

Radiative corrections to elastic-electron proton scattering and uncertainty in proton charge radius

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Abstract. Higher-order QED radiative corrections to elastic electron-proton scattering are discussed. It is shown that they are relevant for high-precision experiments on proton form factor measurements. Analytic results are obtained for next-to-leading second order corrections to the electron line. Light pair corrections are taken into account. The role of the hadronic contribution to vacuum polarization is discussed. Numerical results are given for the conditions of the experiment on proton form factors performed by A1 Collaboration. Preliminary results are also shown for the set-up with reconstruction of momentum transfer from the recoil proton momentum.

1 Introduction

The recent progress in high-precision measurements of proton electromagnetic form factors and, in particular, of the proton charge mean square radius revealed serious difficulties in our studies of this fundamental particle. The very precise result on the proton radius from the muonic hydrogen spectrum [1] appeared to be more than five sigma away from averaged value obtained from the ordinary hydrogen spectrum and from experiments on elastic electron-proton (ep) scattering. The difference is known as the *proton radius puzzle*. Certainly to resolve the problems we have to re-consider all possible effects starting from experimental uncertainties up to new physics contributions. Scrutinizing of the theoretical description of the hydrogen spectra and ep scattering is a part of the list.

In the present report we revisit higher-order effects in elastic electron-proton scattering. As the reference point we use the recent very accurate experiment on the ep elastic scattering performed at the Mainz Microtron (MAMI) [2]. The average point-to-point error in the cross sections measurement was of the order of 3 permille.

Higher order corrections to elastic and inelastic ep scattering were discussed in the literature for a long time. Here we are going to discuss application of the existing theoretical results to concrete experimental set-ups. The high precision of the experimental measurement of the differential ep cross section provides the clear requirement on the magnitude of effects which should be taken into account. We assume that aiming at the one-permille accuracy, we have to treat systematically all relative corrections being at least of the 10^{-4} size.

First, we will consider the conditions of the MAMI experiment. The initial electron energy E is of the order of 1 GeV, $E \gg m_e$. The initial proton is at rest. The momentum transfer squared will be

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considered in the range $0.003 < Q^2 < 1 \text{ GeV}^2$ which was explored in the experiment. Note that the condition $Q^2 \gg m_e^2$ holds for the whole range.

One-loop QED corrections to the process under consideration are well known [3]. They are naturally separated into: i) real and virtual corrections to the electron line; ii) real and virtual corrections to the proton line; iii) interference of amplitudes of the first two types; iv) the effect due to vacuum polarization. Among one-loop corrections, there is still an open discussion about the proper treatment of double photon exchange contributions, see *e.g.* papers [4, 5] and references therein. We agree with the importance of this contribution especially for the region of relatively high momentum transfer. But most of the estimates do not provide any large effect for the proton charge radius problem.

The numerical effect of radiative corrections depends a lot on the experimental conditions of particle registration and kinematics reconstruction. To get the final answer one should include the corrections into the whole procedure of the data analysis. Our task here will be just to present analytic results with simple estimates of their impact. Numerical results below will be presented mostly for the following set-up:

- kinematics is reconstructed from the detected final electron energy and momentum,
- the “bare” event selection, *i.e.* electron is detected without possible accompanying photons,
- the only cut on the lost energy is applied: $p_1^0 - p_2^0 \geq \Delta E$, $\Delta \ll 1$ and $\Delta E \gg m_e$. In Sect. 4 we will also show preliminary estimates for a set-up with the recoil proton momentum measurement.

The magnitude of the relative $O(\alpha)$ corrections is defined by three major factors:

$$\delta^{(1)} = \frac{d\sigma^{(1)}/dQ^2}{d\sigma^{(0)}/dQ^2} \sim \frac{\alpha}{2\pi} \cdot \ln\left(\frac{Q^2}{m_e^2}\right) \cdot \ln \Delta, \quad (1)$$

where $d\sigma^{(0)}/dQ^2$ is the differential Born cross section and $d\sigma^{(1)}/dQ^2$ is the one-loop corrected cross section. The enhancement by the so-called large logarithm $L \equiv \ln(Q^2/m_e^2)$ and by the logarithm of the cut-off parameter make the size of the one-loop correction to be of the order of a few percent. Since the experimental uncertainties are at least one order less, the one-loop corrections were treated in the data analysis with care, see details in [2].

