Nucleon form factors at threshold

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Abstract. The $p\bar{p}$ invariant mass spectra in the processes $e^+e^- \rightarrow p\bar{p}$, $e^+e^- \rightarrow n\bar{n}$, $J/\psi \rightarrow p\bar{p}\omega$, $J/\psi \rightarrow p\bar{p}\rho$, and $J/\psi \rightarrow p\bar{p}\gamma$ close to the $p\bar{p}$ is discussed. The optical potentials for $N\bar{N}$ pair in the superposition of S and D waves (due to tensor forces) at spin unity of the pair, as well as for $N\bar{N}$ pair in the S wave at spin zero of the pair, are proposed. The parameters of the potentials are obtained by fitting the cross sections of $N\bar{N}$ scattering together with the $p\bar{p}$ invariant mass spectra in $e^+e^- \rightarrow p\bar{p}$ annihilation and J/ψ decays. Good agreement with the available experimental data is achieved. Using our potential and the Green's function approach we also describe the cross section of $e^+e^- \rightarrow 6\pi$ and the $\eta'\pi^+\pi^-$ invariant mass spectrum in the decay $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$ in the energy region near the $N\bar{N}$ threshold.

1 Introduction

Investigation of the nucleon-antinucleon interaction in the low-energy region is an actual topic today. Unusual behavior of the cross sections of several processes has been discovered in recent years. For instance, the cross sections of the processes $e^+e^- \rightarrow p\bar{p}$ and $e^+e^- \rightarrow n\bar{n}$ reveal an enhancement near the threshold [1–4]. The enhancement near the $p\bar{p}$ threshold is also observed in the decays $J/\psi(\psi') \rightarrow p\bar{p}\pi^0(\eta)$ [5–7], $J/\psi(\psi') \rightarrow p\bar{p}\omega(\gamma)$ [5, 8–11]. The sharp peak in the vicinity of $N\bar{N}$ threshold has been observed in the cross sections of several processes, i.e., $e^+e^- \rightarrow 6\pi$ [12–16] and $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$ [17]. Strong enhancement of decay probability at low invariant mass of $p\bar{p}$ is observed in the processes $B^+ \rightarrow K^+p\bar{p}$ and $B^0 \rightarrow D^0p\bar{p}$, $B^+ \rightarrow \pi^+p\bar{p}$ and $B^+ \rightarrow K^0p\bar{p}$, $\Upsilon \rightarrow \gamma p\bar{p}$... These effect is similar to that in e^+e^- annihilation. One of the most natural explanation of this enhancement is final state interaction of nucleon and antinucleon. This interaction can be taken into account by means of an optical potential model. Several optical nucleon-antinucleon potentials [18–20] are usually used to describe the interaction in the low-energy region. All these nucleon-antinucleon potentials have been proposed to fit the nucleon-antinucleon scattering data. In our recent paper [21], to fit the parameters of the potential, we have suggested to include all available experimental data in addition to the nucleon-antinucleon scattering data. A simple potential model of $N\bar{N}$ interaction in the partial waves $^3S_1 - ^3D_1$, coupled by the tensor forces, has been suggested. The parameters of this model has been obtained by fitting simultaneously the nucleon-antinucleon scattering data, the cross sections of $p\bar{p}$ and $n\bar{n}$ production in e^+e^- annihilation, and the ratio of electromagnetic form factors of the proton in the timelike region. We have also constructed a simple optical potential model of

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the $N\bar{N}$ interaction in the 1S_0 partial wave [22]. This partial wave should give the most important contribution to the final-state $p\bar{p}$ interaction in the decays $J/\psi(\psi') \rightarrow p\bar{p}\omega(\rho, \gamma)$ in the energy region close to the $p\bar{p}$ threshold. In my report I discuss comparison of our results, obtained on the basis of our optical potentials, with the available experimental data.

