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Meson production in e^+e^- annihilation and tau lepton decays within extended NJL model

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Abstract. The Nambu–Jona-Lasinio model is applied to describe processes of electron-positron annihilation into mesons at center-of-mass energies below 2 GeV and hadronic decays of tau leptons. Contributions of intermediate scalar, vector, and axial-vector mesons in the ground and first radial excited states are taken into account. Comparisons with existing experimental data are performed. Theoretical predictions for several (not yet measured) processes are presented.

1 Introduction

The Nambu–Jona-Lasinio (NJL) model [1] is known to be very successful in description of the spectrum and low-energy interactions of light mesons, see, e.g., reviews [2, 3] and references therein. The model is rather simple, it is based on the idea of spontaneous breaking of the chiral symmetry. The symmetry condition restricts the number of possible interactions between quarks, which allows to have a small number of parameters in the model. It was also demonstrated that the NJL model can be derived from QCD by integration over gluon degrees of freedom and applying the formalism of effective field theories [4]. The success of the model in description of meson interactions at low energies stimulated attempts to extend the area of the NJL model applicability in different directions, including physics of mesons with heavy quarks [5], strong interactions at finite temperature [6], neutron stars properties [7], *etc.* Here we will consider the extension of the NJL model which allows to include the first radial excited states of four light meson nonets (scalar, pseudoscalar, vector and axial-vector ones). This extended NJL model was introduced in refs. [8–10]. The model successfully described spectra and hadronic decay modes of the four nonents of radial exited mesons. Since 2010 a new project devoted to treatment of QED and electroweak processes with these meson states has been started. The energy domain below about 2 GeV, where the dynamics of the first radial meson states is very important, was studied. This energy range is beyond the domain of the standard NJL model applicability. So it is really interesting to check whether the principle of chiral symmetry still allows to provide a good description of mesons interactions at these energies or not. Many processes of electron-positron annihilation and hadronic tau decays were systematically considered within the extended NJL model, see reviews [11, 12] and references therein. Below the main features of the extended model will be described and a few examples of its application will be given.

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2 Extended NJL model

Chirally symmetric four-quark interactions with non-local currents can be written in the following form:

$$\begin{aligned}\mathcal{L}^{(4)}_{\rm int} &= \frac{G_1}{2}\int d^4x \sum_{j=1}^{9}\sum_{i=1}^{2}[J^j_{S,i}(x)J^j_{S,i}(x) + J^j_{P,i}(x)J^j_{P,i}(x)] \\ &- \int d^4x \sum_{j=1}^{9}\sum_{i=1}^{2}[\frac{G_2}{2}J^{J,\mu}_{V,i}(x)J^j_{V,i,\mu}(x) + \frac{G_3}{2}J^{j,\mu}_{A,i}(x)J^j_{A,i,\mu}(x)], \qquad (1)\end{aligned}$$

where G_i are coupling constants. The scalar (S), pseudoscalar (P), vector (V), and axial-vector (A) currents are

$$\begin{aligned}J^j_{S(P),i}(x) &= \int d^4x_1 d^4x_2\ \bar{q}(x_1)F^j_{S(P),i}(x;x_1,x_2)q(x_2), \\ J^{j,\mu}_{V(A),i}(x) &= \int d^4x_1 d^4x_2\ \bar{q}(x_1)F^{j,\mu}_{V(A),i}(x;x_1,x_2)q(x_2), \qquad (2)\end{aligned}$$

where $\bar{q} = (\bar{u},\bar{d},\bar{s})$ is the flavor $SU(3)$ triplet quark field. To describe mesons both in the ground and first radial excited states in was suggested [8] to use simple form factors which in the momentum space read

