

The second frequencies in the wake behind two circular cylinders

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Abstract. The wake behind the cylinders is often characterized by a dominant frequency and dimensionless Strouhal number. However, the experiments performed using the CTA and visualization method and their analysis revealed that other significant frequencies were present in the wake. With the Proper Orthogonal Decomposition analysis, connections between these frequencies and their own eigenmodes were found.

1 Introduction

A wake behind one or more circular cylinders in different configurations and settings is a topic in the field of fluid mechanics, which has been given a lot of attention for decades, yet it is still an open problem. One of the characteristics of the cylinder wake is a vortex shedding frequency. In the case of one cylinder, there is usually mentioned only one dominant frequency, respectively its dimensionless value relative to the size and velocity - the Strouhal number. Roshko [1] measured the frequency of the wake behind the cylinder by a hot wire for Reynolds numbers in the range of 40 to 10,000 for various cylinder diameters a curve of the dependence of Strouhal number on Reynolds number. Williamson in [2] writes about the basic knowledge of wake behind the cylinder. In his work divides wake modes based on the Reynolds number and the phenomena that take place in the wake.

In the case of two interacting circular cylinders in the flow field, there is often found two frequencies in the wake. Zdravkovich in [3] states that one frequency is related to the front cylinder and the other to the rear cylinder. Sumner et al. [4] refer the second frequency to shear layers of cylinders. Ishigai et al. [5] defines the area behind two cylinders in which frequencies are lower and higher. Nevertheless, the measurements made do not fully correspond to these theories.

Therefore, this study focuses on observing the occurrence of second frequencies in both the one-cylinder and the two-cylinder wakes in a staggered configuration. The detailed determination of frequency positions in the wakes by the CTA method and an analysis of visualisation data by POD method showed a possible connection between the second frequencies and the POD modes.

2 Experiment set-up and measurement method.

2.1 Set -up

Measurements were carried out both for the separate cylinder diameter $D = 10$ mm and for the configuration of two cylinders of $D / d = 2.5$ ($D = 10$ mm, $d = 4$ mm). Frequencies were obtained for Reynolds number 500. In the case of two cylinders, 7 mutual positions were measured, which were chosen based on the research by Hwang and Choi present in [6] and Strykowski and Sreenivasan in [7]. The primary objective of these measurements was to investigate the effect of the second cylinder on the occurrence of disturbances and instabilities in the wake of the first cylinder, as mentioned in [8], [9] and [10]. Their coordinates are given in Table 1, pitch ratios (R) and angle of rotation (α).

Table 1. Measured positions of the secondary cylinder.

Position No.	x/D	y/D	R [mm]	α [°]
1	4.3	-1.25	44.8	16
2	3.2	-0.8	33.0	14
3	2.4	-0.8	25.3	18
4	1.95	-0.8	21.1	22
5	2	-0.6	20.9	17
6	1.5	-0.35	15.4	13
7	1.5	0	15.0	0

The 2nd cylinder was always located behind the 1st cylinder. Figure 1 depicts the definitions of the coordinate system and shows positions of the secondary cylinder.

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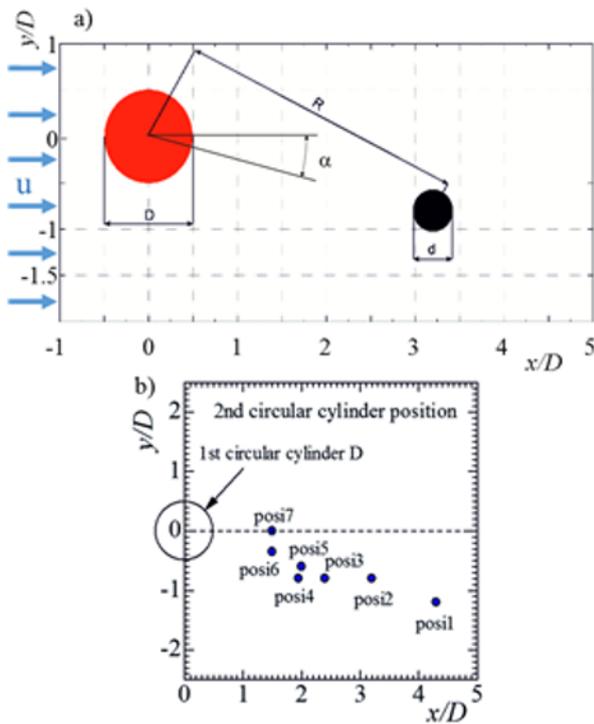


