Determining the thickness of sludge on the heat exchanger tube inside an anaerobic digester

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Abstract. The paper presents a simplified method of determining the thickness of sludge on the walls of the heat exchanger piping in a biogas plant digester. The evaluation of the thickness of a sludge layer is based on the biogas plant operation parameters, including the inlet and outlet temperature of the heat exchanger, mass flow and the geometric characteristics of the heat exchanger and physical parameters of the substrate working inside a fermentation chamber. Measurement of the thickness of the sludge layer on the walls of a heat exchanger is only possible at the time of general cleaning of a digester, which necessitates switching off the biogas plant operation. The paper compares the results of predictions with experimental data of the work of a biogas plant digester located in north-eastern Poland, in Ryboły. The presentation of the obtained numerical results is supplemented by the uncertainty analysis. The significance of undertaking such research lies in its applicational aspects, as during the operation of a biogas plant sludge accumulates on the walls of a digester, which provides additional thermal resistance and reduces the thermal efficiency of the heat exchanger.

1 Introduction

Biogas plants are among the most cost-effective alternative energy sources [1, 2]. Biogas is produced from organic substances that are degraded by microorganisms in the methane fermentation process [3]. The products of the process are methane and carbon dioxide, as well as small amounts of hydrogen sulphide, nitrogen and hydrogen. The growing interest in biogas production means that more and more substrates, both waste and deliberately produced, coming from industry, agriculture or urban areas are used in its production. The most frequently used raw materials include: waste from food production, liquid or solid animal waste (manure, slurry), food leftovers, organic municipal waste, post-slaughter waste, waste from crop production, energy crops (maize, alfalfa) and biomass forest [4-7]. It should be noted that some organic waste, e.g. medical waste, due to the risk of hygienic contamination and the possibility of pathogens, cannot be used as a substrate in fermentation chambers [8, 9]. The residues from the fermentation process can be used as a fertiliser provided they are free from pathogens [10, 11]. The work of a biogas plant also involves many problems related to heat exchange. The main problem is maintaining a constant temperature in the fermentation chambers, especially during the winter. The main causes of the problem related to maintaining a constant temperature inside the fermentation chamber are too thin a layer of thermal insulation of the digester, and inefficient operation of the heat exchanger. The issue of thermal insulation of building partitions has been described in the literature [12-14], while the work of the heat exchanger is presented in the publication [15]. The work of the heat exchanger is mainly influenced by the sludge that appears on the walls of the heat exchanger tubing during the operation of the biogas plant. In the work [15], a simplified model of the simulation of thermal efficiency of the heat exchanger is described, depending on the thickness of the sludge layer on the exchanger walls.

The increase in the thickness of the sludge layer on the walls of the exchanger is an additional thermal resistance and causes a decrease in the thermal efficiency of the heat exchanger. An increase in the efficiency of the exchanger can be achieved by increasing the flow of the heating medium through the exchanger, increasing the heat exchange surface by adding additional loops in the heat exchanger tubing or raising the flow temperature. The first method is limited by the circulation pump operating parameters. The second method involves additional costs of heat exchanger expansion, while the third method can lead to the death of microorganisms in the fermentation chamber due to the temperature increase within the heat exchanger if the temperature of the heat exchanger surface is too high. Examining the thickness of the insulation on the walls of the exchanger is extremely difficult, because it requires switching off the biogas plant operation, emptying the fermentation chamber from the substrate, and then measuring the thickness of the sludge layer on the walls of the exchanger tubing. Due to the inability to

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monitor the thickness of the sludge on the walls of the heat exchanger, it seems appropriate to create a model and procedure to determine the heat exchanger layer based on measurement data such as the thermal efficiency of the heat exchanger in the digester.
The heat exchanger research was performed based on the example of a biogas plant located in the village of Ryboły in the north-eastern part of Poland. The photograph in Fig. 1 shows a general view of fermentation chambers in the biogas plant in Ryboły. A detailed description of the biogas plant can be found in the publications [14, 15].

Fig. 1. View of fermentation chambers in the Ryboły biogas plant.

The present work proposes a method that allows the determination of the thickness of the sludge layer based on the heat exchanger's thermal efficiency measurement. More precisely, the thickness of the sludge layer is inferred from the measured outlet temperature from the exchanger. The obtained estimates turn out to be in good agreement with their true values (known from measurement), which suggests that the method can find a practical application. Moreover, the present study contains uncertainty analysis referring to the parameter being determined (thickness of the sludge layer), in order to control the error of the method.

