Unitarity of neutrino mixing matrix

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Abstract. Neutrinos are neutral leptons and there exist three types of neutrinos (electron neutrinos ν_e , muon neutrinos ν_μ and tau neutrinos ν_τ). These classifications are referred to as neutrinos's "flavors". Oscillations between the different flavors are known as *neutrino oscillations*, which occurs when neutrinos have mass and non-zero mixing. Neutrino mixing is governed by the PMNS mixing matrix. The PMNS mixing matrix is constructed as the product of three independent rotations. With that, we can describe the numerical parameters of the matrix in a graphical form called the *unitary triangle*, giving rise to CP violation. We can calculate the four parameters of the mixing matrix to draw the unitary triangle. The area of the triangle is a measure of the amount of CP violation.

1 Neutrino Oscillation

Neutrino oscillations are a results of neutrino mixing via: the left-handed flavor neutrino fields. Neutrino flavor eigenstates can be expressed as a superposition of the mass eigenstates

$$\nu_{\alpha L} = \sum_{i=1}^n U_{\alpha i} \nu_{iL}, \quad (1)$$

where

$\nu_i (i = 1, 2, 3)$ refers to the mass eigenstates,

$\nu_\alpha (\alpha = e, \mu, \tau)$ refers to the flavor eigenstates.

U is the Pontecorvo-Maki-Nakagawa-Sakata matrix, where each element is a complex number

$$\begin{aligned} U_{PMNS} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_2 & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} P \end{aligned} \quad (2)$$

where

θ_{ij} is the mixing angle, $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$,

CP-violating phases: Dirac phase δ_{CP} has $P = 1$ and Majorana phases ϕ_1, ϕ_2 has $P = \text{diag}(1, e^{i\frac{\phi_1}{2}}, e^{i\frac{\phi_2}{2}})$

2 Oscillation probability

The oscillation probability of flavor transition $\nu_\alpha \rightarrow \nu_\beta$ after distance L in vacuum and with energy E

$$P_{\nu_\alpha \rightarrow \nu_\beta}^{(-)} = |A_{\nu_\alpha \rightarrow \nu_\beta}^{(-)}|^2 = \delta_{\alpha\beta} - 4 \sum_{j>i} \text{Re}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right) \mp 2 \sum_{j>i} \text{Im}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin\left(\frac{\Delta m_{ij}^2 L}{2E}\right), \quad (3)$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$,

- $m_1 < m_2 < m_3$ corresponds to a normal mass order (NO),

- $m_3 < m_1 < m_2$ corresponds to a inverted mass order (IO).

3 Unitarity triangle

The PMNS matrix is unitary and so $U^\dagger U = U U^\dagger = 1$. From this property, we can normalize the matrix elements such that $\sum U_{\alpha i} U_{\alpha i}^* = 1$. So, there are 6 triangles governed by the orthogonal relations

$$\sum U_{\alpha i} U_{\beta i}^* = 0, \quad (\alpha \neq \beta) \quad (4)$$

$$\sum U_{\alpha i} U_{\alpha j}^* = 0, \quad (i \neq j) \quad (5)$$

4 Results

4.1 The unitary triangle with our current neutrino landscape

The current best-fit values from NuFit.org [3] are given in following table. The numbers in the second and third column are assuming Normal Ordering or Inverted Ordering respectively and the numbers given are relative to the respective global minimums.

Table 4.1: The current best fit values from NuFit.org (Global fit framework for ν oscillation) on November 2017.

	Normal Ordering (NO)	Inverted Ordering (IO)
	best fit $\pm 1\sigma$	best fit $\pm 1\sigma$
$\sin^2 \theta_{12}$	$0.307^{+0.013}_{-0.012}$	$0.307^{+0.013}_{-0.012}$
$\sin^2 \theta_{23}$	$0.538^{+0.038}_{-0.069}$	$0.554^{+0.023}_{-0.033}$
$\sin^2 \theta_{13}$	$0.02206^{+0.00075}_{-0.00075}$	$0.02227^{+0.00074}_{-0.00074}$
$\delta_{CP}/^\circ$	234^{+43}_{-31}	278^{+26}_{-29}

The values of PMNS matrix parameters are calculated as

$$U_{normal} = \begin{pmatrix} 0.823 & 0.548 & -0.087 + 0.120 * i \\ -0.323 + 0.073 * i & 0.601 + 0.049 * i & 0.725 \\ 0.456 + 0.068 * i & -0.575 + 0.045 * i & 0.672 \end{pmatrix}$$

$$U_{inverted} = \begin{pmatrix} 0.823 & 0.548 & 0.0206 + 0.148 * i \\ -0.387 + 0.091 * i & 0.554 + 0.060 * i & 0.729 \\ 0.397 + 0.083 * i & -0.622 + 0.055 * i & 0.668 \end{pmatrix}$$

Figure 4.1: The unitary triangle for normal hierarchy and for inverted hierarchy corresponds to the unitary conditions on the first and third columns

4.2 Simple model for estimating uncertainty

These are the 1σ ranges for the magnitudes of the matrix parameters, and the two values correspond to the global minimum and maximum values respectively.

$$|U| = \begin{pmatrix} 0.815 \rightarrow 0.831 & 0.537 \rightarrow 0.559 & 0.146 \rightarrow 0.151 \\ 0.320 \rightarrow 0.393 & 0.536 \rightarrow 0.662 & 0.678 \rightarrow 0.747 \\ 0.425 \rightarrow 0.456 & 0.552 \rightarrow 0.632 & 0.647 \rightarrow 0.721 \end{pmatrix} \quad (6)$$

Figure 4.2: Plot of δ_{CP} with the four oscillation parameters having 1σ uncertainties

4.3 Unitarity test with future neutrino experiments

Figure 4.3: Plot of δ_{CP} with the four oscillation parameters having 1σ uncertainties of $\pm 23^\circ$. (Hyper-K)

Figure 4.4: Plot of δ_{CP} with the four oscillation parameters having with 1σ uncertainties of $\pm 15^\circ$. (Combining Hyper-K and DUNE)

5 Conclusion

- These six unitary triangles have the same area which is equal to half of Jarlskog's invariant, showing the CP violation of the process.
- By reducing uncertainties of δ_{CP} , the size of the allowed region at the C vertex of the unitary triangle at some confidence level will be reduced. The precision of the unitary vertex position will be increased.

References

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