

# Feasibility of measuring EDM in spin transparent colliders

Anatoliy Kondratenko<sup>1,\*</sup>, Mikhail Kondratenko<sup>1</sup>, Yury Filatov<sup>2</sup>, Alexander Kovalenko<sup>3</sup>, Yaroslav Derbenev<sup>4</sup>, and Vasilij Morozov<sup>4</sup>

<sup>1</sup>Science and Technique Laboratory “Zaryad”, Novosibirsk, Russia

<sup>2</sup>Moscow Institute of Physics and Technology, Dolgoprudny, Russia

<sup>3</sup>Joint Institute for Nuclear Research, Dubna, Russia

<sup>4</sup>Jefferson Lab, Newport News, VA, USA

**Abstract.** A new polarization control mode called a Spin Transparency (ST) mode is currently being actively developed for new rings. The ST mode is an intrinsic feature of figure-8 rings such as the JLEIC at Jefferson Lab. A racetrack collider can be converted to the ST mode by inserting two identical Siberian snakes into its opposite straights as at NICA, JINR. The ST mode allows one to develop a completely new approach to the measurement of the EDMs of both protons and deuterons. The idea of the method is to use the significant enhancement of the EDM signal by the interaction of the EDM with arc magnets, which has an interference effect. A unique feature of this technique is the conceptual capability of measuring the EDM signal in the whole energy range of a ring without introducing additional electric fields using only its magnetic fields. We describe an experimental setup and provide estimates of the limiting EDM values that can be measured at the JLEIC and NICA colliders.

## 1 Introduction

The search for the proton and deuteron EDMs has been the goal of accelerators and storage rings for the past few decades. The concept of “frozen spin” was proposed at the US BNL for measuring the proton EDM in a ring consisting of only electric field elements [1]. The idea of this technique is to choose the “magic” energy when there is no effect of the magnetic dipole moment on the spin. However, this technique is not applicable to the search of the deuteron EDM. The ideas of this technique were further developed for measurement of the deuteron EDM using simultaneously magnetic and electric fields [2]. References [3, 4] propose the use of spin rotators based on static combined electro-magnetic fields (Wien filter) for measuring the EDM of protons and deuterons. A technique for the search of the EDM signal in a purely magnetic ring based on the spin rotation in a Wien filter was developed in [5, 6]. The accuracy limit for measuring the EDM in the above technique is determined by lattice element errors and beam orbital emittances. The Juelich Electric Dipole Investigation (JEDI) collaboration actively studies the question of possible systematic false effects using the COSY accelerator (Juelich, Germany) as an example.

Below we propose a technique to measure the proton and deuteron EDMs based on using the ring’s spin transparency mode, which does not require neither additional electric fields nor Wien filters.

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\*e-mail: kondratenkom@mail.ru

## 2 Generalized Thomas-BMT equation

The spin dynamics in electro-magnetic fields  $\vec{E}$  and  $\vec{B}$  is described by the generalized Thomas-BMT equation. The equation for a particle with the magnetic dipole moment (MDM)  $\vec{\mu}$  and the electric dipole moment (EDM)  $\vec{d}$ :

$$\vec{\mu} = (1 + G_M) \frac{e\hbar}{mc} \vec{S}, \quad \vec{d} = G_E \frac{e\hbar}{mc} \vec{S},$$

of a spin  $\vec{S}$  takes the following form:

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}, \quad \vec{\Omega} = -\frac{e}{\gamma mc} \left[ (1 + G_M) \vec{B}_{rest} + G_E \vec{E}_{rest} + \frac{\gamma - 1}{\gamma v^2} \vec{v} \times \vec{E}_{rest} \right],$$

where  $\gamma$  is the Lorentz factor,  $v$  is a speed in units of the light speed  $c$ ,  $\vec{B}_{rest}$  and  $\vec{E}_{rest}$  are the field values in the particle rest frame related to the  $\vec{B}$  and  $\vec{E}$  fields in the laboratory frame by the Lorentz transformations. The first term of the equation describes the spin rotation due the particle MDM, the second one describes the spin rotation due to the EDM, while the third one is due to the Thomas precession.