2 Mathematical formulation

Heat exchangers in the biogas fermentation chambers are made of circular tubes attached to the walls of the loop-shaped fermentation chamber [15-17]. A physical scheme of the piping fragment is shown in Fig. 2.

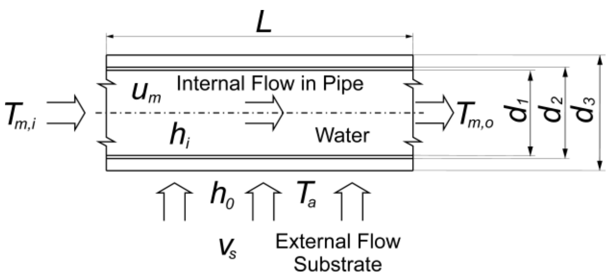

Fig. 2. Schematic diagram of the heat exchanger.

Due to the large ratio of the loop diameter to the heat exchanger's piping diameter, a straight duct was assumed for further consideration.

2.1 Adapting a model for heat exchange

A simplified model, described in [15], was used for calculations, in which the flow inside the heat exchanger pipeline and external flow of the substrate around the heat exchanger tubing are considered (Fig.2). The return temperature $T_{m,o}$ from the heat exchanger is described by the known relationship [18]:

$$T_{m,o} = T_a - \left(T_a - T_{m,i}\right)\exp\left[-\frac{\pi d_3 L}{m c_{pm}} U\right], \quad \left[{}^{o}\mathrm{C}\right] \qquad (1)$$

where T_a is the temperature of the substrate, $T_{m,i}$ is the temperature at the inlet to the heat exchanger, c_{pm} is the specific heat capacity of the working medium in the exchanger, L is the length of a single heat exchanger loop in the digester, d_3 is the outside diameter of the heat exchanger pipe, taking into account the thickness of the sludge, m is the mass flow of the heat exchanger medium, and U is the overall heat transfer coefficient, which is defined by the formula:

$$U = \frac{1}{\frac{1}{h_i} + \frac{d_1}{2k_w}\ln\frac{d_2}{d_1} + \frac{d_1}{2k_{sl}}\ln\frac{d_3}{d_2} + \frac{d_1}{d_3}\frac{1}{h_o}}, \quad \left[\frac{\mathrm{W}}{m^2\,{}^{o}C}\right] \qquad (2)$$

where d_1 is the inner diameter of the heat exchanger, d_2 is the outer diameter of the heat exchanger, k_w is the thermal conductivity of the steel wall of the heat exchanger, k_{sl} is the thermal conductivity of the sludge on the heat exchanger, and h_i and h_o are convective heat transfer coefficients for internal (3) and external (4) flows, respectively:

$$h_i = \frac{\mathrm{Nu}_{d_1} k_m}{d_1} \qquad (3)$$

$$h_o = \frac{\mathrm{Nu}_{d_3} k_s}{d_3} \qquad (4)$$

where k_m is the thermal conductivity of the working medium in the exchanger, k_s is the thermal conductivity of the substrate, Nu_{d1} and Nu_{d3} are Nusselt numbers for internal and external flows, respectively.
In the case of the internal flow, the Nusselt number Nu_{d1} can be determined from the Dittus-Boelter correlation:

$$\begin{aligned} &\mathrm{Nu}_{d_1} = 0.023\,\mathrm{Re}_{d_1}^{4/5}\,\mathrm{Pr}^{0.3}, \\ &\mathrm{Re}_{d_1} \geq 10000, \quad \mathrm{Re}_{d_1} = \frac{4m}{\pi d_1 \mu_m}, \\ &0.6 \leq \mathrm{Pr} \leq 1600, \quad \mathrm{Pr} = \frac{c_{pm}\mu_m}{k_m} \end{aligned} \qquad (5)$$

where Re_{d1} is the Reynolds number of the working medium flow in the heat exchanger tube, Pr is

the Prandtl number, and μ_m is the dynamic viscosity of the working medium in the exchanger.
In the case of the external flow, the Nusselt number Nu_{d3} can be derived from the Churchill-Bernstein correlation:

$$\mathrm{Nu}_{d_3} = 0.3 + \frac{0.62\,\mathrm{Re}_{d_3}^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(\frac{0.4}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_{d_3}}{282000}\right)^{5/8}\right]^{4/5}, \tag{6}$$

$$\mathrm{Re}_{d_3}\mathrm{Pr} \ge 0.2, \quad \mathrm{Re}_{d_3} = \frac{\rho_s \mathrm{v}_s d_3}{\mu_s}, \quad \mathrm{Pr} = \frac{c_{ps}\mu_s}{k_s}$$

where Re_{d3} is the Reynolds number for the substrate flow through the heat exchanger tubes, v_s is the substrate velocity in the fermentation chamber, ρ_s is the substrate density, μ_s is the dynamic viscosity of the substrate, and c_{ps} is the specific heat capacity of the substrate.