2 Annihilation $e^+e^- \rightarrow N\bar{N}$ near the threshold

The Hamiltonian H^I of $N\bar{N}$ interaction for the isospin $I = 0, 1$, spin $S = 1$ of the produced pair, and the total angular momentum $J = 1$ is

$$H^I = \frac{p_r^2}{M} + V_S^I(r)\delta_{L0} + V_D^I(r)\delta_{L2} + V_T^I(r)S_{12}, \quad S_{12} = 6(\mathbf{S} \cdot \mathbf{n})^2 - 4,$$

where $\mathbf{S}$ is the spin operator, $L = 0, 2$ denotes the orbital angular momentum, and $\mathbf{n} = \mathbf{r}/r$. The radial wave functions $u_{nR}^I(r)$ and $w_{nR}^I(r)$, $n = 1, 2$, are the regular solutions of the equations

$$\frac{p_r^2}{M}\chi_n + \mathcal{V}\chi_n = 2E\chi_n, \quad \mathcal{V} = \begin{pmatrix} V_S^I & -2\sqrt{2}V_T^I \\ -2\sqrt{2}V_T^I & V_D^I - 2V_T^I + \frac{6}{Mr^2} \end{pmatrix}, \quad \chi_n = \begin{pmatrix} u_n^I \\ w_n^I \end{pmatrix}.$$

Here M is the proton mass, $E = k^2/(2M)$. Near the threshold, the amplitude $T_{\lambda\mu}^I$ of $e^+e^- \rightarrow N\bar{N}$ annihilation in the non-relativistic approximation (in units $4\pi\alpha/Q^2$, $Q = 2(M + E)$) for the certain isospin channel $I = 0, 1$ reads [23]:

$$T_{\lambda\mu}^I = G_s^I \left\{ \sqrt{2}u_{1R}^I(0)(\mathbf{e}_\mu \cdot \boldsymbol{\epsilon}_\lambda^*) + u_{2R}^I(0) \left[(\mathbf{e}_\mu \cdot \boldsymbol{\epsilon}_\lambda^*) - 3(\hat{\mathbf{k}} \cdot \mathbf{e}_\mu)(\hat{\mathbf{k}} \cdot \boldsymbol{\epsilon}_\lambda^*) \right] \right\},$$

where G_s^I is an energy-independent constant, $\mathbf{e}_\mu$ is a virtual photon polarization vector, corresponding to the projection of spin $J_z = \mu = \pm 1$, $\boldsymbol{\epsilon}_\lambda$ is the spin-1 function of $N\bar{N}$ pair, $\lambda = \pm 1, 0$ is the projection of spin on the nucleon momentum $\mathbf{k}$, and $\hat{\mathbf{k}} = \mathbf{k}/k$. In terms of the form factor G_s^I , the electromagnetic Sachs form factors have the form

$$G_M^I = G_s^I \left[u_{1R}^I(0) + \frac{1}{\sqrt{2}}u_{2R}^I(0) \right], \quad G_E^I = G_s^I \left[u_{1R}^I(0) - \sqrt{2}u_{2R}^I(0) \right], \quad \frac{G_E^I}{G_M^I} = \frac{u_{1R}^I(0) - \sqrt{2}u_{2R}^I(0)}{u_{1R}^I(0) + \frac{1}{\sqrt{2}}u_{2R}^I(0)}.$$

The ratio G_E^I/G_M^I is independent of G_s^I and not zero only because of d -wave ($u_{2R}^I(0) \neq 0$). Our predictions for the process $e^+e^- \rightarrow N\bar{N}$ near the threshold are shown in Fig. 1

3 Virtual $N\bar{N}$ pair production near the threshold.

The total cross section (elastic+inelastic) σ_{tot}^I can be written as [24]

$$\sigma_{\text{tot}}^I = -\frac{2\pi\alpha^2}{M^2Q^2} |G_s^I|^2 \text{Sp} [\text{Im}\mathcal{D}(0, 0|E)], \quad \left(\frac{p_r^2}{M} + \mathcal{V} - 2E \right) \mathcal{D}(r, r'|E) = -\frac{1}{rr'}\delta(r - r'),$$

where $\mathcal{D}(r, r'|E)$ is the Green's function. The solution of this equation has the form

$$\mathcal{D}(r, r'|E) = -Mk \sum_{n=1,2} [\vartheta(r' - r)\chi_{nR}(r)\chi_{nN}^T(r') + \vartheta(r - r')\chi_{nN}(r)\chi_{nR}^T(r')],$$