The particle orbital and spin motion are described using the accelerator reference frame  $\vec{e}_x(z)$ ,  $\vec{e}_y(z)$  and  $\vec{e}_z(z)$ . The radial unit vector  $\vec{r}$  is a function of the three unit vectors  $x$ ,  $y$ , and  $z$  near the design closed orbit  $\vec{r}_c(z)$  given by:

$$\vec{r}(x, y, z) = \vec{r}_c(z) + x \vec{e}_x(z) + y \vec{e}_y(z),$$

where  $z$  is the longitudinal coordinate along the closed orbit and  $x$  and  $y$  are the coordinates of the transverse deviation from it. The longitudinal unit vector  $\vec{e}_z(z) = d\vec{r}_c/dz$  is directed along the closed orbit.

There is a wide class of closed orbits which can have a selection of continuous unit vectors with the frame angular rotation rate determined only by the orbit curvature perpendicular to the curvature plane:

$$\vec{W}_b = \vec{K}(z) = K_x \vec{e}_x + K_y \vec{e}_y = \vec{e}_z \times \frac{d\vec{e}_z}{dz}.$$

The equation of motion in the accelerator frame is obtained by subtracting the frame angular rotation rate  $\vec{W}_b$  from the particle equation of motion (the Lorentz equation). The equations for the particle energy and velocity change are then given by:

$$\frac{d\mathcal{E}}{dz} = -\frac{\mathcal{E}}{c} \vec{\mathbb{B}} \vec{v}, \quad \frac{d\vec{r}}{dz} = \vec{r}' = \left( \vec{\mathbb{B}} + \frac{1}{v} \vec{\mathbb{E}} \times \vec{r} - \vec{K} \right) \times \vec{r},$$

where  $\mathcal{E} = \gamma mc^2$  is the particle energy,  $\vec{\mathbb{B}} = \chi \vec{B}/B\rho$  and  $\vec{\mathbb{E}} = \chi \vec{E}/B\rho$  are the normalized magnetic and electric fields, respectively,  $B\rho = -pc/e$  is the magnetic rigidity, and parameter  $\chi = [(1 + K_x y - K_y x)^2 + x'^2 + y'^2]^{1/2}$ .  $\vec{r}'$  here is a vector composed of the components in the accelerator reference frame.

The generalized Thomas-BMT equation can be analogously written in the accelerator reference frame. However, when describing the spin dynamics, the characteristic vector is the exact direction of the particle velocity. The spin equation has a much simpler form in the natural reference frame connected with the velocity direction of each particle in the beam. The natural reference frame can be expressed in terms of the unit vectors of the accelerator frame in the following way:

$$\vec{e}_1 = \frac{(\vec{e}_y + \vec{r} \times \vec{e}_x) \times \tau}{1 + \tau_z}, \quad \vec{e}_2 = \frac{\tau \times (\vec{e}_x + \vec{e}_y \times \vec{r})}{1 + \tau_z}, \quad \vec{e}_3 = \vec{r}.$$

The unit vectors  $\vec{e}_1$  and  $\vec{e}_2$  are transverse to the particle velocity ( $\vec{e}_1 \vec{v} = \vec{e}_2 \vec{v} = 0$ ). The natural reference frame is only slightly different from the accelerator frame and, in the linear approximation, is related to the accelerator frame as:

$$\vec{e}_1 = \vec{e}_x - \tau_x \vec{e}_z, \quad \vec{e}_2 = \vec{e}_y - \tau_y \vec{e}_z, \quad \vec{e}_3 = \vec{e}_z + \tau_x \vec{e}_x + \tau_y \vec{e}_y.$$

To write the spin equation in the natural frame, the rate of change of the reference frame ( $\vec{e}_1, \vec{e}_2, \vec{e}_3$ ) must be subtracted from the precession rate  $\vec{\Omega}$ . The angular rate of rotation of the natural frame with respect to the stationary frame is

$$\vec{W}_b = \frac{1}{2} \sum_{i=1}^3 \vec{e}_i \times \vec{e}'_i = \vec{v} \times \vec{v}' + \mathfrak{a} \vec{v},$$

where

$$\mathfrak{a} = \vec{e}_2 \frac{d\vec{e}_1}{dz} = \vec{K} \vec{v} + \frac{\tau'_x \tau_y - \tau'_y \tau_x}{1 + \tau_z}$$

describes the oscillations of the unit vectors  $\vec{e}_1$  and  $\vec{e}_2$  about the velocity.