Fig. 1. Set-up of cylinders: a) coordinate system and notation, b) positions of cylinders

2.2 Measurement method

To investigate the frequency occurring in the wake of two methods were used: CTA (Constant Temperature Anemometry) and below (Proper Orthogonal Decomposition). Data obtained from visualization of wakes became the basis for the POD method.

The measurement by method CTA was carried out in an Eiffel type wind tunnel which has a speed range of approximately 0-20 m/s. The intensity of turbulence in the tunnel is around 0.3%. A test section of the tunnel is closed. The length of the test section is 750 mm and its cross section is 300 mm x 200 mm. The test section is equipped with a movable system which enables automatic 3D positioning of the probe. Tunnel control and control of the movable system are performed by software written in a LabView programming environment. The tunnel has an automatically driven moving and measuring system for hot wire anemometry MiniCTA 54T30 (Dantec Dynamics) with 55P16 HW-probe. A maximum solid blockage ratio of two cylinders was 8%, one cylinder of diameter $D = 10$ mm blocked 5%. In these cases, the hot wire probe was positioned in 120 to 140 positions in the cylinders wake in one plane.

The measured velocity fluctuations from the CTA methods were processed in the MATLAB programming environment. The resulting frequency spectrum was obtained using the pwelch function from the MATLAB library.

Visualization measurements were performed in a closed circuit water channel. For visualization of structures and flow around the cylinders, a flow visualization method was chosen a pouring streak method. An ink of different colour was fed into the surface of each

cylinder. It was dosed equally and in such a way that it did not break the boundary layer of the cylinder. A more detailed description, measurement parameters, and settings for this experimental device are given in [10]. For each recorded position and each Reynolds number, image data was obtained, which was then processed using the POD method. The sampling rate of the images was 29.97 fps. Sequences were extracted from each measurement from 800 to 1600 frames per data quality (record of approximately 27 to 54 seconds). These sequences were then processed in the MATLAB program environment, where they were converted to shades of grey.

3 Proper Orthogonal Decomposition (POD)

The POD method used to solve flow in the field of fluid mechanics makes it possible to obtain shapes of flow that correspond to the structures with the greatest energy contribution to the flow. These structures relate to events that make the most contribution to the current energy. [11]. Input data that are processed by the POD method can be a velocity field [12], a vorticity field [13], or a flow visualization [14]. Just the visualization data was used as a basis for the POD method. By using the POD analysis of flow field behind the circular cylinder, it can be obtained dominant frequency occurring in the wake behind the cylinder and also the structure which has the greatest influence on the formation of the wake.

To analyse data using POD snapshot method, it needs to define a fluctuation component as:

$$u_N(x, t) = \sum_{i=1}^N a_i(t) \varphi_i(x) \quad (1)$$

where $u(x, t)$ is the fluctuation component, $a_i(t)$ express the time-dependent POD coefficients, and $\varphi_i(x)$ is the independent ortho-normal functions of POD modes. In this case, a matrix of intensity levels of grey (8-bit images) was used as a fluctuation component. When searching for POD coefficients and modes by the snapshot method, the first step is for N snapshots (N matrix with flow fluctuation component $U = [u_1, u_2 \dots u_N]$) of the covariance matrix C , for which:

$$C = U^T U \quad (2)$$

To find eigenvalues and eigenvectors of matrix C is used the relationship

$$C \mathbf{V}^i = \lambda^i \mathbf{V}^i \quad (3)$$

where \mathbf{V}^i are eigenvectors and λ^i are eigenvalues.

