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Parametrizations of three-body hadronic B- and D-decay amplitudes

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Abstract. A short review of our recent work on amplitude parametrizations of three-body hadronic weak B and D decays is presented. The final states are here composed of three light mesons, namely the various charge $\pi\pi\pi$, $K\pi\pi$ and $KK\bar{K}$ states. These parametrizations are derived from previous calculations based on a quasi-two-body factorization approach where the two-body hadronic final state interactions are fully taken into account in terms of unitary S- and P-wave $\pi\pi$, πK and $K\bar{K}$ form factors. They are an alternative to the isobar-model description and can be useful in the interpretation of CP asymmetries.

1 Introduction

1.1 Motivations: why study three-body hadronic B and D decays?

Three-body hadronic B and D decays provide a rich tool to study not only the Standard Model, QCD, CP violation [1] but also hadron physics. The hadron physics, often characterized by two-body resonances and their interferences, affect weak observables and any reliable determination of the later will require a good knowledge of the final state meson-meson interactions. This can be realized by introducing theoretical constraints such as unitarity, analyticity, chiral symmetry and the use of data from reactions other than B and D decays. Basic Dalitz-plot analyzes rely on sums of relativistic Breit-Wigner amplitudes representing the different possible implied resonances to which some non resonant background amplitude is added. The S-wave resonance contributions are often difficult to fit. Can one go beyond this isobar model approach?

One can replace the sums of relativistic Breit-Wigner components by parametrizations [2] in terms of unitary two-meson form factors keeping the weak-interaction dynamics governing the flavor-changing process via W-meson exchange. These parametrizations are based on published results and motivated by analyzes of high-statistics present and forthcoming data at BES III, LHCb, Belle II, Super c-tau factory .... Up to now there is no three-body decay factorization theorem but major contributions arise from intermediate resonances such

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as $\rho(770)$, $K^*(892)$, $\phi(1020)$ which allows to describe three-body decays as quasi-two-body ones. For instance, for the three-meson final state of the $D^0 \to K_S^0\pi^-\pi^+$ decay, one can introduce quasi-two-body pairs, $[K_S^0\pi^+]_L\,\pi^-$, $[K_S^0\pi^-]_L\,\pi^+$, $K_S^0\;[\pi^+\pi^-]_L$, two of the three mesons forming a state of angular momentum 0 or 1 with $L = S$ or P, respectively.

1.2 QCD quasi-two-body factorization

Decays are mediated by local four-quark operators $O_i(\mu)$ forming the weak effective non-renormalizable Hamiltonian $\mathcal{H}_{\text{eff}}$. Schematically for $B \to M_1 M_2^*(M_2^* \to M_3 M_4)$ one has

$$\langle M_1 M_2^*|\mathcal{H}_{\text{eff}}|B\rangle = \frac{G_F}{\sqrt{2}}\, V_{\text{CKM}} \sum_i C_i(\mu)\langle M_1 M_2^*|O_i(\mu)|B\rangle, \tag{1}$$

where G_F is the Fermi decay constant, V_{CKM} the product of Cabibbo-Kobayashi-Maskawa matrix elements and $C_i(\mu)$ Wilson coefficients renormalized at scale $\mu \sim m_b$ (or m_c in D decays). In the factorization approach [3] with the strong coupling $\alpha_s(\mu)$, i.e. at scale μ,

$$\begin{aligned}\langle M_1 M_2^*|O_i(\mu)|B\rangle = \Big(&\langle M_1|J_1^\nu|B\rangle\langle M_2^*|J_{2\nu}|0\rangle \\ &+\langle M_1|J_3^\nu|0\rangle\langle M_2^*|J_{4\nu}|B\rangle\Big)\left[1+\sum_n r_n\alpha_s^n(\mu)+O\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)\right],\end{aligned} \tag{2}$$

where r_n are strong interaction constant factors and $|0\rangle$ the vacuum state. For the leading order the factorization takes place with either weak quark currents J_1, J_2 or J_3, J_4. The radiative corrections can be evaluatd to a given order $\alpha_s^n(\mu)$. The nonperturbative corrections to the heavy-quark limit $O\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)$ are less reliable for D decays as $m_c \sim m_b/3$; therefore, even though the factorization is more phenomenological for charmed mesons, it can still represent a good starting point.