Thus, the generalized Thomas-BMT equation in the natural reference frame becomes

$$\frac{d\vec{S}}{dz} = \vec{W} \times \vec{S},$$

where

$$\begin{aligned} \vec{W} &= \vec{W}_\perp + \vec{W}_\parallel, & W_\parallel &= (1 + G_M)\mathbb{B}_\parallel + G_E\mathbb{E}_\parallel - \mathfrak{a}, \\ \vec{W}_\perp &= \gamma G_M \vec{\mathbb{B}}_\perp + \gamma v \left( G_M - \frac{1}{\gamma^2 v^2} \right) \mathbb{E} \times \vec{v} + \gamma G_E (\vec{\mathbb{E}}_\perp + \vec{v} \times \vec{\mathbb{B}}). \end{aligned}$$

The above spin equation is equivalent to the generalized Thomas–BMT equation and can serve as a basis for beam polarization calculations in accelerators.

### 3 Spin dynamics in JLEIC in the ST mode

Let us consider the spin dynamics in the ST mode using the figure-8 collider of the JLEIC as an example. For relativistic particles the “direct” contribution of the electric field to the EDM signal is significantly weaker than the contribution of the arc dipoles. To demonstrate the technique, we limit our consideration to writing the spin motion equations in the linear approximation in the absence of additional electric fields. In case of a flat orbit ( $K_x = 0$ ), the angular velocity components  $\vec{W} = \vec{W}_0 + \vec{w}$  in the magnetic field of a ring are given in the natural reference frame by

$$\begin{cases} W_{0x} = 0, \\ W_{0y} = \gamma G_M K_y, \\ W_{0z} = 0, \end{cases} \quad \begin{cases} w_x = -\gamma G_M \tau'_y - \gamma v G_E \tau'_x, \\ w_y = \gamma G_M \tau'_x - \gamma v G_E \tau'_y, \\ w_z = (1 + G_M)(\mathbb{B}_z + K'_y y) + G_M K_y y' + G_E \mathbb{E}_z. \end{cases}$$

Let us switch to the spin reference frame ( $\vec{e}_1^s, \vec{e}_2^s, \vec{e}_3^s$ ), which describes the dynamics of the spins initially directed along the accelerator frame unit vectors ( $\vec{e}_x, \vec{e}_y, \vec{e}_z$ ) when a particle moves along the closed orbit:

$$\vec{e}_1^s + i \vec{e}_3^s = \exp(i \Psi_y) (\vec{e}_x + i \vec{e}_z), \quad \vec{e}_2^s = \vec{e}_y, \quad \Psi'_y(z) = \gamma G_M K_y$$

where  $\Psi_y(z)$  is the phase accumulated by the spin in the arc magnets, which is a periodic function of the ring  $\Psi_y(z) = \Psi_y(z + L)$ .

The components of the spin field  $\vec{\omega}$  in the spin reference frame have the following form:

$$\omega_i = \frac{L}{2\pi} \langle \vec{\omega} \vec{e}_i^s \rangle .$$

The angle brackets here mean averaging over the orbit while  $L$  is the orbit length. When averaging, using the fact that, in the ST mode, the phase  $\Psi_y(z)$  contains only frequencies that are multiples of the revolution frequency, we get that, in a real ring structure, the components of the spin field lie in the plane of the ring ( $\omega_2 = 0$ ) and are equal to

$$\omega_1 + i\omega_3 = G_E \frac{\Delta p}{m} \frac{L}{2\pi} \left\langle D'_x \frac{d}{dz} e^{i\Psi_y} \right\rangle + i(1 + G_M) \frac{L}{2\pi} \langle \mathbb{B}_z e^{i\Psi_y} \rangle + \gamma G_M \frac{L}{2\pi} \left\langle \tau_y \frac{d}{dz} e^{i\Psi_y} \right\rangle ,$$

where  $D_x$  is the radial dispersion function,  $\Delta p$  is the momentum deviation,  $\mathbb{B}_z$  are the solenoid fields for spin control.