Eigenvalues, sorted in descending order ($\lambda^1 > \lambda^2 > \lambda^3 \dots > \lambda^N$), represent the energy contained in the modes, which can be expressed according to the relation:

$$E_i = \frac{\lambda_i}{\sum_{i=1}^N \lambda^i} \quad (4).$$

It should be noted here that in the case of the use of data (images) that do not fluctuate in physical quantities

but only to change the gray shades (case of data from visualization measurements), the meaning of the values cannot be understood as energy but rather as the power of the given phenomenon in a flow.

4 Experimental results and discussions

The measured dominant frequencies by the CTA method and the calculated Strouhal numbers in the case of a single cylinder correspond to the values normally given for a single cylinder at $Re = 500$, namely $Sh \approx 0.21$. From the POD method, dominant frequencies were obtained, followed by Strouhal numbers, frequency analysis of time-dependent coefficients a_i .

The most powerful structures of the investigated flow are contained in the first two modes, therefore their relative power, calculated according to equation 4, contains a total of 27% of the total power. Therefore, it can be assumed that the dominant wake's frequencies can be found from the frequency analysis of the coefficients, even from these modes. This assumption is confirmed by the fact that Strouhal's number is 0.2 for the first two modes. In the other two modes 3 and 4, then $Sh = 0.41$. The Strouhal number corresponds to Strouhal numbers of the other distinct frequencies of the frequency spectra from measurements of the CTA, which were measured in the near wake and its surroundings.

Figure 2 shows the hot wire probe points and highlights those positions where two distinct frequencies were found. Figure 3 shows the frequency spectrum of one of these points.

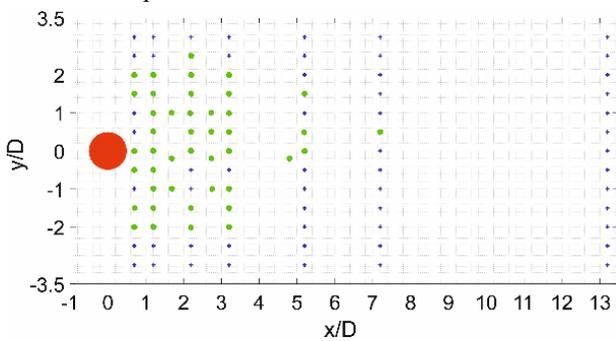


Fig. 2. Positions of hot wire probe behind the single circular cylinder, ● – positions with two strength frequencies, + – positions with only one or any frequencies.

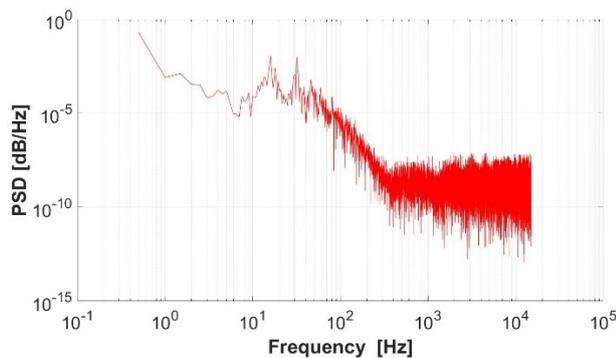


Fig. 3. Example of two strong frequencies behind the single circular cylinder.

Figure 4 shows the first (4a) and third (4b) POD modes. There are clearly visible structures that correspond to the most important events taking place in the wake of the cylinder. The authors Brevis and García-Villalba [14] state that the first two modes (which contain the same shapes but are phase shifted) represent the translational movement of dominant structures, and modes 3 and 4 are related to the transverse movement of structures. The dominant frequencies from this transverse motion correspond to the second frequencies found in the frequency spectra from CTA measurements.