The amplitude $\langle M_1|J_1^\nu|B\rangle$ (= $\langle M_1\overline{B}|J_1^\nu|0\rangle$) is a heavy-to-light transition form factor which can be evaluated within light-front and relativistic constituent quark models, light-cone sum rules, continuum functional QCD and lattice QCD (see Appendix A4 of Ref. [2]). Semi-leptonic decay measurements like $D^0 \to \pi^- e^+\nu_e$ can also allow a phenomenological determination of these form factors.

The matrix element $\langle M_2^*|J_{2\nu}|0\rangle \propto \langle M_3M_4|J_{2\nu}|0\rangle$, where the M_3M_4 resonance, M_2^*, originates from a $\bar{q}q$ pair, corresponds to the M_3M_4 form factor. It has been shown, in Ref. [4], that, using dispersion relations and field theory, this form factor can be fully determined, if the M_3M_4 strong interaction is known at all energies. These form factors are calculated from Muskhelishvili-Omnès equations [5] using two-body data, unitarity, asymptotic QCD and chiral symmetry constraints at low energies.

The term $\langle M_1|J_3^\nu|0\rangle$, related to the M_1 weak decay constant, is known from experiment, e.g. the pion decay constant, f_π or that of the kaon, f_K. It can also be evaluated with lattice-regularized QCD and other nonperturbative approaches.

The matrix element $\langle M_2^*|J_{4\nu}|B\rangle \propto \langle M_3M_4|J_{4\nu}|B\rangle$ corresponding to B meson transitions to two-meson pairs via the M_2^* resonance is the biggest uncertainty in our approach. It could be evaluated from semi-leptonic processes: like $B^0 \to K^+\pi^-\mu^+\mu^-$ or $D^0 \to K^-\pi^+\mu^+\mu^-$. In the derivation of the amplitude presented here it will be related to the $\langle M_2^*[\to M_3M_4]|J_{2\nu}|0\rangle$ form factor. Within the soft-collinear effective theory, the amplitude can be factorized in terms of generalized B-to-two-body form factor and two-hadron light-cone distribution amplitude [6].

1.3 Application to the $D^+ \to K^-\pi^+\pi^+$ decay

In this process, studied in Ref. [7], the final state $\pi^+\pi^+$ interaction can be neglected and the quasi-two-body $[K^-\pi^+]_{S,P}\ \pi^+$ can be introduced. There is no penguin contribution (loop with W meson) and only the effective Wilson-coefficients $a_{1(2)}$ appear in the quasi-two-body factorized amplitude,

$$\langle [K^-\pi^+]_{S,P}\ \pi^+|\mathcal{H}_{\text{eff}}|D^+\rangle = \frac{G_F}{\sqrt{2}}\cos^2\theta_C\Big[a_1\langle [K^-\pi_1^+]_{S,P}|\bar{s}\gamma^\nu(1-\gamma_5)c|D^+\rangle\langle\pi_2^+|\bar{u}\gamma_\nu(1-\gamma_5)d|0\rangle + a_2\langle [K^-\pi_1^+]_{S,P}|\bar{s}\gamma^\nu(1-\gamma_5)d|0\rangle\langle\pi_2^+|\bar{u}\gamma_\nu(1-\gamma_5)c|D^+\rangle\Big] + (\pi_1^+ \leftrightarrow \pi_2^+), \tag{3}$$

θ_C being the Cabbibo angle. The matrix element $\langle [K^-\pi_1^+]_{S,P}|\bar{s}\gamma^\nu(1-\gamma_5)d|0\rangle$ is given by the $K\pi$ form factors. The term $\langle [K^-\pi_1^+]_{S,P}|\bar{s}\gamma^\nu(1-\gamma_5)c|D^+\rangle$ is less straightforward to evaluate. Assuming a dominant intermediate resonance R, it can be written as being proportional to the D to R $[R \to K\pi]$ transition form factor multiplied by the $K\pi$ form factors. This description is a feature of crucial importance to our proposed parametrizations. In Eq. (3), $\langle\pi_2^+(p)|\bar{u}\gamma_\nu(1-\gamma_5)d|0\rangle = -if_\pi p_\nu$ and $\langle\pi_2^+|\bar{u}\gamma_\nu(1-\gamma_5)c|D^+\rangle$ is the $D\pi$ transition form factor.