2.2 Method for determining the thickness of the sludge layer

Equation (1), crucial in the following considerations, allows us to directly calculate the temperature $T_{m,o}$ of the heat exchanger outlet, provided the other model variables and parameters, and especially d_3, are given. Here we invert this reasoning, assuming that $T_{m,o}$ is given while the outer diameter of the sludge layer has to be determined. This is a so-called inverse geometric problem (because the problem consists in searching for some missing geometric characteristics of the model).
From the mathematical point of view, one can treat the formula (1) with U defined by (2) as an implicit equation involving several variables. After some algebraic manipulations, it takes the form

$$\left(\frac{1}{h_i} + \frac{d_1}{2k_w}\ln\left(\frac{d_2}{d_1}\right) + \frac{d_1}{2k_{sl}}\ln\left(\frac{d_3}{d_2}\right) + \frac{d_1}{d_3 h_0}\right)\times \\ \times m c_{pm} \ln\left(\frac{T_{m,i}-T_a}{T_{m,i}-T_a}\right) - \pi L d_1 = 0 \tag{7}$$

It can be solved for d_3 if all the remaining variables are known from measurement or previous calculation and the solution will be unique. Having determined d_3, we easily find the thickness of the sludge layer, s, from the dependence $s=(d_3-d_2)/2$. Since (7) is a non-algebraic equation, only an approximate solution can be found. For the purpose of calculating d_3 from (7), one has to use a numerical procedure like MathCad's built-in function based on the Levenberg-Marquardt method and dedicated to root finding, which was successfully applied by the authors. Typically, such procedures require an initial guess for the solution. Since the sludge layer is by its nature relatively thin, d_2 must be close to the exact solution so it can stand for the first guess in the problem considered.
There appears another possibility to infer the sludge thickness from the experimental data available. Namely, one can make use of the total heat transfer rate defined by

$$q = m c_{pm}\left(T_{m,o} - T_{m,i}\right) \tag{8}$$

Inserting (1) into (8) and after appropriate rearrangement, equation (8) becomes

$$q = m c_{pm}\left(T_a - T_{m,o}\right)\left(\exp\left(\frac{\pi d_3 L}{m c_{pm}}U\right) - 1\right) \tag{9}$$

with U defined by (2). The solution procedure for determining d_3, similar to that based on equation (7), requires q (instead of $T_{m,i}$) to be known.

3 Results and discussion

This section compares the results of calculations with the actual parameters of the biogas plant operation and presents simulation results. Below is a comparison of the results of sludge thickness calculations on the exchanger walls and outlet temperature from the heat exchanger with the results measured during the biogas plant operation. Two variants of biogas plant operation were adopted: (i) after three years of biogas plant operation just before the general biogas plant maintenance and (ii) after general biogas plant maintenance and purification of the heat exchanger tubing from the accumulated sludge. The general maintenance of the biogas plant consists in discontinuing the operation of the biogas plant, then dismantling the pneumatic roof from the fermentation chamber and emptying the chamber of the substrate, and then on cleaning the biogas plant walls and the heat exchanger. An example picture from the cleaning of the digester is in the publication [19]. While cleaning the biogas plant, measurements of the sludge thickness were made on the heat exchanger tubing for the first variant. In the case of the second variant, the thickness of the sludge on the walls of the heat exchanger should be close to zero, due to the short period of operation of the biogas plant after general maintenance, which was three months. It should be noted here that the sludge thickness measurement on the walls of the heat exchanger is possible only when the digester is completely emptied of the sludge, which means that the biogas plant is shut down. Sample photos of the tested heat exchanger and sludge can be found in the publication [15].

