

Properties of $Z_b(10610)$ and $Z_b(10650)$ from an analysis of experimental line shapes

V. Baru^{1,2,3,*}, A. A. Filin⁴, C. Hanhart⁵, A. V. Nefediev^{3,6}, Q. Wang¹, and J.-L. Wynen⁵

¹Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany

²Institute for Theoretical and Experimental Physics, B. Chermushkinskaya 25, 117218 Moscow, Russia

³P.N. Lebedev Physical Institute of the Russian Academy of Sciences, 119991, Leninskiy Prospect 53, Moscow, Russia

⁴Ruhr University Bochum, Faculty of Physics and Astronomy, Institute for Theoretical Physics II, D-44780 Bochum, Germany

⁵Forschungszentrum Jülich, Institute for Advanced Simulation, Institut für Kernphysik and Jülich Center for Hadron Physics, D-52425 Jülich, Germany

⁶National Research Nuclear University MEPhI, 115409, Kashirskoe highway 31, Moscow, Russia

Abstract. The experimental line shapes available in the $B\bar{B}^*$, $B^*\bar{B}^*$, $h_b(1P)\pi$ and $h_b(2P)\pi$ channels are analysed using a theoretical EFT-based framework manifestly consistent with unitarity and analyticity. The line shapes are calculated using a system of coupled channel integral equations with the potential consisting of the one-pion and one-eta meson exchange interactions from the lightest Goldstone boson octet as well as of several contact terms at leading and subleading orders which are adjusted to minimise the overall chi squared. The pole positions of the $Z_b(10610)$ and $Z_b(10650)$ are extracted for the best fits corresponding to $\chi^2/\text{d.o.f.}$ of the order of one.

1 Introduction

The discovery of the two charged resonances $Z_b^+(10610)$ and $Z_b^+(10650)$ by the Belle Collaboration [1] gave evidence of an exotic nature of these bottomonium-like states. These states (below referred to as Z_b and Z_b') were observed as peaks in the invariant mass distributions of the $\Upsilon(nS)\pi^\pm$ ($n = 1, 2, 3$) and $h_b(mP)\pi^\pm$ ($m = 1, 2$) channels in the decays from the vector bottomonium $\Upsilon(10860)$ [1] and later confirmed in the elastic $B^{(*)}\bar{B}^*$ channels [2–4]. The decays of the Z_b 's into conventional bottomonium states and a pion suggest that the minimal quark content of these resonances is four quark. Moreover, the proximity of the Z_b and Z_b' to the $B\bar{B}^*/B^*\bar{B}$ and $B^*\bar{B}^*$ thresholds, respectively, together with the fact that these open-flavour hadronic channels are by far the dominant (S-wave) decay channels of these states is regarded as a strong evidence for a molecular nature of the Z_b 's [5–7]; for an alternative scenario within the tetraquark picture see, e.g., a review [8].

In this contribution we provide a brief summary of an effective field theory (EFT) approach to the $Z_b(10610)$ and $Z_b(10650)$ developed in Ref. [9] and used to analyse the experimental data in various elastic and inelastic channels. The goal of this investigation was to

*e-mail: vadimb@tp2.rub.de

extract the important information about these exotic states such as their pole positions and residues using a theoretical framework that is consistent with chiral and heavy-quark spin (HQSS) symmetries of QCD and that respects constraints on the scattering amplitudes from unitarity and analyticity. In a recent work [10], the same EFT approach was also applied to predict in a parameter-free way the pole positions and the line shapes of the spin partners of the Z_b states.

2 EFT approach for the Z_b molecules

Prior to going to the results we discuss the key features of the EFT approach for hadronic molecules applied to the Z_b states.