Parametrized amplitudes based on quasi-two-body factorization have been given in Ref. [2] in terms of analytic and unitary meson-meson form factors for final states composed of three light mesons, namely the various charge $\pi\pi\pi$, $K\pi\pi$ and $KK\bar{K}$ states. For these hadronic three-body decays we have shown, in previous studies, that this approach is phenomenologically successful. Below, we illustrate these parametrizations for the $B \to K\pi^+\pi^-$ [8–10] and $D^0 \to K_S^0 K^+\ K^-$ [11] for meson-meson final states in S wave. Formulae for meson-meson final states in P wave are given in Ref. [2].

2 Parametrized amplitudes for the $B \to K\pi^+\pi^-$ decays

2.1 Parametrization of the $B \to K[\pi^\pm\pi^\mp]_S$ amplitudes

Let us label the momenta as $B(p_B) \to K(p_1)\pi^+(p_2)\pi^-(p_3)$ with $s_{12} = (p_1+p_2)^2$, $s_{13} = (p_1+p_3)^2$, $s_{23} = (p_2+p_3)^2$ and $s_{12}+s_{13}+s_{23} = m_B^2 + m_K^2 + 2m_\pi^2$. As can be seen from Eq. (1) of Ref. [8] the $B \to K[\pi^+\pi^-]_S$ amplitude can be parametrized in terms of three complex parameters, $b_i^S, i = 1,2,3$, for the different charged states $B = B^\pm, K = K^\pm$ and $B = B^0(\bar{B}^0), K = K^0(\bar{K}^0)$ or K_S^0. For the B^- decays one has

$$\mathcal{A}_S(s_{23}) \equiv \langle K^-\ [\pi^+\pi^-]_S|\mathcal{H}_{\text{eff}}|B^-\rangle = b_1^S\left(M_B^2 - s_{23}\right)F_{0n}^{\pi\pi}(s_{23}) + \left(b_2^S F_0^{BK}(s_{23}) + b_3^S\right)F_{0s}^{\pi\pi}(s_{23}), \tag{4}$$

where $F_0^{BK}(s)$ is the B to K transition form factor (see Refs. [2, 8]). The non-strange scalar form factor $F_{0n}^{\pi\pi}(s)$ contains the contributions of $f_0(500)$, $f_0(980)$ and $f_0(1400)$. Several models are compared in Fig. 8 of Ref. [12]. Although there are large differences, it has been checked by the authors that, with the fitted form factor to obtain the lowest χ^2 for $D^0 \to K_S^0\pi^+\pi^-$, the main conclusions achieved for the $B^\pm \to \pi^+\pi^-\pi^\pm$ in Ref. [13] were unchanged (see Ref. [12] for explanations). The modulus of the Moussallam pion scalar form factor [14], calculated by solving the Muskhelishvili-Omnès equation [5], is close to that of the form factor obtained in Ref. [12], notably below 1 GeV. A plot of the strange scalar form factor $F_{0s}^{\pi\pi}(s)$, which receives the contribution of the $f_0(980)$ and $f_0(1400)$, can be found in Fig. 6 of Ref. [15]. It has been calculated using the Muskhelishvili-Omnès approach.