3.1 Experimental data and calculation results

The calculations were performed for the following values of parameters: m=0.4472 kg/s, v_s=0.005 m/s, k_{sl}=0.5 W/m/K, d_1=0.0563 m, d_2=0.0603 m, L=94.25 m, c_{pm}=4180 J/kg/°C, μ_m=0.000509 kg/m/s, k_m=0.64 W/m/K, k_w=15 W/m/K, c_{ps}=4184 J/kg/°C, μ_s=0.03 kg/m/s, k_s=0.62 W/m/K. In the first variant the measured temperatures were $T_{m,i}$=65.5°C, T_a=36°C and in the second variant: $T_{m,i}$=54°C, T_a=41°C. A comparison of the results of calculations with the actual results is presented in Tab.1.

Table 1. Simulated and measured values of sludge thickness on the walls of the heat exchanger tubing and the temperature of return from the heat exchanger.

Sludge thickness, s		Return temperature, $T_{m,o}$	
measured	simulated	measured	simulated
Values after three years of operation of the digester			
0.012 m	0.01264 m	58 °C	57.78 °C
Values after thorough cleaning of the digester			
-	0.00031 m	46 °C	47.06 °C

In the first variant, the absolute error of the sludge thickness on the heat exchanger was 1mm, while in the case of the second variant the thickness of the sludge is close to zero, which proves the high accuracy of the model. It should be noted here that the thickness of the sludge and the conduction coefficient of the sludge depend mainly on the type of substrate working in the digester. In this case, the presented biogas plant consists of maize silage, chicken manure and potato pulp. At the temperature of the outlet from the heat exchanger, the absolute error was about 1°C.

Based on the reading of the temperature rise on the return from the heat exchanger, the thickness of the sludge layer on the walls of the heat exchanger can be read. Figure 3 presents the simulations of the sludge thickness, s, as a function of the return temperature from the heat exchanger at a constant supply temperature and for different values of the sludge heat transfer coefficient. The predictions were obtained for k_{sl}=0.3, 0.6 and 1.0 W/m/K, while the other parameters were the same as listed in this section, except for $T_{m,i}$=57°C and T_a=40°C.

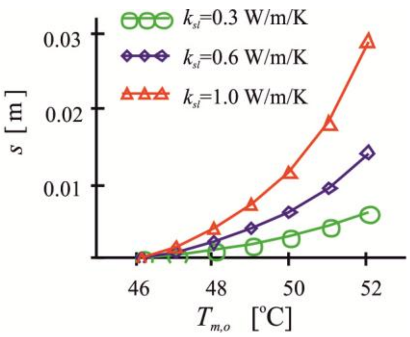

Fig. 3. Thickness of the sludge layer, $s=s(T_{m.o}, k_{sl})$, versus outlet temperature of the heating medium, at a constant supply temperature and for different heat conduction coefficients.

All the resulting graphs shown in Fig. 3 have one point in common, namely the point corresponding to the zero-thickness of the sludge layers, which occurs at about $T_{m,o}$=46.16 °C. The sludge on the exchanger walls is an additional thermal resistance, which results in a lower heat exchange and an increase in the return temperature from the heat exchanger [20-22]. The increase in the value of the heat transfer coefficient of the sludge, which depends on the type of substrate, causes an increase in heat exchange and a drop in the return temperature from the heat exchanger.

3.2 Uncertainty of the results

Whenever a considered variable or parameter depends on some other experimentally specified variables and/or parameters, there appears the question of uncertainty evaluation. In the present problem, we refer this analysis to determining d_3 from equation (7). Let F denote the left-hand side of (7). Formally, F is an implicit function of several variables. However, we decide to pick d_3 and view it as a function of the remaining variables. According to the rule of calculating absolute uncertainty with the use of the total differential, the uncertainty Δd_3 of d_3 can be written as

$$\Delta d_3 = \left[\left(\frac{\partial d_3}{\partial d_1}\Delta d_1\right)^2 + \left(\frac{\partial d_3}{\partial d_2}\Delta d_2\right)^2 + \left(\frac{\partial d_3}{\partial L}\Delta L\right)^2 + \left(\frac{\partial d_3}{\partial T_{m,i}}\Delta T_{m,i}\right)^2 + \left(\frac{\partial d_3}{\partial T_{m,o}}\Delta T_{m,o}\right)^2 + \left(\frac{\partial d_3}{\partial T_a}\Delta T_a\right)^2 + \left(\frac{\partial d_3}{\partial m}\Delta m\right)^2 + \left(\frac{\partial d_3}{\partial h_i}\Delta h_i\right)^2 + \left(\frac{\partial d_3}{\partial h_o}\Delta h_o\right)^2 + \left(\frac{\partial d_3}{\partial c_{pm}}\Delta c_{pm}\right)^2 + \left(\frac{\partial d_3}{\partial k_w}\Delta k_w\right)^2 + \left(\frac{\partial d_3}{\partial k_{sl}}\Delta k_{sl}\right)^2\right]^{\frac{1}{2}} \qquad (10)$$