Below we will present estimations of the leading and next-to-leading higher order corrections. The most important higher order effects are: i) higher order vacuum polarization; ii) cut-off dependence for multiple photon radiation; iii) leading logarithmic corrections in $O(\alpha^n L^n)$, $n \geq 2$; iv) light pair corrections; v) next-to-leading $O(\alpha^2 L^1)$ corrections to the lepton line. As can be seen from the first order, higher order corrections only to the electron line and due to vacuum polarization are numerically relevant.

2 Vacuum polarization

Running of the QED coupling constant can be naturally represented as

$$\alpha(Q^2) = \frac{\alpha(0)}{1 - \Pi(Q^2)}, \quad \Pi(Q^2) = \Pi_e(Q^2) + \Pi_\mu(Q^2) + \Pi_{\text{hadr}}(Q^2) + \dots \quad (2)$$

where $\alpha(0) \equiv \alpha \approx 1/137.036$. A discussion of the relative size of different contributions to $\Pi(Q^2)$ for low Q^2 values can be found in Ref. [6]. The magnitude of $\Pi(Q^2)$ for the range of momentum transfer under consideration is about 0.01. The bulk of the vacuum polarization effect comes from one-loop e^+e^- pair insertion into the photon propagator,

$$\Pi_e(Q^2) = \frac{\alpha(0)}{\pi} \left(\frac{1}{3}L - \frac{5}{9} \right) + \left(\frac{\alpha(0)}{\pi} \right)^2 \left(\frac{1}{4}L + \zeta(3) - \frac{5}{24} \right) + O(\alpha^3). \quad (3)$$

One can note that the $O(\alpha^2)$ contribution is of the next-to-leading order, since it contains only the first power of the large logarithm L . So it makes only a $\sim 10^{-5}$ effect well below the precision tag. The resummation of the vacuum polarization effect gives

$$\delta\sigma_{\text{vac.pol.}} = \sigma^{(0)} \left(\frac{\alpha(Q^2)}{\alpha(0)} \right)^2 = \frac{\sigma^{(0)}}{|1 - \Pi(Q^2)|^2}. \quad (4)$$

Polarization of vacuum by virtual $\mu^+\mu^-$ pairs is not as large as by the e^+e^- ones. But in the bulk of the kinematical domain the suppression is only logarithmic. So, it has to be taken into account at least in the first order in α . For $Q^2 = 1 \text{ GeV}$ it reaches $2 \cdot 10^{-3}$.

Instead of the resummed geometrical series of Eq. 4, the A1 collaboration [2] used exponentiation of the effect of the vacuum polarization by leptons, which gives a small but visible systematic shift from the true value.

The hadronic contribution $\Pi_{\text{hadr}}(Q^2)$ in the running QED coupling constant is rather small at $Q^2 \leq 1 \text{ GeV}$, but at the right edge it is rising steeply and reaches a few permille. Contributions of tau leptons, top quarks and electroweak bosons are obviously numerically negligible in our case. Numerical results of the vacuum polarization contributions are presented in Fig. 1. This figure was obtained with the help of the Fortran package `alphaQED` by F. Jegerlehner [7].

As concerning the hadronic contribution to vacuum polarization, it can be either treated as a part of radiative corrections or as a part of the proton form factor. To our mind, the former treatment has advantages. First, this contribution is always there as for point-like as well as for non-point-like particles. Second, in higher orders it is not factorized out as can be seen already in Eq. (4). Moreover the hadronic contribution should not affect the value of the proton charge radius since it is defined at the zero momentum transfer, where this effect is vanishing. Nevertheless, one can see that the hadronic contribution is numerically important in the given region of momentum transfer. It contributes considerably both to the change of the normalization of the cross section and of its slope in the experimentally accessed domain.