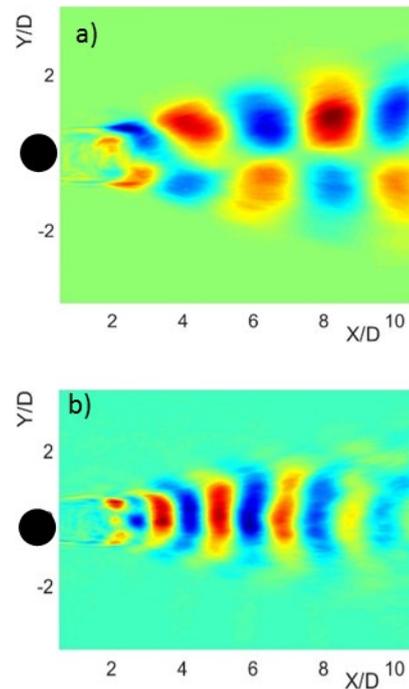


Fig. 4. a) First POD mode of the single circular cylinder, b) third POD mode of the single circular cylinder.

In the case of two cylinders in the staggered configuration, most authors present two significant frequencies. As has already been stated in the introduction, the reasons given are different. From the measurement results presented here, it is evident that the second (and other significant frequencies) are again present in the cylinder's near wakes, as in the case of a single cylinder. Figure 5 shows the hot wire probe points and highlighted positions where at least one additional significant frequency is present for all seven cylinder positions.

