Chiral anomaly transition form factor of $\gamma \to 3\pi$ in nonlocal quark model

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Abstract. Chiral anomaly of gamma transition into three pions are studied in a framework of nonlocal chiral quark model. In the local limit the result is in agreement with chiral perturbative theory and reproduces Wess-Zumino-Witten anomaly. Transition form factor of gamma into three pions in nonlocal quark model has a correction which vanishes in local limit.

1 Introduction

Chiral anomaly plays an important role in description of many processes of particle physics, in particular, which are connected with pseudoscalar mesons and their interaction with external vector or axial-vector fields. One of known process which goes through anomaly is decay of neutral pseudoscalar meson into pair of photons. This process gives main mode of decay and has branching ~ 98.823 % [1].

In case of chiral quark models, anomalous Ward identities are due to Wess-Zumino-Witten (WZW) effective action [2]. Kazuo Fujikawa [3] has shown that the imaginary part of effective action which describes a chiral anomaly goes from Jacobian of productive functional. After integration by quark fields we obtain amplitudes which describe chiral anomalies. Well-known chiral anomaly of pion decay into pair of photons has the following form:

$$A(\pi^0 \to \gamma\gamma) = F_{\gamma\gamma}(M^2_{\pi^0})\epsilon^{\mu\nu\alpha\beta}\epsilon^{\mu}k_1^{\nu}\epsilon^{\alpha}k_2^{\beta}, \tag{1}$$

where ϵ^i_j and k^i_j - polarizations and momenta of photons and

$$F_{\gamma\gamma}(0) = \frac{e^2}{4\pi^2 f_\pi}, \tag{2}$$

with $f_\pi = f_0[1 + O(m_q)] = 92.4\,\text{MeV}$. The correction depends on quark mass and is small in chiral perturbative theory (ChPT) [4] but plays a role in Dalitz decay $\pi^0 \to \gamma e^+e^-$ as shown in reference [5]. On a mass-shell form factor $F_{\gamma\gamma}$ has corrections due to mass of pion.

The considered process of $\pi \to \gamma\gamma$ and $\gamma\pi^\pm \to \pi^\pm\pi^0$ or $e^+e^- \to \gamma^* \to \pi^0\pi^-\pi^+$ are given entirely in terms of the electric charge e and the pion decay constant f_π.

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The process of $\gamma\pi^- \to \pi^-\pi^0$ has also a connection to the WZW anomalous effective action and amplitude of reaction has a form

$$A(\gamma\pi^- \to \pi^-\pi^0) = -iF(s,t,u)_{3\pi}\epsilon^{\mu\nu\alpha\beta}\epsilon^\mu p_0^\nu p_1^\alpha p_2^\beta, \tag{3}$$

where ϵ^μ is polarization of incident photon and p_i – momenta of pions. In the chiral limit in low-order by quark-loops, the form factor of this amplitude is independent of the Mandelstam variables s, t, u and has a simple form [6]:

$$F_{3\pi}(0,0,0) = \frac{e}{4\pi^2 f_\pi^3} = 9.72\,\mathrm{GeV}^{-3}. \tag{4}$$

The process that described this amplitude was measured for the first time at the IHEP accelerator (Serpukhov) at the 40 GeV negative-pion beam [7, 8]. The experiment was based on pion pair production by pions in the nuclear Coulomb field via the Primakoff reaction:

$$\pi^- + (Z,A) \to \pi^{-\prime} + (Z,A) + \pi^0. \tag{5}$$

The estimated value of $F_{3\pi}$ from the experiment is

$$F_{3\pi}^{\mathrm{exp}} = (12.9 \pm 0.9 \pm 0.5)\,\mathrm{GeV}^{-3}. \tag{6}$$

New experimental data of Primakoff reaction $\pi\gamma \to \pi\pi$ can be extracted from the data which obtained at CERN COMPASS experiment and may be measured on upgraded version of the experiment facility.

2 Nonlocal model

In paper [9] the method for introducing of the gauge fields into the nonlocal model was developed and a nonlocal chiral quark model (NCQM) was proposed. The Lagrangian density of this model is:

$$\mathcal{L}_{NCQM}(x,y) = \bar{\psi}(x)\delta(x-y)\hat{\partial}\psi(y) + \bar{\psi}(x)\Sigma_\pi(x,y)\psi(y), \tag{7}$$

where

$$\begin{aligned}\Sigma_\pi(x,y) = \Sigma(x-y)\Big[1 &- \frac{i}{F_0}\gamma_5[\pi(x)+\pi(y)] - \\ &- \frac{1}{2F_0^2}[\pi^2(x)+\pi^2(y)+\pi(x)\pi(y)+\pi(y)\pi(x)] + O(\pi^3)\Big]\end{aligned} \tag{8}$$

is the term containing nonlocal fermion mass and pion-fermion interaction terms. Interaction of fermions with gauge fields can be introduced by Wilson $\mathcal{P}$-exponent:

$$\psi(x) = \mathcal{P}\exp\left[i\int_x^y dz^\mu A_\mu(z)\right]\psi(y). \tag{9}$$

The gauge field introduced into the nonlocal action gives infinite number of interaction vertices of fermions with arbitrary number of gauge field.