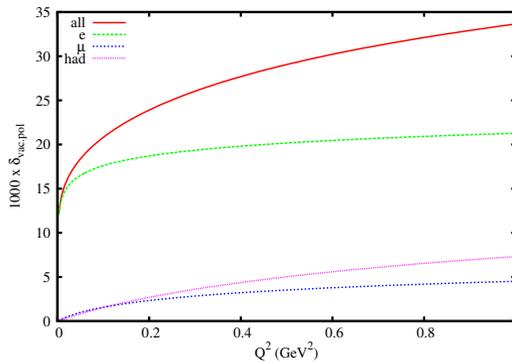


Figure 1. Vacuum polarization corrections due to electrons (e), muons (μ), hadrons (had), and the combined resummed effect (all).

3 Higher-order corrections to electron line

The Yennie-Frautschi-Suura theorem [8] claims that multiple emission of soft photons can be resummed into an exponent. By construction in the case of independent emission of soft photons, the

maximal energy of each photon is limited independently. But in the given experimental set-up, we have a cut-off on the total lost energy. The corresponding effect was discussed *e.g.* in [9]. For double soft photon emission in gives the following shift:

$$e^{\delta_{\text{soft}}} \rightarrow e^{\delta_{\text{soft}}} - \left(\frac{\alpha}{\pi}\right)^2 \frac{\pi^2}{3} (L-1)^2. \quad (5)$$

At $Q^2 = 1 \text{ GeV}^2$ this leads to a visible relative change of the cross section of about $-3.5 \cdot 10^{-3}$ which is comparable with the experimental precision. It is worth to note that this shift doesn't depend on the cut-off value, but depends on Q^2 .

To have the theoretical precision under control we can estimate the effect also for the leading logarithmic photonic correction in higher orders, see details in [10]. The proper exponentiation of radiative corrections in the leading logarithmic approximation is based on the exact solution of QED renormalization group equations, see [11].

The contribution of (virtual and real) e^+e^- pairs can be easily estimated within the QED leading logarithmic approximation (LLA) [11–13]:

$$\delta_{\text{pair}}^{\text{LLA}} = \frac{2}{3} \left(\frac{\alpha}{2\pi} L\right)^2 P_{\Delta}^{(0)} + \frac{4}{3} \left(\frac{\alpha}{2\pi} L\right)^3 \left\{ (P^{(0)} \otimes P^{(0)})_{\Delta} + \frac{2}{9} P_{\Delta}^{(0)} \right\} + O(\alpha^2 L, \alpha^4 L^4) \quad (6)$$

The so-called Δ -parts of splitting functions can be found in [12, 13]. Note that only non-singlet pair corrections are relevant in the given kinematics.

In order to control the precision of theoretical estimates we can compute the complete set of next-to-leading order corrections to the given process by means of the renormalization group approach to QED [11]. The NLO QED structure functions were first introduced in [14]. The corresponding fragmentation functions were used first in [15, 16]. For the kinematics with a cut on the lost energy we can follow the approach developed in [17]. A compact analytic formula for $O(\alpha^2 L)$ correction to ep scattering was presented in [10].

Relative higher order QED corrections to the electron line

$$\delta_i = \frac{d\sigma^{(i)}/dQ^2}{d\sigma^{(0)}/dQ^2}$$

are shown in Fig. 2 for the beam energy $E = 800 \text{ MeV}$ and the maximal lost energy $\Delta E = 10 \text{ MeV}$. Index i runs over: i) “2,LLA”, *i.e.* pure photonic $O(\alpha^2 L^2)$ corrections; ii) “2,NLA”, *i.e.* the sum of pure photonic $O(\alpha^2 L^2)$ and $mathcal{O}(\alpha^2 L^1)$ corrections; iii) “pair”, *i.e.* the leading and next-to-leading pair corrections; iv) “diff.”, *i.e.* the shift from the exponentiated one-loop result:

$$\delta_{\text{diff.}} = \frac{d\sigma^{\text{NLO}}}{d\sigma^{(0)}} + \delta_{\text{LLA}}^{(3)} + \delta_{\text{LLA,pair}}^{(3)} + \delta_{\text{LLA}}^{(4)} - \left[\exp\{\delta^{(1)}\} - 1 - \delta^{(1)} \right]. \quad (7)$$

One can see that the difference between the exponentiated one-loop result and the more advanced treatment of higher order corrections is several times more than the claimed experimental precision. Such a large the systematic effect should be taken into account in future experiments. An adequate treatment of all other relevant effects (hadronic vacuum polarization, double photon exchange, radiative corrections to the proton line, details of the experimental set-up, *etc.*) is also required.

