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Constraining Non-linear Dirac Equations with Neutrino Oscillations

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Abstract. Considering the phenomenological studies of non-linear quantum models, we use an axiomatic approach to modify the Dirac Lagrangian. We apply constraints such as Hermiticity, locality, universality, etc to obtain various generic modified energy dispersion relations. After-which, we use the parameters from the neutrino oscillations to obtain bounds on these new modified dispersion relations.

1 Motivation

The usefulness of non-linear equations could be seen in quantum cosmology, where a weak non-linearity could replace the Big Bang singularity with a bounce [1]. This motivates us to ask if a similar generalisation with the Dirac equation could be used to model new physics. In the search for these non-linear Dirac equations, we have taken an information theoretic approach. That is, we use the maximum entropy principle to deduce the probability distribution which gives the least biased description of the system's state [2].
Using information theoretic arguments, we construct an action of the form,

$$S = \int d^4x \, (\mathscr{L}_0 + \mathscr{F}) \tag{1}$$

where $\mathscr{L}_0 = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$ is the usual Dirac Lagrangian. ψ and its adjoint $\bar{\psi}$ are the Dirac spinor wavefunctions. The information measure is given by $I = \int d^4x \mathscr{F}$. In finding the new equations of motion, we use the Lagrange multiplier method[1]. This simultaneously minimizes the information measure and gives us a generalized Dirac equation. We can interpret these non-linear Dirac equations as encoding potential new physics at higher energies. We then proceed to apply constraints to the information measure to obtain various forms of $\mathscr{F}$ as in [3]. It is important to note that the non-linearity here is not demanded but a result of information theoretic generalisation [3].

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[1]the multiplier is implicit in $\mathscr{F}$

2 Information Theoretic Approach

2.1 Conditions

We assume that ψ and $\bar{\psi}$ contract in the natural way in $\mathscr{F}$ to give us scalars. The information measure should satisfy the following conditions [3]:

[C1] Homogeneity: The information measure should be homogeneous and is invariant under the following scaling, $\mathscr{F}(\lambda\psi, \lambda\bar{\psi}) = \lambda^2 \mathscr{F}(\psi, \bar{\psi})$. This allows the wavefunction ψ to be freely normalised and subsequently minimizes our deformation.

[C2] Uncertainty: The information measure $\mathscr{F}$ should decrease as ψ approaches a uniform value. This causes the uncertainty in the location of the quantum particle to be at a maximum. This enforces that $\mathscr{F}$ contains derivatives of ψ. We also note that $\mathscr{L}_0$ already contains such derivatives and our assumption appears to be natural.

[C3] Locality: All dependence of the wavefunction ψ and its adjoint $\bar{\psi}$ is at the same space-time point and has a finite number of derivatives. Thus we consider,

$$\mathscr{F} = \frac{N}{D} \tag{2}$$

where N is a polynomial of the wavefunction, which has a finite number of derivatives. The denominator D is also a polynomial. Both N and D satisfies [C1].

[C4] Positivity: The information measure $\mathscr{F}$ is an inverse uncertainty measure, and thus is positive for generic ψ. Hence $\mathscr{F}$ is positive and real.

[C5] Minimisation: The maximum uncertainty principle demands that the information measure $\mathscr{F}$ should take the minimum value when one extremizes the action S.

2.2 Form of the Information Measure

We consider a class of solutions in the following form,

$$\mathscr{F} = \frac{(P(\psi, \bar{\psi}))^{n+1}}{(Q(\psi, \bar{\psi}))^n} \tag{3}$$

where P, Q are linear combinations of single bilinears.
In order to satisfy [C4], we have the additional condition [3]:

$$\begin{cases} n \text{ is odd,} & Q(\bar{\psi},\psi) \text{ is positive} \\ n \text{ is even,} & P(\bar{\psi}, \psi) \text{ is positive} \end{cases}$$

where n is an integer greater than 0.

2.3 Explicit examples

We consider four examples of non-linear Dirac Lagrangian with the form in Equation (3).

$$\mathscr{L}_1 = \mathscr{L}_0 + \frac{(i\bar{\psi}\gamma^\nu\partial_\nu - i(\partial_\nu\bar{\psi})\gamma^\nu\psi)^2}{4A_\mu\bar{\psi}\gamma^\mu\psi} \tag{4}$$

$$\mathscr{L}_2 = \mathscr{L}_0 + \frac{(B_\nu\partial^\nu(\bar{\psi}\psi))^2}{A_\mu\bar{\psi}\gamma^\mu\psi} \tag{5}$$

$$\mathscr{L}_3 = \mathscr{L}_0 + \frac{(A_\mu \bar{\psi}\gamma^\mu \psi)^3}{(\bar{\psi}\psi)^2} \tag{6}$$

$$\mathscr{L}_4 = \mathscr{L}_0 + \frac{(\bar{\psi}\gamma^5 \psi)^2}{A_\mu \bar{\psi}\gamma^\mu \psi} \tag{7}$$

where $A_\mu = (A, 0, 0, 0)$ is a time-like constant positive background field and $B_\nu = (0, \mathbf{B})$ is a constant space-like background field. These background fields give a measure of Lorentz violation [4].

2.4 Modified Energy Dispersion relations

We have listed the modified dispersion relations in Table 1.