The spin field consists of three terms each having its own nature. The contribution of the EDM to the spin field is determined by the first term, which is proportional to the momentum deviation. The second term is due to the effect of the control solenoids. The third term determines the coherent part of the resonance strength  $\omega_{coh}$ , which is related to the closed orbit excursion.

After compensation of the coherent resonance part by small solenoids, the ring becomes equivalent to an ideal one, which has no errors in the construction and setup of its magnetic elements. The spin field is then determined only by one term:

$$\omega_1 + i\omega_3 = G_E \frac{\Delta p}{m} \frac{L}{2\pi} \left\langle D'_x \frac{d}{dz} e^{i\Psi_y} \right\rangle .$$

In the linear approximation, when averaging over the synchrotron oscillations, the dependence of the spin field  $\vec{\omega}$  on the EDM term ( $\langle \Delta p \rangle = 0$ ) disappears. The dependence on the EDM term appears only in the next, second-order approximation. The largest contribution to the spin field is related to the spread of the spin precession phase  $\Psi_y$  when the spin makes a large number of turns ( $\gamma G_M \gg 1$ ) in the ring arcs. The average spin field lying in the orbit plane becomes:

$$\omega_1 + i\omega_3 = i\gamma v^3 G_E \left( \frac{\Delta p}{p} \right)^2 \frac{L}{2\pi} \left\langle D'_x \Psi_y \frac{d}{dz} e^{i\Psi_y} \right\rangle \equiv G_E k_E \left( \frac{\Delta p}{p} \right)^2 .$$

Taking into account the betatron oscillations in the second order of the averaging method gives a vertical component of the spin field proportional to the vertical emittance [7]

$$\omega_2 = \omega_{emitt} .$$

The ‘‘EDM signal’’ is related to the synchrotron oscillations and is determined by the ring structure. Let us emphasize that the contribution to the EDM signal from the structure depending on the spin rotation angle in the arc magnets (energy) has interference character. Thus, there is a possibility of manifold enhancement of the EDM signal by the correlated contributions of all arc dipoles of the ring.

## 4 Measurement of the EDM at the JLEIC

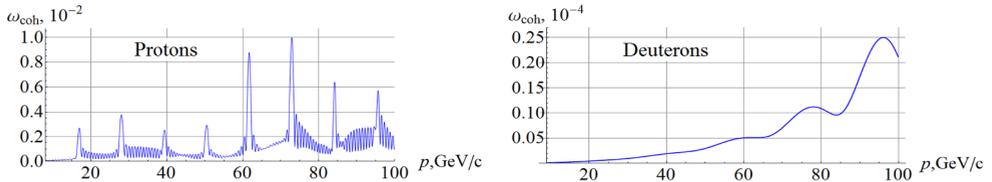
Measurement of the EDM signal can be done by increasing the momentum spread after compensation of the coherent part of the spin field. The number of turns  $N_{rev}$  needed to rotate the polarization by an angle  $\alpha_s$  is determined by:

$$G_E k_E \left( \frac{\Delta p}{p} \right)^2 = \frac{\alpha_s}{2\pi} \frac{1}{N_{rev}}, \quad N_{rev} = \frac{v T_{exp}}{L},$$