$$\begin{aligned}F^j_{S,2}(\vec{k}^2) &= \tau^j c^j_S f_j(\vec{k}^2), \qquad F^j_{P,2}(\vec{k}^2) = i\gamma_5\tau^j c^j_P f_j(\vec{k}^2), \\ F^{j,\mu}_{V,2}(\vec{k}^2) &= \gamma^\mu\tau^j c^j_V f_j(\vec{k}^2), \qquad F^{j,\mu}_{A,2}(\vec{k}^2) = \gamma_5\gamma^\mu\tau^j c^j_A f_j(\vec{k}^2). \qquad (3)\end{aligned}$$

where τ^j are the Gell-Mann matrices and

$$f_j(\vec{k}^2) \equiv 1 + d_j\vec{k}^2. \qquad (4)$$

The slope parameters $d_u = -1.784\ \mathrm{GeV}^{-2}$ and $d_s = -1.727\ \mathrm{GeV}^{-2}$ are fixed by requiring that the inclusion of excited meson states doesn't change the values of quark condensates. The coefficients c^j_U are fitted using the physical masses of excited mesons.

Taking into account the 6-quark 't Hooft interaction [13] and performing the standard hadronization procedure, one gets the Lagrangian of the extended NJL model with quark-meson interactions. Diagonalization of the mass terms of the ground and excited states leads to the appearance of the mixing of physical mesons. Here we present the part which describes pions:

$$\begin{aligned}\mathcal{L}(\pi,q) &= \bar{q}(p')\Big[A_\pi\gamma_5[\tau^3\pi^0(k) + \sqrt{2}\tau^+\pi^+(k) + \sqrt{2}\tau^-\pi^-(k)] - A_{\pi'}\gamma_5[\tau^3(\pi^0)'(k) \\ &+ \sqrt{2}\tau^+(\pi^+)'(k) + \sqrt{2}\tau^-(\pi^-)'(k)]\Big]q(p), \qquad k = p'-p, \quad \tau^\pm = \frac{\tau^1 \pm i\tau^2}{\sqrt{2}}, \\ A_\pi &= g_{\pi_1}\frac{\sin(\alpha+\alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2}f(\vec{k}^2)\frac{\sin(\alpha-\alpha_0)}{\sin(2\alpha_0)}, \\ A_{\pi'} &= g_{\pi_1}\frac{\cos(\alpha+\alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2}f(\vec{k}^2)\frac{\cos(\alpha-\alpha_0)}{\sin(2\alpha_0)},\end{aligned}$$

where we see the mixing angles $\alpha_0 \approx 59.12°$ and $\alpha \approx 59.48°$. The coupling constants $g_{\pi_1} = 7.34$ and $g_{\pi_2} = 12.54$ describe the interactions of the ground and excited pion states with quarks. The complete Lagrangian of the extended NJL model and details of its derivation can be found in review [12].

3 Meson production in e^+e^- annihilation

Let us discuss for example how the process $e^+ + e^- \rightarrow K^+ + K^-$ is described within the extended NJL model, see details in Ref. [14]. The corresponding Feynman diagrams are given in Fig. 1.

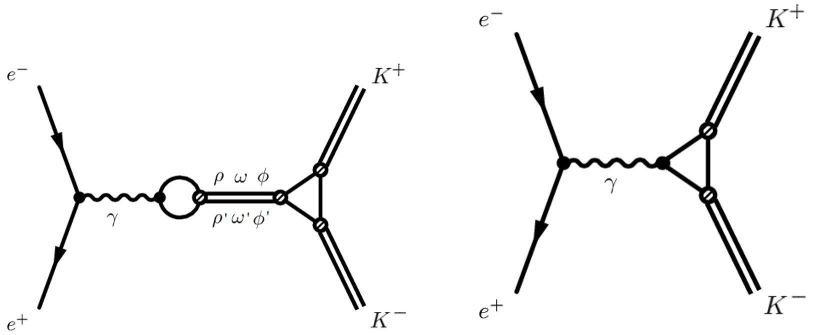

Figure 1. Amplitudes with (left) and without (right) intermediate vector mesons.