In terms of the original amplitude [8] one has[1], $F_0^{B\to f_0(980)}(m_K^2)$ being the B to $f_0(980)$ transition form factor evaluated at m_K^2 [2],

$$b_1^S = \frac{G_F}{\sqrt{2}}\left[\chi f_K F_0^{B\to f_0(980)}(m_K^2)\, U - \tilde{C}\right], \tag{5}$$

where $\tilde{C} = f_\pi F_\pi \left(\lambda_u P_1^{GIM} + \lambda_t P_1\right)$, $\lambda_u = V_{ub}V_{us}^*$, $\lambda_t = V_{tb}V_{ts}^*$, F_π is the $B\pi$ form factor at $m_\pi^2 = 0$, P_1^{GIM}, P_1 complex charming penguin parameters and U is a short-distance contribution given in terms of CKM matrix element multiplied by effective Wilson coefficients. The fitted parameter χ represents the strength of the non-strange pion form factor contribution, furthermore. Its value can be estimated from the $f_0(980)$ decay properties [8]. A summary of the models for the scalar-isoscalar pion form factor can be found in Appendix A4 of Ref. [2] and, as just noted above, see also the recent determination of Ref. [15] and the review talk [16].

2.2 Parametrization of the $B \to [K\pi^\pm]_S\pi^\mp$ amplitudes

In terms of the two complex parameters c_1^S, c_2^S (see Eq. (68) of Ref. [10]) one has

$$\mathcal{A}_S(s_{12}) \equiv \langle \pi^-\ [K^-\pi^+]_S|\mathcal{H}_{\text{eff}}|B^-\rangle = (c_1^S + c_2^S\, s_{12})\,\frac{F_0^{B\pi}(s_{12})F_0^{K\pi}(s_{12})}{s_{12}}, \tag{6}$$

where $F_0^{K\pi}(s)$ (contribution of $K_0^*(800)$ or κ and of $K_0^*(1430)$, see e.g. Fig. 7 of Ref. [12]) and $F_0^{B\pi}(s)$ are the $K\pi$ and $B\pi$ scalar form factors, respectively. This parametrization has been used with success in the amplitude analysis [17] of the Dalitz-plot distribution of the LHCb $\bar{B} \to K_S^0\pi^+\pi^-$ data. One has [10]

$$c_1^S = \frac{G_F}{\sqrt{2}}(M_B^2 - m_\pi^2)(m_K^2 - m_\pi^2)\left[\lambda_u\left(a_4^u(S) - \frac{a_{10}^u(S)}{2} + c_4^u\right) + \lambda_c\left(a_4^c(S) - \frac{a_{10}^c(S)}{2} + c_4^c\right)\right], \tag{7}$$

where $\lambda_c = V_{cb}V_{cs}^*$. The $a_i^{u(c)}(S)$, $i = 4, 10$ are the leading order effective Wilson coefficients including vertex and penguin corrections. The $c_4^{u(c)}$ are free fitted parameters simulating nonperturbative and higher order contributions to the penguin diagrams. Models for the $F_0^{K\pi}(s)$ form factor are described in Ref. [2], see also some complementary aspects in Ref. [16].

3 $D^0 \to K_S^0[K^+K^-]_S$ and $D^0 \to [K_S^0K^\pm]_S K^\mp$ parametrized amplitudes

The $[K^+K^-]$ pairs can have isospin 0 or 1 but the $[K_S^0K^\pm]$ ones have isospin 1. The $f_0(980)$, $f_0(1400)$, $a_0(980)^0$ and $a_0(1450)^0$ contribute to the following parametrized amplitude

$$\mathcal{A}_{S,0}^0(s_{23}) = h_1^S\left(m_{D^0}^2 - s_{23}\right)F_{0n}^{K\bar{K}}(s_{23}) + h_2^S\left(m_{K^0}^2 - s_{23}\right)F_{0s}^{K\bar{K}}(s_{23}) + h_3^S\left(m_{D^0}^2 - s_{23}\right)G_0^{K\bar{K}}(s_{23}). \tag{8}$$

where s_{23} is the energy squared of the K^+K^- pair while s_{12} is associated to the $K_S^0K^-$ pair and s_{13} to the $K_S^0K^+$ one. The decay amplitude associated with the $a_0(980)^-$ and $a_0(1450)^-$ resonances, can be parametrized as:

$$\mathcal{A}_{S,-}^0(s_{12}) = \left(h_4^S + h_5^S\, s_{12}\right)G_0^{K\bar{K}}(s_{12}). \tag{9}$$

