

$B_s \rightarrow K^{*0}$ decay form factors from covariant confined quark model

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Abstract. We evaluate $B_s \rightarrow K^{*0}$ transition form factors in the full kinematical region within the covariant confined quark model. The calculated form factors can be used to calculate the $B_s \rightarrow K^{*0}\mu^+\mu^-$ rare decay branching ratio, which was recently measured by LHCb collaboration.

Recent measurements of rare B-decays show deviations with respect to Standard Model predictions [1–5]. The $b \rightarrow s\ell^+\ell^-$ and $b \rightarrow d\ell^+\ell^-$ processes are forbidden at tree-level in Standard Model and sensitive to New Physics contributions in loops. The $b \rightarrow d$ transition is suppressed than $b \rightarrow s$ due to CKM matrix elements. However it is interesting to study decays proceeding via flavour-changing neutral-current (FCNC) transition. The $b \rightarrow d$ transition decays are observed by LHCb collaboration for $B^+ \rightarrow \pi^+\mu^+\mu^-$ [6, 7] and $\Lambda^0 \rightarrow \rho\pi^-\mu^+\mu^-$ [8] decays. Recently the LHCb Collaboration [9] reported about the measurement of branching ratio of $B_s \rightarrow K^{*0}\mu^+\mu^-$ decay.

The $B_s \rightarrow K^{*0}$ transition form factors were studied in the light-cone sum rule [10, 11] and lattice QCD [12] techniques. In view of these development, we calculate $B_s \rightarrow K^{*0}$ form factors within the covariant confined quark model (CCQM).

The covariant confined quark model [13] is an effective quantum field approach to hadronic interactions based on an interaction Lagrangian of hadrons interacting with their constituent quarks. The value of the coupling constant follows from the compositeness condition $Z_H = 0$, where Z_H is the wave function renormalization constant of the hadron. Matrix elements of the physical processes are generated by a set of quark loop diagrams according to the $1/N_c$ expansion. The ultraviolet divergences of the quark loops are regularized by including vertex functions for the hadron-quark vertices. These function also describe finite size effects related to the non-pointlike hadrons. The quark confinement [14] is built-in through an infrared cutoff on the upper limit of the scale integration to avoid the appearance of singularities in matrix elements. The infrared cutoff parameter λ is universal for all processes. The covariant confined quark model has limited number of parameters: the light and heavy constituent quark masses, the size parameters which describe the size of the distribution of the constituent quarks inside the hadron and the infrared cutoff parameter λ . They are determined by a fit to available experimental data.

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In calculations we used next values of the model parameters which are shown in Eq. (1):

$m_{u/d}$	m_s	m_c	m_b	λ	Λ_{B_s}	Λ_{K^*0}	m_{B_s}	m_{K^*0}	
0.241	0.428	1.67	4.68	0.181	2.05	0.81	5.367	0.896	GeV

(1)

Below, we list the definitions of the dimensionless invariant transition form factors together with the covariant quark model expressions that allow one to calculate them. We closely follow the notation used in our papers [15, 16]:

$$\begin{aligned}
& \langle V(p_2, \epsilon_2)_{[\bar{q}_3 q_2]} | \bar{q}_2 O^\mu q_1 | P_{[\bar{q}_3 q_1]}(p_1) \rangle = \\
& = N_c g_P g_V \int \frac{d^4 k}{(2\pi)^4 i} \bar{\Phi}_P(- (k + w_{13})^2) \bar{\Phi}_V(- (k + w_{23})^2) \\
& \times \text{tr} \left[O^\mu S_1(k + p_1) \gamma^5 S_3(k) \not{\epsilon}_2^\dagger S_2(k + p_2) \right] \\
& = \frac{\epsilon_v^\dagger}{m_1 + m_2} \left(-g^{\mu\nu} P \cdot q A_0(q^2) + P^\mu P^\nu A_+(q^2) + q^\mu P^\nu A_-(q^2) \right. \\
& \left. + i \epsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta V(q^2) \right), \tag{2}
\end{aligned}$$

$$\begin{aligned}
& \langle V(p_2, \epsilon_2)_{[\bar{q}_3 q_2]} | \bar{q}_2 (\sigma^{\mu\nu} q_\nu (1 + \gamma^5)) q_1 | P_{[\bar{q}_3 q_1]}(p_1) \rangle = \\
& = N_c g_P g_V \int \frac{d^4 k}{(2\pi)^4 i} \bar{\Phi}_P(- (k + w_{13})^2) \bar{\Phi}_V(- (k + w_{23})^2) \\
& \times \text{tr} \left[(\sigma^{\mu\nu} q_\nu (1 + \gamma^5)) S_1(k + p_1) \gamma^5 S_3(k) \not{\epsilon}_2^\dagger S_2(k + p_2) \right] \\
& = \epsilon_v^\dagger \left(- (g^{\mu\nu} - q^\mu q^\nu / q^2) P \cdot q a_0(q^2) + (P^\mu P^\nu - q^\mu P^\nu P \cdot q / q^2) a_+(q^2) \right. \\
& \left. + i \epsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta g(q^2) \right). \tag{3}
\end{aligned}$$

We use $P = p_1 + p_2$ and $q = p_1 - p_2$ and the on-shell conditions $\epsilon_2^\dagger \cdot p_2 = 0$, $p_i^2 = m_i^2$. Since there are three quark species involved in the transition, we have introduced a two-subscript notation $w_{ij} = m_{q_j} / (m_{q_i} + m_{q_j})$ ($i, j = 1, 2, 3$) such that $w_{ij} + w_{ji} = 1$. The form factors defined in Eqs. (2)-(3) satisfy the physical requirement $a_0(0) = a_+(0)$, which ensures that no kinematic singularity appears in the matrix element at $q^2 = 0$.