- $B^{(*)}\bar{B}^{(*)}$ scattering near the poles of the scattering amplitudes is nonperturbative and requires a resummation of the effective potential to all orders. The potential is treated perturbatively to a given order in Q/Λ_h , where Q denotes relevant soft scales and $\Lambda_h \approx 1$ GeV represents a chiral symmetry breaking scale. For the problem at hand, the soft scale Q corresponds to the binding momenta, the pion mass and the momentum scale generated by the splitting between the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds

$$p_{\text{typ}} = \sqrt{m\delta} \simeq 500 \text{ MeV}, \quad (1)$$

where $\delta = m_* - m \approx 45$ MeV with m (m^*) denoting the B (B^*) meson mass. Simultaneously with the chiral EFT expansion, the potential is expanded around the HQSS limit. Since $\Lambda_{\text{QCD}}/M_b \approx 0.04 \ll 1$, it suffices to include only the B^*-B mass splitting while all interaction vertices can be constructed in line with HQSS [9].

- Employing the so-called Weinberg counting [11] that is widely used in a similar chiral EFT approach in few-nucleon systems (see Ref. [12] for a review), one finds that at leading order (LO) the heavy-meson potential consists of two momentum-independent, $\mathcal{O}(Q^0)$, contact interactions and the one-pion exchange (OPE). This type of approach with nonperturbative pions was employed in Refs. [13, 14] in the context of spin partner states of hadronic molecules in the c - and b -quark sectors. However, it was realized in Refs. [9, 10] that the high-momentum contributions from the S -wave-to- D -wave $B\bar{B}^* \rightarrow B^*\bar{B}^*$ (and vice versa) OPE transitions generate the line shapes that show a significant dependence on a regulator. To remove this dependence a promotion of the $\mathcal{O}(Q^2)$ S -wave-to- D -wave (S - D) counter term to LO is required. The effect from the other contact interactions at the order $\mathcal{O}(Q^2)$, (namely, from two S -wave-to- S -wave terms) on the line shapes in the 1^{+-} channel is marginal in line with the assumed power counting.
- The effect of the inelastic channels $\Upsilon(nS)\pi$ ($n = 1, 2, 3$) and $h_b(mP)\pi$ ($m = 1, 2$) is included via their coupling to the S -wave $B^{(*)}$ -meson pairs. Meanwhile, the direct interactions between the pion and heavy quarkonia [15] and the coupling of inelastic channels to the D -wave $B^{(*)}$ -meson pairs are suppressed [9].
- To extend the approach to the SU(3) sector, the inclusion of the whole pseudoscalar Goldstone-boson octet at LO is required. In the SU(2) sector, however, the effect from the explicit treatment of the η meson exchange is negligible.

3 Formalism

The partial-wave-projected effective potential in the elastic channels reads

$$(V_{\text{eff}})_{\alpha\beta} = (V_{\text{eff}}^{\text{CT}})_{\alpha\beta} + (V^\pi)_{\alpha\beta} + (V^\eta)_{\alpha\beta}, \quad (2)$$

where the effective contact interaction potential $V_{\text{eff}}^{\text{CT}}$ is composed of the elastic, $V_{\text{NLO}}^{\text{CT}}$, and inelastic, δV , contributions, and V^π and V^η stand for the OPE and one η meson exchange (OEE), respectively. The indices α and β depend on the particle channel and for $J^{PC} = 1^{+-}$ are defined as

$$\alpha, \beta = \{B\bar{B}^*({}^3S_1, -), B\bar{B}^*({}^3D_1, -), B^*\bar{B}^*({}^3S_1), B^*\bar{B}^*({}^3D_1)\}, \quad (3)$$

where the individual partial waves are labelled as ${}^{2S+1}L_J$ with S , L , and J denoting the total spin, the angular momentum, and the total momentum of the two-meson system, respectively. The C-parity eigenstates are defined as

$$B\bar{B}^*(\pm) = \frac{1}{\sqrt{2}} (B\bar{B}^* \pm B^*\bar{B}) \quad (4)$$

and correspond to the convention for the C-parity transformation $\hat{C}M = \bar{M}$. The explicit form of the potentials can be found in Refs.[9, 10].