4 Recoil Proton Set-Up

A new experiment on electron-proton scattering is proposed at MAMI in Mainz. The idea is to reconstruct the kinematics from measurement of the recoil proton momentum. This will allow to

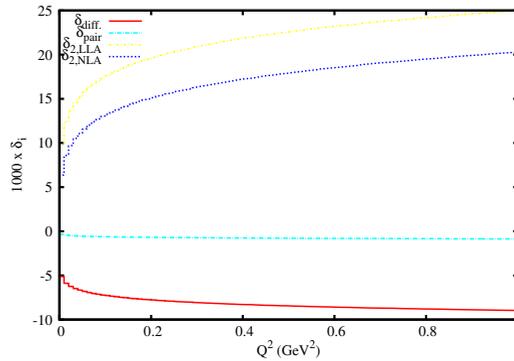


Figure 2. Relative higher order QED corrections to electron line in ep scattering cross section vs momentum transferred squared.

Table 1. Higher order leading logarithmic corrections for the proton measurement set-up.

Q^2 [GeV]	0.001	0.01	0.02
δ_1^{LLA}	$-2.7 \cdot 10^{-4}$	$-1.2 \cdot 10^{-3}$	$-2.0 \cdot 10^{-3}$
δ_2^{LLA}	$-0.7 \cdot 10^{-5}$	$-3.0 \cdot 10^{-5}$	$-4.6 \cdot 10^{-5}$
$\delta_{2+3+pairs}^{LLA}$	$-0.8 \cdot 10^{-5}$	$-3.5 \cdot 10^{-5}$	$-5.4 \cdot 10^{-5}$

access the extremely low range of momentum transfer $0.001 < Q^2 < 0.02 \text{ GeV}^2$. It is also very important that both experimental and theoretical uncertainties in extraction of the charge proton radius from this measurement will be completely different from the ones in the conventional ep scattering with observation of the final electron’s energy and momentum. We stress that effects of higher order corrections do depend very much on experimental set-up. In Table 1 we give preliminary numerical estimates for the relative corrections within the leading logarithmic approximation in the n^{th} order

$$\delta_n(Q^2) = \frac{d\sigma_n^{LLA}(Q^2)/dQ^2}{d\sigma^{\text{Born}}(Q^2)/dQ^2} \quad (8)$$

One can see that they are much less than the ones for the conventional set-up. This happens because the leading logarithmic corrections affect mainly the electron kinematics. In particular, the Kinoshita-Lee-Nauenberg theorem provides the complete cancellation of the large log contributions for the final state radiation.

5 Outlook

In conclusion we would like to stress the importance of accurate treatment of radiative corrections in modern high-precision experiments. As one can see, the naïve exponentiation of one-loop corrections and missing of hadronic vacuum polarization led to a very large systematic error in the result of the MAMI experiment [2]. Nevertheless, only after taking into account of the missed effects within the data analysis procedure one would conclude on the resulting shift in the proton charge radius value. At the present moment, one can say only that the value will be reduced just because the slope of the Q^2 distribution of nett missed correction is positive.

The most adequate way to treat radiative corrections in a modern experiment in particle physics is to perform Monte Carlo simulations of the process under consideration (and of backgrounds to it). An advanced Monte Carlo code ESEPP for simulation of elastic electron-proton scattering process was presented in [18]. The code contains an extended library of proton form factors, vacuum polarization contributions, the complete one-loop QED corrections, the dependence on m_e^2/Q^2 ratio (not complete), a parameterization of the double photon exchange. Our plans are to include the higher order QED corrections described above into the ESEPP program. Further tests and tuned comparisons will be performed. In particular new estimates of theoretical uncertainties should be done different experimental conditions of ep scattering studies.

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