employed covariant confined quark model with built-in infrared confinement for computing weak decays of charmed strange mesons. In Tab. 1, we present our results on leptonic branching fractions for $\ell = e, \mu, \tau$. We also compare our findings with the PDG average data and it is observed that our results for electron channel satisfies the experimental constraint. For muon and tau channel, our results are within the uncertainty predicted in PDG data [37]. Overall, our results on leptonic decays of D_s^+ mesons matches well with the PDG data.

Having determined form factors in Eq. (3), we compute the semileptonic branching fractions using Eq. (5). In Fig. (3), we plot the differential branching fractions in the entire range of momentum transfer and we compute the semileptonic branching fractions by numerical integration and tabulate in Tab. 3. It is observed that our results are a bit higher than the BESIII [9] and CLEO [11] data but it is still within the range predicted by *BABAR* collaborations [10]. We found the ratio of the branching fraction for muon channel to the electron channel $\mathcal{R}_{\mu/e} = 0.95$ which is consistent with the BESIII data suggesting no violation of lepton flavor universality. This work was further extended and the detailed study can be found in our recent paper [40].

Acknowledgement

We thank our collaborators Prof. M. A. Ivanov, Prof. J. G. Körner, Prof. P. Santorelli and Dr. C. T. Tran for their support and consent to publish part of the outcome of our collaboration. J.N.P. acknowledges financial support from University Grants Commission of India under Major Research Project F.No. 42-775/2013(SR).

References

[1] J. D. Richman, P. R. Burchat, Rev. Mod. Phys. **67**, 893 (1995).
[2] J. P. Alexander *et al*. (CLEO Collaboration), Phys. Rev. D **79**, 052001 (2009).
[3] P. Naik *et al*. (CLEO Collaboration), Phys. Rev. D **80**, 112004 (2009).
[4] P. U. E. Onyisi *et al*. (CLEO Collaboration), Phys. Rev. D **79**, 052002 (2009).
[5] P. del Amo Sanchez *et al*. (*BABAR* Collaboration), Phys. Rev. D **82**, 091103 (2010).
[6] A. Zupanc *et al*. (Belle Collaboration), JHEP **09**, 139 (2013).
[7] M. Ablikim *et al*. (BESIII Collaboration), Phys. Rev. D **94**, 072004 (2016).
[8] M. Ablikim *et al*. (BESIII Collaboration), arXiv:1811.10890.

[9] M. Ablikim *et al*. (BESIII Collaboration), Phys. Rev. D **97**, 012006 (2018).
[10] B. Aubert *et al*. (BABAR Collaboration), Phys. Rev. D **78**, 051101 (2008).
[11] J. Hietala, D. Cronin-Hennessy, T. Pedlar, I. Shipsey, Phys. Rev. D **92**, 012009 (2015).
[12] G. C. Donald, C.T.H. Davies, J. Koponen, G.P. Lepage (HPQCD Collaboration), Phys. Rev. D **90**, 074506 (2014).
[13] S. Fajfer, J.F. Kamenik, Phys. Rev. D **71**, 014020 (2005).
[14] S. Fajfer, J. F. Kamenik, Phys. Rev. D **72**, 034029 (2005).
[15] T. Sekihara, E. Oset, Phys. Rev. D **92**, 054038 (2015).
[16] L. R. Dai, X. Zhang, E. Oset, Phys. Rev. D **98**, 036004 (2018).
[17] D. Melikhov, B. Stech, Phys. Rev. D **62**, 014006 (2000).
[18] H. Y. Cheng, X. W. Kang, Eur. Phys. J. C **77**, 587 (2017).
[19] N. R. Soni, J. N. Pandya, Phys. Rev. D **96**, 016017 (2017).
[20] M. Ablikim *et al*. (BESIII Collaboration), Phys. Rev. Lett. **122**, 011804 (2019).
[21] G. V. Efimov, M. A. Ivanov, Int. J. Mod. Phys. A **4**, 2031 (1989).
[22] G. V. Efimov, M. A. Ivanov, *The Quark confinement model of hadrons* (IOP, Bristol, 1993).
[23] A. Faessler, T. Gutsche,M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, Eur. Phys. J.direct **4**, 18 (2002).
[24] T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, Phys. Rev. D **8**, 034010 (2010).
[25] S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov, A. Liptaj, P. Santorelli, C. T. Tran, Phys. Rev. D **99**, 014042 (2019).
[26] M. A. Ivanov, J.G. Körner, C. T. Tran, Phys. Part. Nucl. Lett. **14**, 669 (2017).
[27] M. A. Ivanov, J. G. Körner, C. T. Tran, Phys. Rev. D **95**, 036021 (2017).
[28] M. A. Ivanov, J. G. Körner, C. T. Tran, Phys. Rev. D **94**, 094028 (2016).
[29] T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, P. Santorelli, C. T. Tran, Phys. Rev. D **98**, 053003 (2018).
[30] T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, Phys. Rev. D **98**, 074011 (2018).
[31] T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, Phys. Rev. D **96**, 054013 (2017).
[32] F. Goerke, T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, Phys. Rev. D **96**, 054028 (2017).
[33] F. Goerke, T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, P. Santorelli, Phys. Rev. D **94**, 094017 (2016).
[34] T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, K. Xu, Phys. Rev. D **96**, 114004 (2017).
[35] S. Weinberg, Phys. Rev. **130**, 776 (1963).
[36] A. Salam, Nuovo Cim. **25**, 224 (1962).
[37] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).
[38] T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, P. Santorelli, N. Habyl, Phys. Rev. D **91**, 074001 (2015), [Erratum: Phys. Rev.D **91**,119907(2015)].
[39] M. A. Ivanov, J. G. Körner, C. T. Tran, Phys. Rev. D **92**, 114022 (2015).
[40] N. R. Soni, M. A. Ivanov, J. G. Körner, J. N. Pandya, P. Santorelli, C. T. Tran, Phys. Rev. D **98**, 114031 (2018).