To arrive at the scattering amplitudes which contain the information about the Z_b poles, the effective potential is iterated to all orders within Lippmann-Schwinger-type integral equations. The partial-wave-decomposed coupled-channel Lippmann-Schwinger-type equations read

$$T_{\alpha\beta}(M, p, p') = V_{\alpha\beta}^{\text{eff}}(p, p') - \sum_{\gamma} \int \frac{d^3q}{(2\pi)^3} V_{\alpha\gamma}^{\text{eff}}(p, q) G_{\gamma}(M, q) T_{\gamma\beta}(M, q, p'), \quad (5)$$

where α , β , and γ label the basis vectors defined in Eq. (3), and the two-body propagator takes the form

$$G_{\gamma} = (q^2/(2\mu_{\gamma}) + m_{1,\gamma} + m_{2,\gamma} - M - i\epsilon)^{-1}, \quad (6)$$

where $m_{1,\gamma}$ and $m_{2,\gamma}$ are the masses of the $B^{(*)}$ mesons in the channel γ , μ_{γ} is their reduced mass and M defines the total energy of the system. The convolution of the amplitudes (5) with the (S-wave) point-like source result in the production amplitudes which are used in Ref. [9] to analyse the line shapes.

4 Results

Our most advanced calculation involves (apart the OPE and OEE potentials) the following set of low-energy constants (LECs) extracted from the best fits to data: two S - S and one S - D elastic LECs together with five effective couplings to the inelastic channels $\Upsilon(nS)\pi$ ($n = 1, 2, 3$) and $h_b(mP)\pi$ ($m = 1, 2$) at order $O(Q^0)$ plus two S - S elastic LECs at order $O(Q^2)$. All low-energy constants were fixed from a combined fit to the experimental line shapes in the decays $\Upsilon(10860) \rightarrow B\bar{B}^*\pi$, $B^*\bar{B}^*\pi$, $h_b(1P)\pi\pi$, and $h_b(2P)\pi\pi$ which proceed via the excitation of the $Z_b(10610)$ and $Z_b(10650)$ exotic states as well as from the total rates for the decays $\Upsilon(10860) \rightarrow \Upsilon(nS)\pi\pi$ ($n = 1, 2, 3$). The line shapes in the $\Upsilon(10860) \rightarrow \Upsilon(nS)\pi\pi$ channels could not be included in the analysis so far since they require a proper treatment of the two-pion final-state interaction. We find that the quality of the line shape description by the pionful fits turns out to be better than that by the contact fit at $O(Q^0)$ that is reflected in the change of the $\chi^2/\text{d.o.f.}$ from 1.29 for the contact fit to 0.83 for the most advanced pionful fit (called fit G in Ref. [9]). In Fig.1 we present the results for our most advanced pionful fit including the uncertainties which correspond to a 1σ deviation in the parameters including correlations. Based on these results we extract the pole positions and the residues of the Z_b states summarized in Table 1. In order to extract these quantities, in the vicinity of a pole