Figure 1. Left picture: the cross sections of $p\bar{p}$ (red line) and $n\bar{n}$ (green line) production; right picture: G_E^p/G_M^p for proton. The experimental data are from BaBar [2], CMD3 [4], and SND [3].

where χ^T denotes transposition of χ and $\chi_{nN}(r)$ is the non-regular solutions of the wave equation [21]. The inelastic cross section is the difference between the total cross section and elastic cross section:

$$\sigma_{\text{ann}}^I = \sigma_{\text{tot}}^I - \sigma^I, \quad \sigma^I = \frac{2\pi\beta\alpha^2}{Q^2} |G_s^I|^2 \left[|u_{1R}^I(0)|^2 + |u_{2R}^I(0)|^2 \right].$$

Data on $e^+e^- \rightarrow 4\pi$ do not demonstrate strong energy dependence in the vicinity of the $N\bar{N}$ threshold [25, 26]. Data on $e^+e^- \rightarrow 5\pi$ are not accurate enough for all channels [13]. The cross section of $e^+e^- \rightarrow 6\pi$ near the threshold in the energy region between 1.7 GeV and 2.1 GeV, see Fig. 2, is approximated by the formula

$$\sigma_{6\pi} = A\sigma_{\text{ann}}^1 + B \cdot E + C,$$

where the best coincidence is for $A = 0.56$, $B = 0.012$ nb/MeV, $C = 4.96$ nb. The coefficient A agrees with the data of $p\bar{p} \rightarrow \text{pions}$ annihilation at rest, where 6π give $\sim 55\%$ of $I = 1$ contribution [27].

Figure 2. Comparison of our predictions with the experimental data.

4 Decays $J/\psi \rightarrow p\bar{p}\pi^0(\eta)$, $J/\psi \rightarrow p\bar{p}\rho(\omega)$, $J/\psi, \psi(2S) \rightarrow p\bar{p}\gamma$, $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$

Notations: $\mathbf{k}$ is the momentum of meson (photon) in the J/ψ rest frame, $\mathbf{p}$ is the proton momentum in the $p\bar{p}$ center-of-mass frame, M is the invariant mass of the $p\bar{p}$ pair. Dominant contribution to the

amplitudes of $J/\psi \rightarrow p\bar{p}\pi^0(\eta)$ decays is given by the state of $p\bar{p}$ pair with $J^{PC} = 1^{--} (^3S_1)$. Then [22]

$$\frac{d\Gamma^I}{dM d\Omega_k} = \frac{\mathcal{G}_I^2 p k^3 \mathcal{F}}{2^9 \pi^4 m_{J/\psi}^4} [1 + \cos^2 \vartheta_k], \quad \mathcal{F} = |u_{1R}^I(0)|^2 + |u_{2R}^I(0)|^2.$$

where ϑ_k is the angle between $\mathbf{n}$ and $\mathbf{k}$, $\mathbf{n}$ is directed along the electron momentum in the beam. The angular part of this distribution does not depend on the features of the $p\bar{p}$ interaction. The distributions over the angle ϑ_p between $\mathbf{p}$ and $\mathbf{n}$ and over the angle ϑ_{pk} between $\mathbf{p}$ and $\mathbf{k}$ read:

$$\frac{d\Gamma^I}{dM d\Omega_p} = \frac{\mathcal{G}_I^2 p k^3 \mathcal{F}}{2^7 3\pi^4 m_{J/\psi}^4} [1 + \gamma^I P_2(\cos \vartheta_p)], \quad \frac{d\Gamma}{dM d\Omega_{pk}} = \frac{\mathcal{G}_I^2 p k^3 \mathcal{F}}{2^7 3\pi^4 m_{J/\psi}^4} [1 - 2\gamma^I P_2(\cos \vartheta_{pk})].$$

$$\gamma^I = \left[|u_{2R}^I(0)|^2 - 2\sqrt{2} \text{Re}[u_{1R}^I(0)u_{2R}^{I*}(0)] \right] / (4\mathcal{F}).$$

where $P_2(x) = \frac{3x^2-1}{2}$ is the Legendre polynomial. The invariant mass spectra of $J/\psi \rightarrow p\bar{p}\pi^0$ and $J/\psi \rightarrow p\bar{p}\eta$ decays are shown in Fig. 3. Dominant contribution to the amplitude of $J/\psi \rightarrow p\bar{p}\rho(\omega)$