The amplitude of the process at energies below 2 GeV contains contributions of several different intermediate states:

$$T = \frac{16\pi\alpha_{em}}{s} l^\mu \Big[B_{(\gamma)} + B_{(\rho+\rho')} + +B_{(\omega+\omega')} + e^{i\pi}B_{(\phi+\phi')}\Big]_{\mu\nu}(p_{K^+} - p_{K^-})^\nu, \tag{5}$$

where $s = (p(e^-) + p(e^+))^2$, $l^\mu = \bar{e}\gamma^\mu e$ is the lepton current. The NJL model can not describe a relative phase between different states. Thus, we take the phase ($e^{i\pi}$ factor in the ϕ mesons) from e^+e^- annihilation experiments [15].

It is interesting to look at the sum of $V = \rho, \omega, \phi$ and $V' = \rho', \omega', \phi'$ vector meson contributions:

$$B_{(V+V')\mu\nu} = r_V\Big[\frac{C_V}{g_V}\frac{g_{\mu\nu}s - p_\mu p_\nu}{M_V^2 - s - i\sqrt{s}\Gamma_V(s)} I_2^{a_V a_K a_K} + \frac{C_{V'}}{g_V}\frac{g_{\mu\nu}s - p_\mu p_\nu}{M_{V'}^2 - s - i\sqrt{s}\Gamma_{V'}(s)} I_2^{b_V a_K a_K}\Big]. \tag{6}$$

The numerical coefficients $r_\rho = r_{\rho'} = 1/2$, $r_\omega = r_{\omega'} = 1/6$, $r_\phi = r_{\phi'} = 1/3$ came from the flavor $SU(3)$ algebra. The energy dependence of an intermediate vector meson width is approximated by

$$\Gamma_V(s) = \Gamma_V \frac{s}{M_V^2}\left(\frac{\beta(s, M_K)}{\beta(M_V^2, M_K)}\right)^3, \qquad \beta(s, M_K) = \sqrt{1 - 4{M_K}^2/s}. \tag{7}$$

Numerical coefficients C_V are obtained from the quark loops in the photon transitions into the intermediate vector mesons,

$$C_V = \frac{1}{\sin\left(2\theta_a^0\right)}\left[\sin\left(\theta_a + \theta_a^0\right) + R_V \sin\left(\theta_a - \theta_a^0\right)\right], \qquad R_V = \frac{I_2^f(m_1, m_2)}{\sqrt{I_2(m_1, m_2) I_2^{f^2}(m_1, m_2)}}, \tag{8}$$

where m_1 and m_2 are the masses of the $u(d)$ or s quarks depending on the quark structure of the intermediate vector meson. Integrals I_2 are taken over quark loop momenta with the cut-off $\Lambda = 1.03$ GeV.

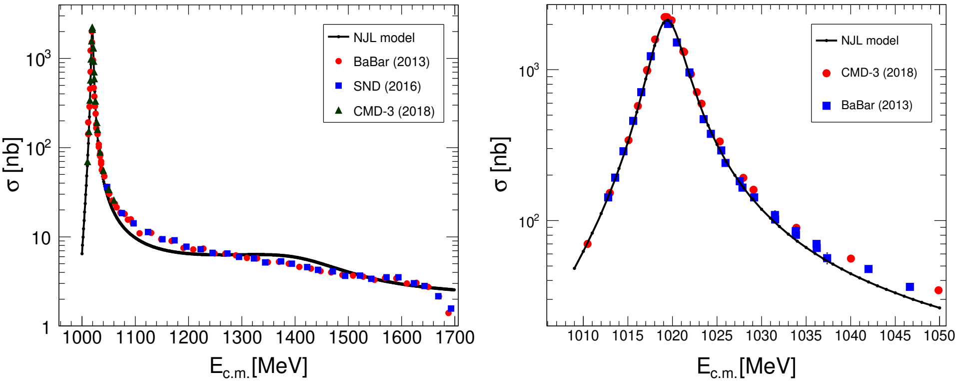

Figure 2. The cross section of $e^+e^- \to K^+K^-$ vs. center-of-mass energy. The ϕ meson peak region is zoomed (right).

