

Parton Branching Process with Supersymmetric Particles at LHC Energies

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Abstract. A 2-step parton branching model is proposed to describe the potential presence of supersymmetric particles in multiplicity distributions. This model gives a reasonable description of the data obtained at 13TeV by the ATLAS Collaboration across the maximum pseudorapidity range of $\eta \leq 2.5$.

1 Introduction

The statistical study of charged particle multiplicity distributions reveals correlations about the underlying dynamics of multiparticle production. The characteristics of the statistical distribution which best describes the data can then be used to infer the corresponding characteristics of the production mechanism. It is in this spirit that the current project was undertaken in a bid to reveal the presence of any supersymmetric particles via a phenomenological approach.

One of the authors (Y.Y. Zhang) has used the resulting probability distribution to describe the multiplicity data obtained by the CMS Collaboration at 7TeV [1]. The values of the model parameters obtained from this earlier study are used as a basis for comparison for the current study using data from the ATLAS Collaboration at 13TeV.

2 SUSY-Induced Multiplicity Distribution (SIMD)

As the name suggests, the SIMD assumes a 2-stage branching model to describe the resulting charged particle multiplicity distribution as measured in the various experiments at the LHC. The model takes a fixed number of super-symmetric and normal ancestor partons and follows them via various branching processes as described by [1, 2]. In the first stage, a fixed number of SUSY and normal partons begin branching with high primary virtuality $Y = Y_{max}$. This process will continue until the virtuality decreases to the mass scale of the SUSY partons $Y = Y_{SUSY}$. For simplicity's sake, we assumed the branching of $g \rightarrow g + g$ for the normal partons and $\tilde{q} \rightarrow \tilde{q} + g$ and $\tilde{g} \rightarrow \tilde{g} + g$ for the squark and gluino branching respectively.

The second stage commences upon the virtuality reaching $Y = Y_{SUSY}$. Henceforth, the SUSY particles will branch into normal partons and the Lightest Supersymmetric Partner (LSP denoted by $\tilde{\chi}_1^0$). The simplest SUSY on-shell branching is assumed. A squark will branch into a normal quark with an LSP via $\tilde{q} \rightarrow q + \tilde{\chi}_1^0$. On the other hand, a gluino will produce a quark, an antiquark and an LSP via $\tilde{g} \rightarrow q + \bar{q} + \tilde{\chi}_1^0$.

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2.1 Mathematical Derivation

The 2-stage branching model contains the following dominant branches. The first branch corresponds to the simple birth process while the remaining 3 branches correspond to the Poisson process. Each branch is associated with its corresponding branching probabilities:

$$\begin{aligned}
 A &: g \rightarrow g + g \\
 \tilde{A} &: q \rightarrow q + g \\
 I &: \tilde{q} \rightarrow \tilde{q} + g \\
 J &: \tilde{g} \rightarrow \tilde{g} + g
 \end{aligned} \tag{1}$$

To derive the probability function of the SIMD $p_{SIMD}(n, Y)$, one can work from obtain the generating function of the distribution. Once that is known, the probability functions at individual multiplicities n is only a matter of taking repeated derivatives. A more rigorous approach to obtain the generating function is to solve the following differential equation which describes phase 1, with δ , γ , q and g denoting respectively the initial number of squarks, gluinos, quarks and gluons.

$$\begin{aligned}
 \frac{dp_n}{dY} &= -Anp_n + A(n-1)p_{n-1} - (q\tilde{A} + \delta I + \gamma J)p_{n-1} \\
 &= -Anp_n + A(n-1)p_{n-1} - \nu_1 p_{n-1}
 \end{aligned} \tag{2}$$

The solution of the above differential difference equation then serves as the initial condition to the differential equation of phase 2 described by the following equation to obtain an expression for p_{SIMD} .

$$\begin{aligned}
 \frac{dp_n}{dY} &= -Anp_n + A(n-1)p_{n-1} - (q + \delta + 2\gamma)\tilde{A}p_n + (q + \delta I + 2\gamma J)p_{n-1} \\
 &= -Anp_n + A(n-1)p_{n-1} - (q + \delta + 2\gamma)\tilde{A}p_n + \nu_2 p_{n-1}
 \end{aligned} \tag{3}$$

Alternatively, a faster way to obtained the generating function will be to separately derive the generating function for the SUSY plus quark branching denoted by P_{SUSY} , and that for the parton birth process in (1) denoted by P_G . Denoting the ratio $k_1 = \frac{\nu_1}{A}$ and $k_2 = \frac{\nu_2}{A}$, we have

$$\begin{aligned}
 P_{SIMD} &= P_{SUSY} \times P_G \\
 &= \frac{\left[(1 - e^{A(Y-Y_1)})x + e^{A(Y-Y_1)} \right]^{k_1 - k_2}}{\left[(1 - e^{AY})x + e^{AY} \right]^{k_1}} \times \left[\frac{x}{(1 - e^{AY})x + e^{AY}} \right]^g
 \end{aligned} \tag{4}$$

By using the formula $p(n) = \frac{1}{n!} \left. \frac{d^n P}{dx^n} \right|_{x=0}$, we obtain the probability for SIMD.