In order to express the uncertainty defined by (10) in a ready-to-use form, we apply the implicit differentiation rule to calculate the first needed derivative:

$$\frac{\partial d_3}{\partial d_1} = -\frac{\dfrac{\partial F}{\partial d_1}}{\dfrac{\partial F}{\partial d_3}} \qquad (11)$$

and the rest of the derivatives are obtained using the same rule (and therefore not presented here).

As shown before, the theoretical results presented in Tab. 1 generally agree well with the measured values. In contrast to them, the predictions shown in Fig.3 could not be verified experimentally. Therefore, a proper uncertainty analysis is presented, using the following values: $\Delta d_1=\Delta d_2$=0.00005 m, ΔL=0.005 m, Δm=0.001 kg/s, Δc_{pm}=105 J/kg/°C, Δk_w=0.1 W/m/K, Δk_{sl}=0.05 W/m/K, $\Delta T_{m,i}=\Delta T_{m,o}=\Delta T_a$=0.25°C. Besides, it is assumed that the convective heat transfer coefficients h_i and h_o are known with errors up to 20%.

The results of calculations based on formula (10) are shown in Tab. 2. The percentage uncertainties corresponding to a very high drop of the temperature at the outlet from the heat exchanger turned out to be quite large, which should not be surprising as the exact value of the sludge thickness, s, is close to zero. In view of possible applications of the present research, special attention should be given to the case of a small drop of the temperature $T_{m,o}$, resulting from a thick layer of sludge accumulated on the walls of the digester.

The uncertainty estimates corresponding to this case remain at an acceptable level.

Table 2. Simulated values of the sludge thickness (s) and estimated uncertainties of the predictions (Δs) for k_{sl} = 0.3, 0.6 and 1.0 W/m/K

$T_{m,o}$ [°C]	k_{sl} [W/m/K]	s [m]	Δs [m]	$\Delta s/s$ [%]
47	0.3	0.000425	0.000276	64.9
	0.6	0.000894	0.000563	62.9
	1.0	0.001598	0.000974	60.9
48	0.3	0.001035	0.000298	28.7
	0.6	0.002199	0.000598	27.2
	1.0	0.003985	0.001046	26.3
49	0.3	0.001802	0.000339	18.8
	0.6	0.003876	0.000659	17.0
	1.0	0.007154	0.001167	16.3
50	0.3	0.002804	0.000412	14.7
	0.6	0.006129	0.000769	12.5
	1.0	0.011577	0.001384	12.0
51	0.3	0.004175	0.000537	12.9
	0.6	0.009328	0.000975	10.5
	1.0	0.01818	0.001814	10.0
52	0.3	0.006175	0.000755	12.2
	0.6	0.014232	0.001391	9.8
	1.0	0.029018	0.00277	9.5

4 Conclusions

The paper presents a numerical method of determining the thickness of sludge on the surface of a heat exchanger as a function of the fermentation chamber operation parameters. The sludge accumulated on the surface of the heat exchanger tubes significantly affects the thermal efficiency of the heat exchanger. The mathematical model of the problem is based on the equation relating the following variables to one another: inlet and outlet temperature (of the working medium), substrate temperature and the average overall heat transfer coefficient, the latter containing the thickness of sludge which we need to determine. In terms of computation, the problem is reduced to solving a non-algebraic equation with one unknown, which we perform with the use of a commercial computational program. The obtained numerical results were verified by comparison to the real experimental values and they turned out to be sufficiently accurate. Besides, based on the conducted numerical experiment, we present theoretical predictions of the thickness of the sludge layer, and these results agree with the physics of the considered problem. Moreover, the calculated uncertainties of the obtained results were at an acceptable level and therefore could be considered credible. The presented study shows an alternative proposal for determining the thickness of sludge making use of the total heat transfer rate of the heat exchanger. Finally, the developed method has some potential for supporting the work of a biogas plant.

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