Theoretical estimates obtained with the help of the extended NJL model in comparison with experimental results [16–18] are shown Fig. 2. One can see that the model quite well describes the data at the peak, but there are some deviations above it. And certainly, the theoretical description fails at 1700 MeV where the second radial excited *rho* meson state is situated.

Using the same set of model parameters the following processes were described (see review [12] and references therein):

- $e^+e^- \to [\pi, \pi(1300)]\gamma$
- $e^+e^- \to [f_1(1285), a_1(1260)]\gamma$
- $e^+e^- \to \omega(782)\pi^0$
- $e^+e^- \to K^{\pm}[K^{*\mp}(892), K^{*\mp}(1410)]$
- $e^+e^- \to [\eta, \eta'(958)]2\pi$
- $e^+e^- \to [\eta, \eta'(958), \eta(1295), \eta(1475)]\gamma$
- $e^+e^- \to [\pi, \pi(1300)]\pi$
- $e^+e^- \to \rho(770)\eta$
- $e^+e^- \to [\eta, \eta^{'}(958)][\phi(1020), \phi(1680)]$

Square brackets denote here the list of possible channels, *e.g.*, $e^+e^- \to [\pi, \pi(1300)]\pi$ means that both $e^+e^- \to 2\pi$ and $e^+e^- \to 2\pi(1300)$ were described. A good agreement with experimental data was observed for all processes that have been already studied experimentally. Theoretical predictions are made for the rest of processes, which can be verified in the running and future experiments. In particular, the following reactions are certainly of interest: $e^+e^- \to \pi(1300)\gamma$, $e^+e^- \to [\eta'(958), \eta(1295), \eta(1475)]\gamma$, $e^+e^- \to [f_1(1285), a_1(1260)]\gamma$, $e^+e^- \to \pi(1300)\pi$, $e^+e^- \to K^{\pm}K^{*\mp}(1410)$, $e^+e^- \to \eta'(958)\phi(1020)$, $e^+e^- \to \eta, \phi(1680)$, $e^+e^- \to \eta'(958)2\pi$.

3.1 Meson production in τ lepton decays

Note that interactions of mesons with leptons (and photons) are described within the NJL model via the standard couplings of quarks to photons and W bosons. So, hadronic modes of *tau* lepton decays can be described within the same model without any change of its structure and parameters. Since the *tau* lepton mass is about 1.777 GeV, including of the first radial excited states of mesons is crucial for getting a good theoretical description of hadronic τ decay modes.

Table 1. Comparison of NJL model predictions for τ lepton decay branching fractions with experimental data.