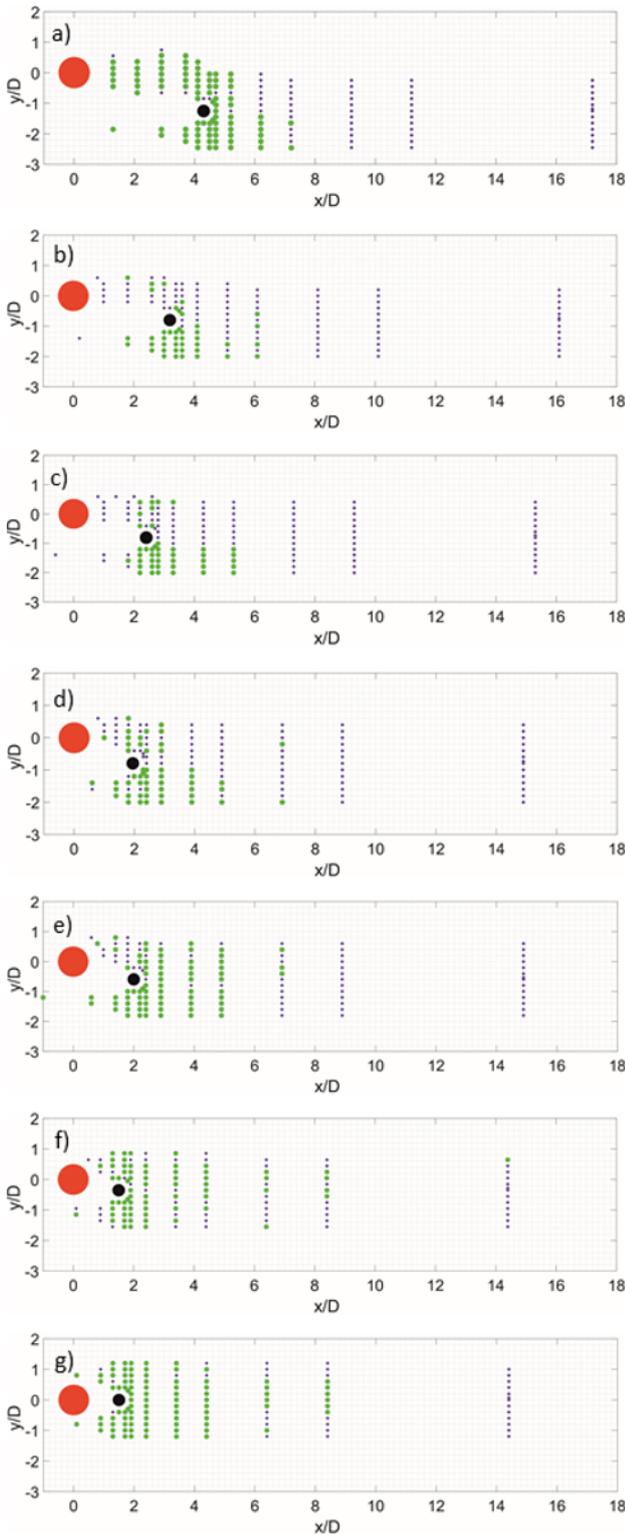


Fig. 5. Positions of hot wire probe behind the two circular cylinders, ● – positions with two strength frequencies, + - positions with only one or any frequencies, a) position 1, b) position 2, c) position 3, d) position 4, e) position 5, f) position 6, g) position 7

The figure shows that in the case of positions 1, 6 and 7 in the near wake of the front cylinder occurs substantially more locations with the detected significant frequencies other than the case, in particular, the positions 2 and 3 and partly 4 and 5. The detailed analysis of the frequency spectre showed that it is mainly at positions 2

and 3 that it is difficult to identify one strong dominant frequency in the near wakes of cylinders because disturbances and significant interactions between the shear layers of the two cylinders occur in this region. This is also apparent from the third (Figure 6) and fourth modes. Here the structures are ambiguous and Strouhal's numbers of frequency are considerably smaller than in the first and second modes.

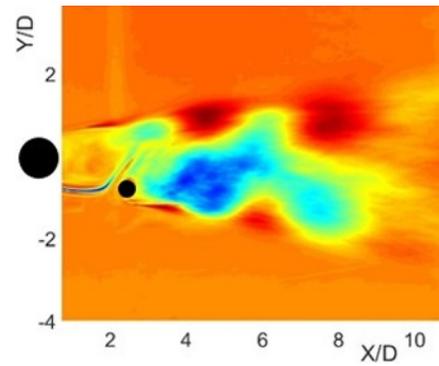


Fig. 6. The third POD mode of position 3

The more phenomena taking place in the near wake of the two cylinders is evidenced by the size of the relative power, which is usually in the 3rd and 4th mode significantly lower than in the first two modes, but in this case, it is only slightly lower. For position 2, the relative power of the first mode is 11.4%, the second mode is 10.7%, and the third mode is 7.5%. In position 3, the first mode reaches 8.1%, the other mode 7.7% and the third mode 7.3%. For this reason, it cannot be said that the wake for these configurations is characterized by one or two significant frequencies. In other positions, the 3rd and 4th mode contains frequencies corresponding to the second frequencies found in the frequency spectra from CTA measurements. Again, there are clearly recognizable shapes that belong to the transverse movement of coherent structures. Figure 7 shows an example for position 5.

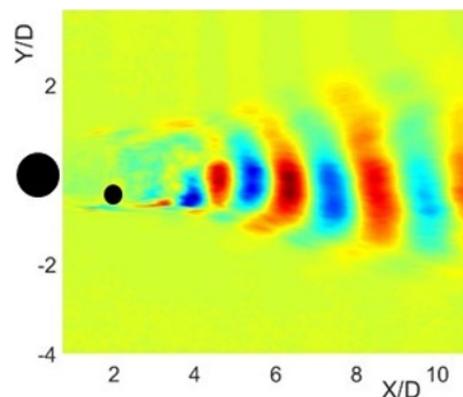


Fig. 7. The third POD mode of position 5.

5 Conclusion

It was found from the measurements by the method of CTA in a large network of measurement points that in wake both for single circular cylinder and two circular cylinders in the staggered configuration there are two significant frequencies in the near wake. The calculated Strouhal numbers for the measured frequencies were compared with the results of the POD analysis. Here, a possible association of second frequencies with the third and fourth POD modes can be found, which represent a transverse movement of coherent structures. At the same time, measurements and analyzes have shown that interaction of shear layers of cylinders and vortices formed by each of the cylinders also affects the occurrence of strong frequencies in the wake.

Although these contexts need to be further explored, the use of the POD method for the identification of phenomena in the flow field appears to be a very useful tool.

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