2.1 Nonlocal quark model

In what follows we will operate in the framework of the nonlocal chiral quark model. Its Lagrangian in $SU(2)\times SU(2)$ modification has the following form [10, 11]:

$$\mathcal{L}_{N\chi QM} = \bar{q}(x)(i\hat{\partial} - m_c)q(x) + \frac{G}{2}[J^a_S(x)J^a_S(x) + J^a_P(x)J^a_P(x)], \tag{10}$$

where $q(x)$ are the quark fields, m_c is the diagonal matrix of the quark current masses[1], G is the four-quark coupling constant. This model is nonlocal version of local linear NJL model. Unlike NCQM model vertices of interaction between pion and quark fields have a nonlocal structure.

The nonlocal structure of the model is introduced via the nonlocal quark currents:

$$J^a_{S,P}(x) = \int d^4x_1 d^4x_2\, f(x_1)f(x_2)\,\bar{q}(x-x_1)\,\Gamma^a_{S,P}q(x+x_2), \quad \Gamma^a_S = \tau^a, \quad \Gamma_P = i\gamma^5\tau^a, \tag{11}$$

where $f(x)$ is a form factor reflecting the nonlocal properties of the QCD vacuum and τ^a are the Pauli matrices. For simplicity here we do not consider an extended model that includes other structures besides the pseudoscalar (P) and scalar (S) ones.

In the momentum space general vertex of quark-antiquark-pion interaction obtained from the above Lagrangian has the following form:

$$\Gamma^a_{qq\pi}(p_1, p_2) = if(p_1)f(p_2)\gamma_5\tau^a,$$

where p_1 and p_2 are momenta of coming in and out quarks.

The nonlocal vertex of the quark-antiquark interaction with external gauge field which generated within the gauge approach [9, 10] can be written in the following way:

$$\Gamma_\mu(q) = \gamma_\mu - (p_2 + p_1)_\mu m^{(1)}(p_1, p_2), \tag{12}$$

where p_1 and $p_2 = p_1 + q$ are momenta of quarks, q is momentum of external field and $m^{(1)}(k, q)$ is finite difference derivation:

$$m^{(1)}(p, k) = \frac{m(p^2) - m(k^2)}{p^2 - q^2},$$

with

$$m(p^2) = m_c + m_d f^2(p^2). \tag{13}$$

For numerical calculations we have used the Gaussian form factor $f(k) = \exp(\frac{k^2}{2\Lambda^2})$ where Λ is cutoff parameter. In SU(2) version of model we have four parameters: current m_c and dynamical m_d masses of quark, four-quark interaction constant G and cutoff parameter Λ. One parameter m_d we can keep free and change in band of physical dynamical mass of quark extracted from the experimental data or lattice calculations. Two parameters: current m_c and G or Λ can be fitted on pion mass and width of pion decay into two photons. The last parameter can be obtained from the gap equation [11, 13].

[1] We consider the isospin limit where masses of quarks u and d are equal.

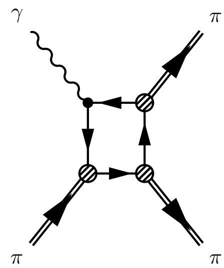

Figure 1. Feynman diagram which describes transition form factor $\gamma^*\pi^- \to \pi^0\pi^-$. All vertices are nonlocal

3 Transition form factor

Generally, the amplitude of $\gamma^* \to \pi^+\pi^0\pi^-$ processes can be written as sum of six diagrams. One of those diagrams is shown Fig. 1. Other diagrams can be produced by permutations of pion legs. There are three groups of diagrams that are symmetric with respect to change of direction of quark momenta $k \to -k$. The amplitude of transition gamma into three pions can be written as follows:

$$A(\gamma \to \pi^+\pi^0\pi^-) = -iF_{3\pi}(s,t,u)\epsilon^{\mu\nu\alpha\beta}\epsilon^\mu p_0^\nu p_1^\alpha p_2^\beta, \tag{14}$$

where p_i are momenta of pions, ϵ^μ is polarization of photon and $F_{3\pi}(s,t,u)$ is a Lorentz scalar function of the Mandelstam variables which is defined from three types of diagrams in different kinematics:

$$F_{3\pi}(s,t,u) = F_1(s,t,u) + F_2(t,s,u) + F_3(u,t,s), \tag{15}$$

where s,t,u are Mandelstam invariance variables. The first function of transition form factor can be calculated from the Feynman diagrams Fig. 1 and in case of nonlocal model has the following form:

$$\begin{aligned} F_1(s,t,u) = eN_c \int \frac{d_E^4k}{(2\pi)^4} & \frac{g_\pi(p_0^2)g_\pi(p_1^2)g_\pi(p_2^2)f_k f_{k+p_1}^2 f_{k-p_0}^2 f_{k-p_0-p_2}\mathrm{Tr}_f[Q(\pi^-\pi^0\pi^+ + \pi^+\pi^0\pi^-)]}{D(k)D(k+p_1)D(k-p_0)D(k-p_0-p_2)} \\ & \times 4\{m(k^2)[A+1-B] - m((k-p_0)^2)[C+A] \\ & + m((k+p_1)^2)C + m((k-p_0-p_2)^2)B\}, \end{aligned} \tag{16}$$

where $D(k) = k^2 + m^2(k^2)$, $f_k = f(k^2)$, Q is a charge matrix of quark, $\pi^i = \pi^a\tau^a/\sqrt{2}$ where π^a is the matrix of pion fields, and d_E^4k means that we perform calculations in Euclidean space. Coefficient functions A, B, C can be found in [12]. Structure functions $F_2(s,u,t)$ and $F_3(t,s,u)$ can be obtained from $F_1(s,t,u)$ by the following replacements:

$$F_2(t,s,u) = F_1(s,t,u)(p_0 \leftrightarrows -p_1, \pi^0 \leftrightarrows \pi^+), \tag{17}$$

$$F_3(u,t,s) = F_1(s,t,u)(p_0 \leftrightarrows -p_2, \pi^0 \leftrightarrows \pi^-). \tag{18}$$

In chiral limit when current mass of quark m_c is equal to zero this form factor takes the form [12]:

$$F_{3\pi}(0,0,0) = \frac{eN_cN_f}{f_\pi^3}\int \frac{d_E^4k}{(2\pi)^4}\left[\frac{4m^4(k^2) - 4m'(k^2)m^3(k^2)k^2}{D(k)^4}\right], \tag{19}$$

where $f_\pi = g_\pi/m_d$ and $m'(k^2) = \dfrac{\partial m(k^2)}{\partial k^2}$.

In local limit of the model when parameter of nonlocality $\Lambda \to \infty$, $f(k^2) \to 1$ and $m'(k) = 0$, $m(k^2) = m_d$. Obtained integral from eq.(19) can be solved analytically:

$$\int_0^\infty dk^2 \frac{k^2 m^4}{(k^2+m^2)^4} = \frac{1}{6}. \tag{20}$$

The Eq. (19) reproduces the WZW form factor [2]:

$$F_{3\pi} = \frac{eN_c N_f}{24\pi^2 f_\pi^3} = \frac{e}{4\pi^2 f_\pi^3} \simeq 9.72\,(0.09)\,\mathrm{GeV}^{-3}. \tag{21}$$

In the nonlocal quark model in the low energy limit and when current quark mass is equal to zero we can pick out the local term which follows from the WZW action and correction to it which are connected with nonlocality of the model:

$$F_{3\pi}(0,0,0) = \frac{e}{4\pi^2 f_\pi^3} + \frac{eN_c N_f}{f_\pi^3}\int \frac{d_E^4 k}{(2\pi)^4}\left[\frac{4m'(k^2)m^3(k^2)k^2}{D(k)^4}\right], \tag{22}$$

the second term vanishes in the local limit.

4 Conclusion

Transition $\gamma^* \to 3\pi$ form factor plays an important role in understanding of chiral anomaly and pseudoscalar meson properties. We showed that nonlocality of model gives a correction to the WZW term of anomaly $\gamma^* \to 3\pi$. It is needed to study of mixing scheme of η and η' mesons. It is also important for g-2 muon anomaly investigation where/how this process contributes to hadron vacuum polarization. In addition, it is necessary for dispersion analysis of light-by-light contribution calculations [14].

Recently there was shown that behavior of transition form factor will be changed with taking into account the intermediate vector meson resonance exchange [15, 16]. A similar picture also has a place for the processes $\pi\gamma^* \to \pi\pi/K\bar{K}$. We are planning to consider a role of the lightest vector meson in these reactions in framework of the extended $SU(2)\times SU(2)$ model. It is also interesting in connection with future measurement of the exclusive $\gamma p \to \pi^+\pi^- p$ reaction of photoproduction at the GlueX experiment at Jefferson laboratory [17].

In future work we plan to calculate Dalitz decays of η and η' mesons into $\gamma\pi^+\pi^-$ or $e^+e^-\pi^+\pi^-$ which are of significant interest to experimentators. We have a plan to study $e^+e^- \to 3\pi$ taking into account vector meson exchanges. The study of these processes is now in progress. The vector mesons plays an important role for processes under consideration and should be included in the extended $SU(3)\times SU(3)$ model.

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