$$\begin{aligned}
 p_{SIMD}(n, Y) &= e^{A(Y-Y_1)(k_1 - k_2)} e^{-AY(k_1 + g)} (1 - e^{-AY})^{n-g} \frac{\Gamma(n + k_1)}{\Gamma(k_1 + g)\Gamma(n - g + 1)} \\
 &\quad \times {}_2F_1 \left[k_2 - k_1, g - n, 1 - k_1 - n, \frac{e^{AY} - e^{AY_1}}{e^{AY} - 1} \right]
 \end{aligned} \tag{5}$$

The Gamma function is defined as $\Gamma(x) = \int_0^{\infty} s^{x-1} e^{-s} ds$ while ${}_2F_1$ above refers to Gauss's Hypergeometric Function. Equation (5) can be further simplified if we condense some of the parameters. We let $l = e^{AY}$, $m = e^{AY_1}$. Using these substitutions we have a slightly more compact form as follows:

$$P_{SIMD}(n, Y) = \left(\frac{l}{m}\right)^{k_1 - k_2} \left(1 - \frac{1}{l}\right)^{n-a} l^{-k_1} \frac{\Gamma(n + k_1)}{\Gamma(k_1 + a)\Gamma(n - a + 1)} \times {}_2F_1\left[k_2 - k_1, g - n, 1 - k_1 - n, \frac{l - m}{l - 1}\right] \quad (6)$$

The distribution in (6) is used to describe the multiplicity data at 13TeV from ATLAS [3]. In SIMD, l and m are functions of the virtualities Y and Y_1 respectively. As such, these 2 parameters can be used as proxies for the values of the virtualities. Equal values of l and m imply that SIMD is unable to detect any data from supersymmetric branching processes. This may mean that the assumptions of SIMD does not correctly model the actual branching process, or the energy from which the data is obtained is too low to detect supersymmetric branching. In other words, the energy scale at the collider is lower than that of the LSP. The parameters k_1 and k_2 refers to the relative amplitudes of probability of the Poisson branching process during phase 1 and phase 2 respectively.

3 Description of Multiplicity Distribution by SIMD

As seen from the Table 1, the differences in the l and m values are within 2 standard deviations. Statistically, there is a significant probability that this difference stems from statistical fluctuations rather than the actual physical branching mechanism. In addition, this difference should not be analysed in isolation. By comparing across the different values obtained from the fit at 7TeV, it is seen that the fit at 13TeV comes with a higher χ^2/dof value as well as a huge fractional uncertainty in the value of k_2 . In view of these factors, this study is unable to make a conclusive claim on the presence of supersymmetric branching processes at current LHC energies.

$\sqrt{s}(TeV)$	l	m	k_1	k_2	g	χ^2/dof
7	22.5	22.5	1.48	1.60	0	130.68/121
13	18.9 ± 0.2	19.4 ± 0.2	0.58 ± 0.06	0.03 ± 0.03	0.19 ± 0.05	206.9/76

Table 1: Best fit parameters using SIMD on CMS 7TeV data and ATLAS 13TeV data. The CMS data is obtained at $\eta < 2.4$ while the ATLAS data is obtained for $\eta < 2.5$.

4 Future Work

The SIMD has been derived under the assumption of a fixed initial number of supersymmetric particles prior to the commencement of the branching process between collision events. This was initially done in the pursuit of simplicity for physical realism. Although this assumption leads to a closed form for the probability distribution function seen in (6), it is physically untenable and artificial. There is no physical argument as to why the initial number should be fixed. On this front, looking at a Poisson distributed number of initial number of particles

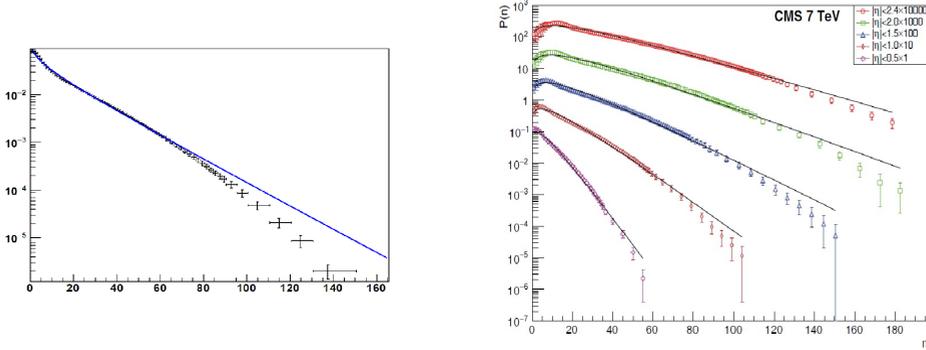


Figure 1: Plots of charged particle multiplicity distribution obtained at 13TeV by the ATLAS collaboration (left) and at 7TeV by the CMS collaboration (right) [4]. The data from ATLAS is obtained with $\eta < 2.5$ while that from CMS covers the different η values up to $\eta < 2.4$. Both plots were described using the SIMD with the parameters given in Table 1.

would be a more natural alternative. Analogous to solving differential equations with different initial conditions result in different outcomes, modifying the initial number of partons will change the form of the probability function. This is an academic exercise worth pursuing in lieu of collisions at higher energies, where supersymmetric particles are currently postulated to reside.

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