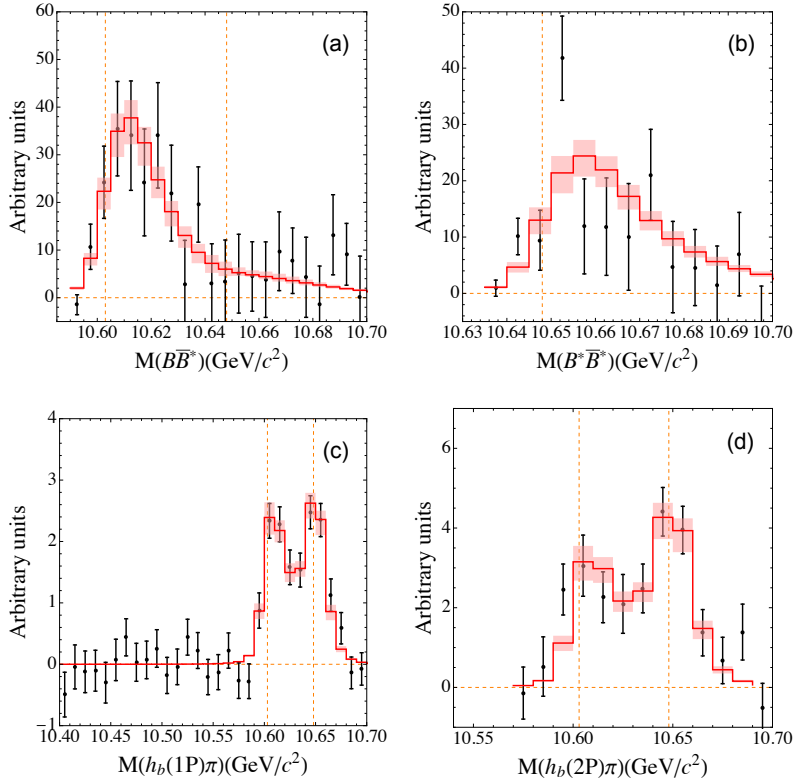


Figure 1. The fitted line shapes for the most advanced pionful fit of Ref. [9] with the uncertainties corresponding to a 1σ deviation in the parameters of the fits. The experimental data are from Refs. [1, 4].

located at $M = M_{R_\alpha}$ the elastic scattering amplitude $T_{\alpha\alpha}(M, p, p')$ given in Eq. (5) is written as

$$T_{\alpha\alpha} = \frac{g_\alpha^2}{M^2 - M_{R_\alpha}^2} \approx \frac{g_\alpha^2}{2M_{R_\alpha}} \frac{1}{M - M_{R_\alpha}}, \quad (7)$$

where g_α^2 and M_{R_α} stand for the residue and the pole position in the channel α , respectively. We note that the poles shown in Table 1 reside on the Riemann Sheets which have the shortest path to the physical sheet and reveal themselves as above threshold resonances; for a detailed discussion on how to identify the relevant poles in a multichannel scattering problem we refer to Ref. [10], see also a review [16].

5 Conclusions

In this contribution we gave a brief summary of the EFT approach for hadronic molecules that was employed in Refs. [9, 10] to systematically analyze the line shapes relevant for the bottomonium-like states $Z_b(10610)$ and $Z_b(10650)$ and to extract their pole positions and residues. The advantages of an EFT approach like this are as follows: it is consistent with underlying chiral and heavy-quark symmetries, systematically improvable and allows for a theoretical error estimate. In addition, relying on HQSS, the approach allows to predict in a

Table 1. The pole positions E_R and the residues g^2 in various S -wave $B^{(*)}\bar{B}^*$ channels for the most advanced pionful fit [10]. The energy E_R is given relative to the nearest open-bottom threshold quoted in the second column. Uncertainties correspond to a 1σ deviation in the parameters allowed by the fit to the data in the channels with $J^{PC} = 1^{+-}$ where the $Z_b^{(\prime)}$ states reside. The theoretical uncertainty from the truncation of the EFT expansion was estimated for these states to be about 1 MeV [10].

State	Threshold	E_{pole} w.r.t. threshold [MeV]	Residue at E_{pole}
Z_b	$B\bar{B}^*$	$(-2.3 \pm 0.5) - i(1.1 \pm 0.1)$	$(-1.2 \pm 0.2) + i(0.3 \pm 0.2)$
Z_b'	$B^*\bar{B}^*$	$(1.8 \pm 2.0) - i(13.6 \pm 3.1)$	$(1.5 \pm 0.2) - i(0.6 \pm 0.3)$

parameter-free way the spin partners of the Z_b with the quantum number J^{++} , as demonstrated in a recent work [10].

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