

Heavy-quark spin-symmetry partners of $Z_b(10610)$ and $Z_b(10650)$ molecules

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Abstract. Heavy-quark spin-symmetry (HQSS) partners of the isovector bottomonium-like states $Z_b(10610)$ and $Z_b(10650)$ are predicted within the molecular picture. Treating both Z_b 's as shallow bound states, we solve the system of coupled-channel integral equations for the contact plus one-pion exchange (OPE) potentials to predict the location of the partner states with the quantum numbers J^{++} ($J = 0, 1, 2$). In particular, we predict the existence of a narrow tensor 2^{++} state residing a few MeV below the $B^* \bar{B}^*$ threshold. It is emphasised that the tensor part of the OPE potential in combination with HQSS breaking due to the nonvanishing B^*-B mass splitting has a significant impact on the location of this partner state.

1 Introduction

The experimental discovery and further studies by the Belle collaboration of the bottomonium-like $J^{PC} = 1^{+-}$ resonances $Z_b(10610)$ and $Z_b(10650)$ [1–3] revealed very peculiar properties of these states. Indeed, they reside in the vicinity of the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds, respectively, and couple to the corresponding hadronic channels in S waves. Moreover, both Z_b resonances decay predominantly to these open-flavour channels. Such properties indicate that a molecular component in the wave function of these states should be significant [4], see also Ref. [5] for a review. It should be stressed also that, unlike the isoscalar charmonium-like molecular candidate, the $X(3872)$, both Z_b states are isovectors, that precludes these molecules from mixing with conventional $\bar{b}b$ quarkonia.

As follows from the heavy-quark spin symmetry (HQSS), in the limit when the masses of heavy quarks become infinitely large, the Z_b states should have spin partners. First predictions for the partner states were made in Ref. [4] employing the decomposition of the $B^{(*)}\bar{B}^{(*)}$ wave functions in terms of

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the light and heavy-quark degrees of freedom. Similar results were deduced in the effective field theory (EFT) approach [6] under the assumption that the $B^{(*)}\bar{B}^{(*)}$ molecules are formed via iterations of contact S-wave interactions.

In a recent work [7], we investigated how these predictions changed, if the HQSS violating corrections driven by the B^*-B mass splitting were included (for a similar study of the spin partners of the X(3872) see Ref. [8]). It was emphasised that the main source of the HQSS violating corrections originated from the one-pion exchange (OPE) potential, iterations of which led to a significant shift of the resonance poles to the complex plane. In this contribution, we review the main findings of Ref. [7].

2 Description of the approach

In a recent series of works [7–11] we proposed a chiral EFT based approach for studying long-range light-quark dynamics of various molecular candidates. The approach benefits from an explicit inclusion of the relevant momentum scales related with the binding energies of the states, SU(3) Goldstone boson exchanges, heavy meson-antimeson coupled-channel dynamics, three-body effects as well as HQSS breaking via the heavy-meson hyperfine mass splitting. The implications of this approach to the spin partners of the bottomonium-like states $Z_b(10610)$ and $Z_b(10650)$ treated as loosely bound states of $B\bar{B}^*$ and $B^*\bar{B}^*$ mesons, respectively, are investigated in Ref. [7] and briefly discussed in this contribution.

The information about poles of the states can be directly inferred from the scattering amplitudes which are extracted from the nonperturbative solutions of the coupled-channels integral equations of the Lippmann-Schwinger type,

$$a_{ij}^{(JPC)}(p, p') = V_{ij}^{(JPC)}(p, p') - \sum_n \int dk k^2 V_{in}^{(JPC)}(p, k) G_n(k) a_{nj}^{(JPC)}(k, p'), \quad (1)$$

where

$$G_n = \left(k^2 / (2\mu_n) + m_{1,n} + m_{2,n} - \sqrt{s} - i\epsilon \right)^{-1}, \quad \mu_n = \frac{m_{1,n}m_{2,n}}{m_{1,n} + m_{2,n}} \quad (2)$$

are the propagator and the reduced mass of the heavy meson-antimeson pair in the given channel evaluated in its centre of mass. Here, the indices i, j , and n label the basis vectors, which enumerate the heavy meson-antimeson states corresponding to various particle channels in various partial waves for the given quantum numbers J^{PC} , namely

$$\begin{aligned} 0^{++} : & \quad \{B\bar{B}(^1S_0), B^*\bar{B}^*(^1S_0), B^*\bar{B}^*(^5D_0)\}, \\ 1^{+-} : & \quad \{B\bar{B}(^3S_1, -), B\bar{B}^*(^3D_1, -), B^*\bar{B}^*(^3S_1), B^*\bar{B}^*(^3D_1)\}, \\ 1^{++} : & \quad \{B\bar{B}(^3S_1, +), B\bar{B}^*(^3D_1, +), B^*\bar{B}^*(^5D_1)\}, \\ 2^{++} : & \quad \{B\bar{B}(^1D_2), B\bar{B}^*(^3D_2), B^*\bar{B}^*(^5S_2), B^*\bar{B}^*(^1D_2), B^*\bar{B}^*(^5D_2), B^*\bar{B}^*(^5G_2)\}. \end{aligned} \quad (3)$$

The C-parity of the states is indicated explicitly in parentheses in Eq. (3) whenever necessary. The potential $V_{ij}^{(JPC)}(p, p')$ includes the contact term as well as SU(3) Goldstone boson exchange interactions in the channel with the given quantum numbers J^{PC} and in the given partial wave.