Table 1. Modified energy dispersion relations from our constructed generalised Dirac Lagrangians.

Lagrangian	Modified Energy Dispersion Relation
$\mathscr{L}_1$	$E^2 = p^2 + m^2 - \frac{2m^2}{AE}$
$\mathscr{L}_2$	$E^2 = p^2 + m^2$
$\mathscr{L}_3$	$E^2 = p^2 + m^2 - \frac{2A^3E^3}{m^2}$
$\mathscr{L}_4$	$E^2 = p^2 + m^2$

3 Results of Constraining Additional Parameters with Neutrino Oscillation

We will ignore $\mathscr{L}_2$ and $\mathscr{L}_4$ as they do not modify the neutrino oscillation probability. For $\mathscr{L}_1$ and $\mathscr{L}_3$, we assume Δm^4_{32} to have the same order of magnitude as $(\Delta m^2_{32})^2$, where $\Delta m_{32}^{\;4} \equiv {m_3}^4 - {m_2}^4$ and $\Delta m_{32}^2 \equiv {m_3}^2 - {m_2}^2$. From [5], we have $\sin^2(2\theta_{23}) = 0.999$, $E \sim 0.6$ [GeV] and $\Delta m^2_{32} = 2.463^{+0.071}_{-0.070} \times 10^{-3}$ [eV$^2/c^4$]. These gives a bound on the order of magnitude of the additional parameters as shown in Table 2. With these constrains, we plot the modified probability curve in Figure 1.

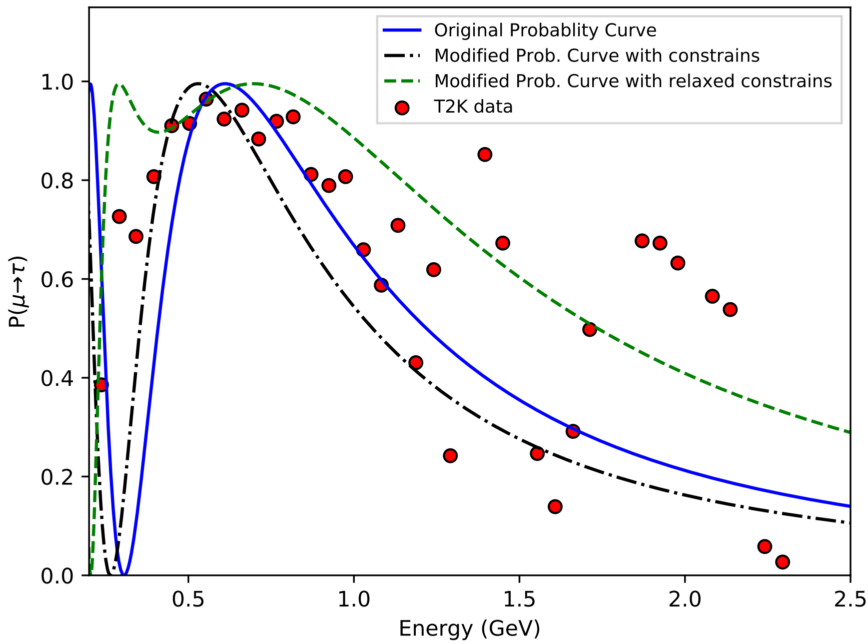
Table 2. The table below shows the bounds on the additional parameters. These bounds are calculated from the experimentally determined range of Δm^2_{32} and our additional parameter must lie in this range. Here we define $M_{32} \equiv m_3^2 m_2^2$.

Lagrangian	Additional Parameter
$\mathscr{L}_1$	$A \sim 10^2 \left[\frac{1}{\text{GeV}}\right]$
$\mathscr{L}_3$	$\frac{A}{M_{32}} \sim 10^{-1}\left[\frac{e\text{V}}{\text{GeV}^4}\right]$

If we relax the constrain on the energy E and let $\Delta m^2_{32} \sim 10^{-3}$ eV$/c^4$ and A to be any positive value[2], we get an interesting result for $\mathscr{L}_1$ as shown in the modified probability curve (with relaxed constrains) in Figure 1. It gives a better fit to the data and has some distinctive features that can be tested in the future experiments.

[2] any positive value outside the order of magnitude given in Table 2.

Figure 1. For $\mathscr{L}_1$, we have the transition probability of ν_μ neutrino in the graph below. For the constrained case, we have $A = 999$ [1/GeV] & $\Delta m^2_{32} = 0.0022$ [eV^2]. The modified probability equation with constrains has a smaller χ^2 value of 26.10 as compared to the original probability curve's 33.75. For the relaxed constrain case, we have $A = 0.04$ [1/GeV] & $\Delta m^2_{32} = 0.0041$ [eV^2]. The modified probability equation with relaxed constrains has a smaller χ^2 value of 6.41 as compared to the original probability curve. The modified probability curve with relaxed constraints gives the best fit.



4 Conclusion

We have discussed the methodology of obtaining the non-linear Dirac lagrangian from the information-theoretic arguments. We have obtained bounds on the various parameters with the current data from [5]. By relaxing certain parameters, we have produced a better fit for the oscillation probability with distinctive features. Note that the above analysis is a rough fitting to the data.
For future work, a more detailed fitting could be done to see whether the distinctive features remains and the analysis can be applied to other modified dispersion relations obtained from different physical motivations.

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