Figure 3. Left picture: $J/\psi \rightarrow p\bar{p}\pi^0$ decay; right picture: $J/\psi \rightarrow p\bar{p}\eta$ decay. The red band corresponds to our previous parameters of the potential and the green band corresponds to the refitted model. The phase space behavior is shown by the dashed curve.

and $J/\psi, \psi(2S) \rightarrow p\bar{p}\gamma$ decays is given by the state of $p\bar{p}$ pair with $J^{PC} = 1^{-+} (^1S_0)$ [28]. Only one function $u_R^I(0)$ contributes:

$$\frac{d\Gamma^I}{dM d\Omega_p d\Omega_k} = \frac{\mathcal{G}_I^2 p k^3}{2^{10} \pi^5 m_{J/\psi}^4} |u_R^I(0)|^2 [1 + \cos^2 \vartheta_k].$$

The corresponding invariant mass spectra are shown in Figs. 4 and 5.

The $\eta'\pi^+\pi^-$ invariant mass spectrum in the decay $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$ decay near the $N\bar{N}$ threshold is given by the formulas

$$\frac{d\Gamma_{\text{tot}}^I}{dM} = -\frac{\mathcal{G}_I^2 k^3}{2^4 3\pi^3 m_p m_{J/\psi}^4} \text{Im}\mathcal{D}^I(0, 0|E), \quad d\Gamma_{\text{inel}}^I/dM = d\Gamma_{\text{tot}}^I/dM - d\Gamma_{N\bar{N}}^I/dM,$$

We approximate it as

$$d\Gamma_{\gamma\eta'\pi^+\pi^-}/dM = A d\Gamma_{\text{inel}}^0/dM + B \cdot E + C,$$

Figure 4. Left picture: $J/\psi \rightarrow p\bar{p}\omega$ decay; right picture: $J/\psi \rightarrow p\bar{p}\rho$ decay. The phase space behavior is shown by the dashed curve.

Figure 5. Left picture: $J/\psi \rightarrow p\bar{p}\gamma$ decay; right picture: $\psi(2S) \rightarrow p\bar{p}\gamma$ decay. The phase space behavior is shown by the dashed curve.

where A , B and C are some fitting parameters. The coefficient $A \approx 4 \cdot 10^{-3}$ is the estimation of the branching ratio of the decay $p\bar{p} \rightarrow \eta'\pi^+\pi^-$ at rest. This value is very close to the branching ratio $3.46 \cdot 10^{-3}$ measured in the experiment. Comparison with the experiment is shown in Fig. 6.

Conclusion

- Unusual phenomena are related to interaction at large distances ("nuclear physics" of elementary particles).
- The results of SND and CMD-3 obtained at e^+e^- collider VEPP-2000 give an important contribution to our understanding of these phenomena.
- Using the data on $e^+e^- \rightarrow N\bar{N}$ annihilation and the data on $N\bar{N}$ scattering, we describe simultaneously: the cross section of $e^+e^- \rightarrow \text{mesons}$, the decays $J/\psi \rightarrow p\bar{p}\omega$, $J/\psi \rightarrow p\bar{p}\gamma$, $\psi(2S) \rightarrow p\bar{p}\gamma$, $J/\psi \rightarrow p\bar{p}\pi^0$, $J/\psi \rightarrow p\bar{p}\eta$ and $J/\psi \rightarrow \gamma N\bar{N} \rightarrow \gamma\eta'\pi^+\pi^-$ with good precision. We have obtained the predictions for the decay $J/\psi \rightarrow p\bar{p}\rho$ which has not been measured yet.

Figure 6. The $\eta'\pi^+\pi^-$ invariant mass spectrum in the decay $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$. The thin line shows the contribution of non- $N\bar{N}$ channels. Vertical dashed line is the $N\bar{N}$ threshold.

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