At leading order the contact terms contribute to S waves only and they are related by the heavy-quark spin symmetry [12, 13], that gives

$$V_{\text{LO}}^{(0^{++})} = \frac{1}{4} \begin{pmatrix} 3C_1 + C'_1 & -\sqrt{3}(C_1 - C'_1) \\ -\sqrt{3}(C_1 - C'_1) & C_1 + 3C'_1 \end{pmatrix}, \quad V_{\text{LO}}^{(1^{+-})} = \frac{1}{2} \begin{pmatrix} C_1 + C'_1 & C_1 - C'_1 \\ C_1 - C'_1 & C_1 + C'_1 \end{pmatrix}, \quad (4)$$

$$V_{\text{LO}}^{(1^{++})} = V_{\text{LO}}^{(2^{++})} = C_1, \quad (5)$$

where $\{C_1, C'_1\}$ are the low-energy constants (contact terms) and the other (vanishing) non S-wave transitions were omitted. Then, the OPE potentials (before partial-wave projection) connecting the isovector heavy-meson $B^{(*)}B^{(*)}$ pairs in the initial and final state read

$$\begin{aligned}
 V_{B\bar{B}^* \rightarrow B^* \bar{B}}(\mathbf{p}, \mathbf{p}') &= \frac{2g_b^2}{(4\pi f_\pi)^2} (\boldsymbol{\epsilon}_1 \cdot \mathbf{q})(\boldsymbol{\epsilon}_2^* \cdot \mathbf{q}) \left(\frac{1}{D_{BB\pi}(\mathbf{p}, \mathbf{p}')} + \frac{1}{D_{B^*B^*\pi}(\mathbf{p}, \mathbf{p}')} \right), \\
 V_{B\bar{B}^* \rightarrow B^* \bar{B}^*}(\mathbf{p}, \mathbf{p}') &= -\frac{2\sqrt{2}g_b^2}{(4\pi f_\pi)^2} (\mathbf{A}_1 \cdot \mathbf{q})(\boldsymbol{\epsilon}_2^* \cdot \mathbf{q}) \left(\frac{1}{D_{BB^*\pi}(\mathbf{p}, \mathbf{p}')} + \frac{1}{D_{B^*B^*\pi}(\mathbf{p}, \mathbf{p}')} \right), \\
 V_{B\bar{B} \rightarrow B^* \bar{B}^*}(\mathbf{p}, \mathbf{p}') &= \frac{2g_b^2}{(4\pi f_\pi)^2} (\boldsymbol{\epsilon}_1^* \cdot \mathbf{q})(\boldsymbol{\epsilon}_2^* \cdot \mathbf{q}) \left(\frac{2}{D_{BB^*\pi}(\mathbf{p}, \mathbf{p}')} \right), \\
 V_{B^* \bar{B}^* \rightarrow B^* \bar{B}^*}(\mathbf{p}, \mathbf{p}') &= \frac{4g_b^2}{(4\pi f_\pi)^2} (\mathbf{A}_1 \cdot \mathbf{q})(\mathbf{A}_2 \cdot \mathbf{q}) \left(\frac{2}{D_{B^*B^*\pi}(\mathbf{p}, \mathbf{p}')} \right),
 \end{aligned} \tag{6}$$

where \mathbf{p} (\mathbf{p}') denotes the centre-of-mass momentum of the initial (final) heavy-meson pair, $\mathbf{q} = \mathbf{p} + \mathbf{p}'$ is the pion momentum, $\mathbf{A} = \frac{i}{\sqrt{2}}(\boldsymbol{\epsilon} \times \boldsymbol{\epsilon}^*)$ with $\boldsymbol{\epsilon}$ and $\boldsymbol{\epsilon}^*$ standing for the polarisation vectors of the initial and final B^* mesons, respectively, and $f_\pi = 92.2$ MeV is the pion decay constant. Furthermore, the pion coupling to the B mesons, g_b , was extracted with the help of heavy-flavour symmetry from the observable $D^* \rightarrow D\pi$ decay width, and it agrees within 10% with the recent lattice QCD determination of the $B^*\pi$ coupling constant [14]. Therefore, the OPE potential does not bring any additional free parameter.