Process	NJL (Br)	Experiment (Br)	NJL Ref.
$\tau \to \pi\nu_\tau$	11.04%	$(10.82 \pm 0.05)\%$	[12]
$\tau \to \pi(1300)\nu_\tau$	9.8×10^{-5}	$(10 \div 19) \times 10^{-5}$	
$\tau \to K^*(892)\nu_\tau$	1.15%	$(1.2 \pm 0.07)\%$	[20]
$\tau \to K^*(1410)\nu_\tau$	0.23%	$(0.15 + 1.4 - 1)$	
$\tau \to K_1(1270)\nu_\tau$	0.4%	$(0.47 \pm 0.11)\%$	
$\tau \to K_1(1650)\nu_\tau$	2.99×10^{-4}	-	
$\tau \to a_1(1260)\nu_\tau$	14.1%	-	
$\tau \to a_1(1640)\nu_\tau$	0.63%	-	
$\tau \to \pi^-\pi^0\nu_\tau$	24.76%	$(25.49 \pm 0.09)\%$	[21]
$\tau \to \pi\omega(782)\nu_\tau$	1.85%	$(1.95 \pm 0.06)\%$	[22]
$\tau \to \eta\pi^-\nu_\tau$	4.72×10^{-6}	$< 9.9 \times 10^{-5}$	[23]
$\tau \to \eta'(958)\pi^-\nu_\tau$	3.74×10^{-8}	$< 4 \times 10^{-6}$	
$\tau \to K^-\pi^0\nu_\tau$	4.13×10^{-3}	$(4.33 \pm 0.15) \times 10^{-3}$	[24]
$\tau \to \eta K^-\nu_\tau$	1.45×10^{-4}	$(1.55 \pm 0.08) \times 10^{-4}$	[25]
$\tau \to \eta'(958)K^-\nu_\tau$	1.25×10^{-6}	$< 2.4 \times 10^{-6}$	
$\tau \to K^0K^-\nu_\tau$	1.27×10^{-3}	$(1.48 \pm 0.05) \times 10^{-3}$	[26]
$\tau \to \rho(770)\eta\nu_\tau$	1.44×10^{-3}	-	[27]
$\tau \to \bar{K}^{*0}(892)\pi^-\nu_\tau$	1.78×10^{-3}	$(2.2 \pm 0.5) \times 10^{-3}$	[28]
$\tau \to f_1(1285)\pi^-\nu_\tau$	3.98×10^{-4}	$(3.9 \pm 0.5) \times 10^{-4}$	[29]
$\tau \to \eta 2\pi\nu_\tau$	1.46×10^{-3}	$(1.39 \pm 0.07) \times 10^{-3}$	[30]
$\tau \to \eta'(958)2\pi\nu_\tau$	9×10^{-7}	$< 1.2 \times 10^{-5}$	

Some results of the extended NJL model predictions for different τ lepton decay branching fractions are given in Table 1. The corresponding experimental results [19] (where available) are listed in the third column. References to the papers with details on the corresponding NJL model calculations are in the fourth column. One can see that the theoretical predictions reasonably well agree with the data. That allows us to expect that the predictions for not yet observed decay modes are reliable. In particular, it is interesting to look at the decays $\tau \to \eta\pi^-\nu_\tau$ and $\tau \to \eta'\pi^-\nu_\tau$ which belong to the so-called second-class current type. In the NJL model the amplitudes of these processes are suppressed by the difference of the up and down quark masses, see details in Ref. [23].

4 Conclusions

It is found that the extended NJL model rather well describes processes of electron-positron annihilation into mesons at the center of mass energy below 2 GeV and hadronic modes of τ lepton decays. It is important to note that the model parameters were fixed and retained being the same for all considered processes. The typical deviations of the model predictions from existing experimental results for inclusive observables in such processes doesn't exceed 10%. That allows to expect that the provided theoretical predictions for a number of not yet observed annihilation channels and tau decay modes are reliable. The predictions contribute to the physical programs of running and future experiments.

There are also a few specific observations concerning meson dynamics in the given energy domain. In particular, we found important to implement 4-by-4 mixing of $\eta(550)$, $\eta'(958)$, $\eta(1295)$, and $\eta(1475)$ pseudoscalar mesons. On more interesting conclusion is done about the applicability of the vector meson dominance (VMD) model. Namely, the NJL model reproduces the results of VMD for vector mesons in the ground states, but that is not held for the corresponding excited vector mesons.

In spite of the general good agreement of the model predictions with experimental data, there are several cases where the model fails. In such cases we certainly meet phenomena which go beyond the scope of the model. In particular, it appeared difficult to describe within the model the masses and/or interactions of such mesons as $\eta(1405)$, $a_0(980)$, $a_1(1410)$, $f_0(1500)$ which are known as candidates to tetraquarks, glueballs or other exotics states. In a sense, the problem of the model to incorporate such meson provides an additional indication of the exotic nature of the states.

As the result, we see that the very simple NJL model demonstrated the ability to describe a really wide range of meson states and their interactions even at energies reaching 2 GeV. This is certainly a non-trivial success. We believe that its main origin is the application of spontaneous chiral symmetry breaking mechanism.

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