The amplitude carrying contributions from $a_0(980)^+$ and $a_0(1450)^+]$ reads

[1] The interested reader will find, in Appendix B of Ref. [2], the corresponding relations for the other parameters.

$$\mathcal{A}^0_{S,+}(s_{13}) = \left[h^S_6 \frac{F^{DK}_0(s_{13})}{s_{13}} + h^S_7 (m^2_K - s_{13})\right] G^{K\bar{K}}_0(s_{13}). \quad (10)$$

Models for the $F^{K\bar{K}}_{0n(s)}(s)$ form factors entering Eq. (8) have been derived in Ref. [18, 19] (see their Figs. 1) solving three coupled channels viz. $\pi\pi$, $K\bar{K}$ and 4π (effective 2π-2π or $\sigma\sigma$ or $\rho\rho$ ...) and imposing chiral symmetry constraints. The $F^{K\bar{K}}_{0s}(s)$ form factor has also been calculated in a dispersive approach in Ref. [15] (see their Fig. 7).

In Eqs. (9) and (10), the scalar-isovector $G^{K\bar{K}}_0(s)$ form factor, built in Ref. [20] from a unitary S-wave coupled channel $(\eta\pi, K\bar{K})$ model, is plotted in their Fig. 7. This model, derived from the Muskhelishvili-Omnès equation [5], imposes the presence of the $a_0(980)$ and $a_0(1450)$ and includes asymptotic QCD and chiral symmetry constraints. Models for the transition form factor $F^{DK}_0(s)$ in Eq. (10) can be found in Ref. [2]. The above complex h^S_i coefficients are given in terms of the original amplitudes in Appendix B of Ref. [2].

4 Concluding remarks

Alternatives to isobar Dalitz-plot model for weak D, B decays into various $\pi\pi\pi$, $K\pi\pi$ and $KK\bar{K}$ charge states have been presented in Ref. [2]. Let us recall that isobar parametrizations do not respect unitarity and extraction of strong CP phases should be taken with caution. Furthermore S-wave resonance contributions are hard to fit.

Our parametrizations, although not fully three-body unitary, are based on a sound theoretical application of QCD factorization to a hadronic quasi-two-body decay. They assume that final three-meson state are preceded by intermediate resonant states which is justified by phenomenological and experimental evidence. Analyticity, unitarity, chiral symmetry plus correct asymptotic behavior of the two-meson scattering amplitude in S and P waves are implemented via analytical and unitary S- and P-wave $\pi\pi$, πK and $K\bar{K}$ form factors entering in hadronic final states of our amplitude parametrizations.

These parametrized amplitudes can be readily used adjusting parameters in a least-square fit to the Dalitz plot for a given decay channel and employing tabulated form factors as functions of momentum squared or energy. The reproduction of the Dalitz-plot data might require some adjustment of the meson-meson form factors. The addition of phenomenological amplitudes (contributions of higher interacting waves, in particular D waves or J=2 resonances), and possible three-body rescattering effects may be necessary.

We have exemplified here expressions for the $B \to K\pi^+\pi^-$ [8–10] and $D^0 \to K^0_S K^+ K^-$ [11] for meson-meson final states in S wave. In Ref. [2] one can find other explicit amplitude expressions for meson-meson final states in S and P wave for $B^\pm \to \pi^+\pi^-\pi^\pm$, $B \to K\ \pi^+\pi^-$, $B^\pm \to K^+K^-K^\pm$, $D^+ \to \pi^-\pi^+\pi^+$, $D^+ \to K^-\pi^+\ \pi^+$, $D^0 \to K^0_S\ \pi^+\ \pi^-$. Previous studies have shown that this approach is successful. In addition, expressions for $D^0 \to K^0_S\ K^+K^-$ are also given in Ref. [2]. We have derived preliminary parametrized amplitudes for the $B^\pm \to K^+K^-\pi^\pm$ decays [1, 21] and for the $B^0 \to K^0_S\ K^+K^-$ process presently analyzed by the LHCb collaboration.

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