In the nonrelativistic limit for the B and B^* mesons, the time-ordered three-body propagators take the form

$$D_{H_1 H_2 \pi}(\mathbf{p}, \mathbf{p}') = 2E_\pi(\mathbf{q}) \left(m_1 + m_2 + \frac{p^2}{2m_1} + \frac{p'^2}{2m_2} + E_\pi(\mathbf{q}) - \sqrt{s} - i\epsilon \right), \tag{7}$$

where $E_\pi(\mathbf{q}) = \sqrt{q^2 + m_\pi^2}$ with m_π being the pion mass, m_1 and m_2 stand for the masses of the H_1 and H_2 mesons, respectively, which label either B or B^* mesons in Eq. (6), and \sqrt{s} is the energy of the system. Finally, the partial wave projections of the OPE potential (6) (which contribute to $V_{ij}^{(JPC)}(p, p')$ in Eq. (1)) were evaluated with the help of the projection operators listed explicitly in Ref. [15]. The one- η exchange (OEE) can be calculated straightforwardly from the expressions given above if one makes the following replacements: (i) the pion mass by the η mass, and (ii) the pion coupling constant g_b by the η coupling constant $g_b/\sqrt{3}$, as follows from the SU(3) chiral Lagrangian which can be found, for example, in Ref. [16]; (iii) all potentials in Eq. (6) should be multiplied by (-1) since the isospin coefficient for the η meson has a different sign.

3 Results and discussions

As our starting point, we assume both Z_b and Z'_b to be shallow bound states and treat their binding energies as input parameters. This is consistent with the analysis of Ref. [17] where it is demonstrated that both Z_b 's are compatible with bound state poles as soon as threshold effects are included properly. This assumption allows us to adjust both low-energy constants, C_1 and C'_1 , in the leading-order contact potential — see Eqs. (4)-(5). For definiteness, we fix the binding energies of the Z_b 's to be

$$E_B(B^* \bar{B})[Z_b] = 5 \text{ MeV}, \quad E_B(B^* \bar{B}^*)[Z'_b] = 1 \text{ MeV}, \tag{8}$$

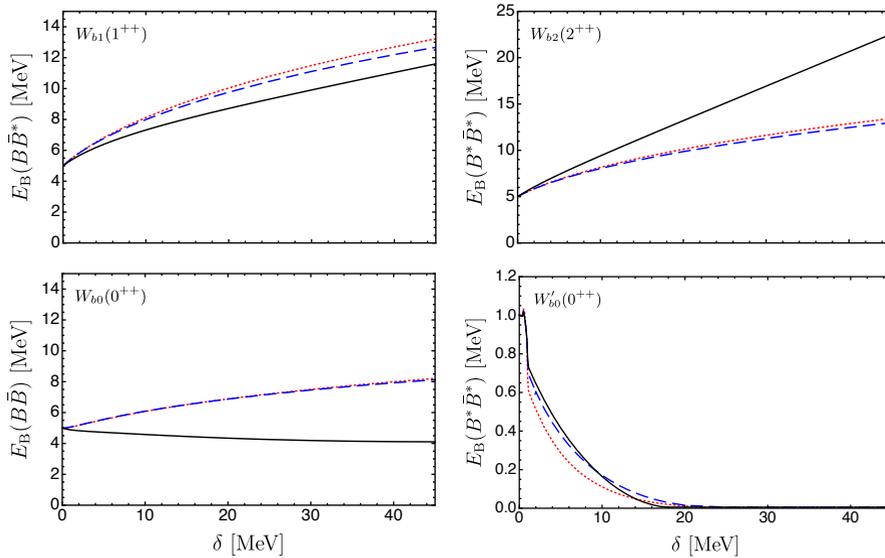


Figure 1. Evolution of the binding energies of the Z_b 's spin partners as functions of the mass splitting δ between the B^* and B mesons. The red dotted curves correspond to the pionless (purely contact) theory; the blue dashed curves are obtained for the central (S -wave) part of the OPE included; the black solid curves represent the results for the problem with the contact plus full OPE interactions, including tensor forces. The physical mass splitting corresponds to the right edge of the plots. Please note the different scales for each of the plots.

in line with Ref. [17]. Here and in what follows, the binding energies of the states are always defined relative to their reference thresholds, as stated in Eq. (8) in parentheses, namely

Threshold:	Z_b 's and their spin partners W_{bJ} :	
$B\bar{B}$	$W_{b0}(0^{++})$,	(9)
$B\bar{B}^*$	$Z_b(1^{+-})$, $W_{b1}(1^{++})$,	
$B^*\bar{B}^*$	$Z'_b(1^{+-})$, $W'_{b0}(0^{++})$, $W_{b2}(2^{++})$.	