The form factors are calculated in the full kinematical region of momentum transfer squared and results of our numerical calculations are with high accuracy approximated by the parametrization

$$F(q^2) = \frac{F(0)}{1 - as + bs^2}, \quad s = \frac{q^2}{m_1^2}, \tag{4}$$

where the relative error is less than 1%. The values of $F(0)$, a , and b are listed in Tab. 1.

Table 1. Parameters for the approximated form factors in Eq. (4)

	A_0	A_+	A_-	V	a_0	a_+	g
$F(0)$	0.30	0.21	-0.23	0.24	0.21	0.21	0.21
a	-0.64	-1.47	-1.55	-1.60	-0.69	-1.48	-1.61
b	-0.28	0.44	0.52	0.56	-0.23	0.45	0.57

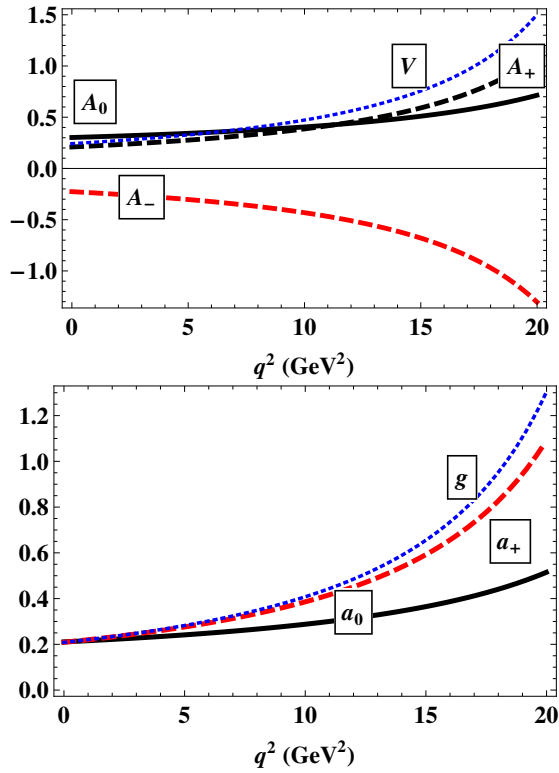


Figure 1. The q^2 -dependence of the vector and axial form factors (upper plot) and tensor form factors (lower plot) for $B_s \rightarrow K^{*0}$ decay

The curves are depicted in Fig. 1.

The obtained errors of the fitted parameters were of the order of 10%. Indeed it follows that form factors at $q^2 = 0$ were calculated with 10% uncertainties. This implies at least 10% uncertainty in form factors in the full kinematical region of momentum transfer squared.

For reference it is useful to relate the above form factors to those used, e.g., in [10] (we denote them by the superscript c). The relations read as follows:

$$\begin{aligned}
 A_0 &= \frac{m_1 + m_2}{m_1 - m_2} A_1^c, & A_+ &= A_2^c, \\
 A_- &= \frac{2m_2(m_1 + m_2)}{q^2} (A_3^c - A_0^c), & V &= V^c, \\
 a_0 &= T_2^c, & g &= T_1^c, & a_+ &= T_2^c + \frac{q^2}{m_1^2 - m_2^2} T_3^c.
 \end{aligned} \tag{5}$$

We note in addition that the form factors (5) satisfy the constraints

$$\begin{aligned}
 A_0^c(0) &= A_3^c(0) \\
 2m_2 A_3^c(q^2) &= (m_1 + m_2) A_1^c(q^2) - (m_1 - m_2) A_2^c(q^2).
 \end{aligned} \tag{6}$$

Since $a_0(0) = a_+(0) = g(0)$ we display in Tab. 2 the form factors $A_0^c(0) = (m_1 - m_2)[A_0(0) - A_+(0)]/(2m_2)$, $A_1^c(0) = A_0(0)(m_1 - m_2)/(m_1 + m_2)$, $A_2^c(0) = A_+(0)$, $T_1^c(0) = g(0)$ and $T_3^c(0) =$

Table 2. The form factors at maximum recoil $q^2 = 0$

	$V^c(0)$	$A_0^c(0)$	$A_1^c(0)$	$A_2^c(0)$	$T_1^c(0)$	$T_3^c(0)$
CCQM	0.24 ± 0.02	0.18 ± 0.02	0.21 ± 0.02	0.21 ± 0.02	0.21 ± 0.02	0.14 ± 0.01
[10]	0.31	0.36	0.23	0.18	0.26	0.14

$\lim_{q^2 \rightarrow 0} (m_1^2 - m_2^2)(a_+ - a_0)/q^2$ obtained in our model and compare them with those from light-cone sum rule [10].

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