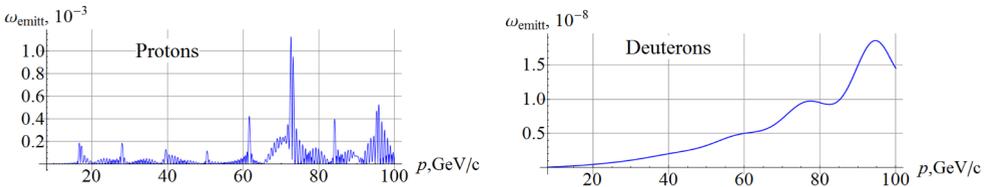
where  $T_{exp}$  is a time of experiment. The number of turns  $N_{rev}$  is limited by the incoherent part of the resonance strength and cannot be greater than  $\omega_{emitt}^{-1}$ . The limiting precision of the EDM measurement occurs in the region of the interference maximum  $k_E$  and minimum  $\omega_{emitt}$ :

$$G_E^{lim} = \frac{\alpha_s}{2\pi} \frac{\omega_{emitt}}{k_E (\Delta p/p)^2}.$$

Figs. 1 and 2 show the values of  $\omega_{coh}$  and  $\omega_{emitt}$  for the JLEIC lattice [7], which indicate that the limiting precision of the EDM measurement at the JLEIC is  $G_E \sim 10^{-5}$  for protons and  $G_E \sim 10^{-6}$  for deuterons. Our calculations assume:  $\Delta p/p = 10^{-3}$ ,  $\alpha_s = 0, 1$  and that the EDM measurement takes place in the region of the interference maximum  $k_E$  ( $\sim 10^3$  for protons and  $\sim 10$  for deuterons) and minimum  $\omega_{emitt}$  ( $\sim 10^{-6}$  for protons and  $\sim 10^{-9}$  for deuterons).



**Figure 1.** Coherent part of the resonance strength for protons and deuterons at the JLEIC



**Figure 2.** Incoherent part of the resonance strength for protons and deuterons at the JLEIC

## 5 Measurement of the EDM at the NICA collider

The spin dynamics at the NICA collider with two solenoidal snakes becomes equivalent to the dynamics in a figure-8 ring and the EDM measurement method can be analogous. For the NICA collider, this estimate gives the limiting precision of the EDM measurement of  $G_E \sim 10^{-4}$  for protons and  $G_E \sim 10^{-5}$  for deuterons. These calculations assume that  $\Delta p/p = 3 \cdot 10^{-3}$ ,  $\alpha_s = 0, 1$  and that the EDM measurement takes place in the region of the interference maximum  $k_E$  ( $\sim 100$  for protons and  $\sim 1$  for deuterons) and minimum  $\omega_{emitt}$  ( $\sim 10^{-5}$  for protons and  $\sim 10^{-8}$  for deuterons).

An additional issue with the ST mode at the NICA collider is the necessity to account for a strong betatron oscillation coupling and alignment errors of strong snake solenoids.

## 6 Further enhancement of the limiting EDM measurement precision

To further enhance the limiting precision of the EDM measurement, one must complete a detailed analysis of:

- the system for compensation of the coherent part of the spin resonance strength, which has all three components, using control solenoids (3D spin rotators);
- the system for compensation of the incoherent part of the spin resonance strength, which is determined by beam emittances;
- the impact of the betatron oscillation coupling on the spin dynamics in the ST mode;
- the contribution of higher-order orbital and spin resonances;
- the collider optics allowing one to increase the interference maxima of the EDM signal;
- the EDM signal measurement methods;
- the beam polarization measurement methods.

## Conclusion

In conclusion, let us summarize the main results presented in this paper.

- A theoretical analysis of the spin dynamics of a particle with the MDM and EDM in ST mode rings is completed in the linear approximation.
- It is proposed to measure the EDM at the JLEIC and NICA colliders using the method based on the interference enhancement of the EDM signal in the whole energy range using only the magnetic fields of the ST mode ring.
- Estimates of the EDM signal in the ST modes of the JLEIC and NICA colliders are presented demonstrating the possibility of measuring the EDM of proton and, especially, deuteron beams.
- Questions are formulated about the study of further enhancement of the limiting precision of the EDM measurement in ST mode rings.

**Acknowledgement.** This material is based on the work supported in part by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under contracts DE-AC05-06OR23177 and DE-AC02-06CH11357.

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