The impact of HQSS violation on the location of the spin partner states is illustrated in Fig. 1, where variations of the spin-partners binding energies are shown versus the hyperfine mass splitting δ between the B^* and B meson masses. It should be stressed that the contact terms are re-fitted for each value of δ to provide the given binding energies of the Z_b and Z'_b states used as input — see Eq. (8), as if we lived in different worlds, in which the value of the mass splitting had some fixed values in the considered interval. In line with the findings of Refs. [8, 18], in the strict HQSS limit (i.e. when $\delta = 0$), Z_b 's and their spin partners W_{bJ} 's populate two families of states, the members of which are exactly degenerate in the mass,

$$E_B^{(0)}[W_{b0}] = E_B^{(0)}[W_{b1}] = E_B^{(0)}[W_{b2}] = E_B^{(0)}[Z_b] \quad \text{and} \quad E_B^{(0)}[W'_{b0}] = E_B^{(0)}[Z'_b], \quad (10)$$

where the superscript (0) indicates the strict HQSS limit. Then, as δ grows from zero to the physical value of 45 MeV, the spin partners from the first family (W_{b0} , W_{b1} , and W_{b2}) tend to become more bound while the state W'_{b0} turns to be virtual. It is seen that the binding energies of the partner states W_{b1} and W_{b2} exhibit significant HQSS violation at the physical value of the mass splitting. The most

	$Z_{b1}(1^{+-})$	$Z'_{b1}(1^{+-})$	$W_{b0}(0^{++})$	$W'_{b0}(0^{++})$	$W_{b1}(1^{++})$	$W_{b2}(2^{++})$
E_B [MeV]	5 (input)	1 (input)	5.3 ± 1.7	—	12.4 ± 0.6	19.8 ± 2.2
Γ_B [MeV]	—	—	—	—	—	4.6 ± 1.0
E_B [MeV]	1 (input)	1 (input)	0.7 ± 0.5	—	3.8 ± 0.1	10.2 ± 1.8
Γ_B [MeV]	—	—	—	—	—	6.2 ± 1.1

Table 1. The binding energies and the widths of the spin partners for the two sets of the Z_b and Z'_b binding energies used as input. The uncertainty in the results is due to the variation of the cutoff in the Lippmann-Schwinger equations from 800 to 1500 MeV.

pronounced effect is observed for the tensor spin partner W_{b2} where the binding energy at the physical value of δ is a factor four larger (≈ 20 MeV) than in the strict HQSS limit. It is remarkable that the large portion of this rise is due to the off-diagonal (S-D) tensor-force transitions from OPE, the fact which signals the importance of the nonperturbative inclusion of the pion dynamics. On the other hand, the S-wave central part of the OPE interaction can be almost fully absorbed into the redefinition of the low-energy constants and, therefore, does not have any impact on the location of the spin partners. The effect of the η meson also appears to be marginal, see Ref. [7] for further details.

The potentially largest source of the theoretical uncertainty in making predictions for the partner states could be associated with the input parameters for the Z_b states. Indeed, in the recent coupled-channel analysis of the experimental line shapes including inelastic channels [19, 20] the interpretation of the Z_b 's as virtual states with the excitation energy ≈ 1 MeV was proposed. Due to technical reasons we are not yet able to use virtual states as input. However, we can anyhow investigate, how the locations of the spin partner states evolve as the input masses are varied. In Table 1, we confront our results for two different sets of binding energies used as input for the Z_b states. The most striking prediction corresponds to the tensor partner W_{b2} which remains bound due to sizeable HQSS violating effects even when the binding energy of the Z_b state is reduced. The width of the tensor state to the open-flavour channels is predicted and appears to be only slightly sensitive to the input, since it is largely controlled by pion dynamics. Indeed, the width is given by the transitions to lighter open flavour channels in D-waves which would have vanished in the theory without meson exchanges, especially without pions. Although we have not yet included inelastic channels in our study, the similarity to the case of the Z_b 's, where inelastic channels play only a minor role, could imply that the open-flavour contributions to the width of the W_{b2} should be the dominant ones.

In summary, we propose a systematic chiral EFT-based approach for studying the long-range dynamics of hadronic molecules and for predicting their spin partners on the basis of the heavy-quark spin-symmetry (HQSS). We emphasise that having quantitative control over HQSS violating effects is mandatory for making reliable predictions for the molecular partner states. The molecular partners of the $Z_b(10610)$ and $Z_b(10650)$ states are predicted with the special focus on the tensor partner W_{b2} , where the effects of pion coupled-channel dynamics are especially important. Due to effects of HQSS violation governed by pionic transitions, this state remains bound even for the vanishing binding energies of the Z_b 's and has a few MeV width to the open-flavour channels. It is, therefore, expected to be detectable in experimentally measured line shapes.

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