

Atom-atom entanglement in a not-resonant two-photon Tavis-Cummings model

Marya O. Guslyannikova^{*}, and *Eugene K. Bashkirov*

Samara University, 34 Moskovskoe shosse Str., Samara 443086 Russia

Abstract. The entanglement between two two-level atoms (qubits) interacting not-resonantly with a one mode of thermal field in a lossless cavity via effective degenerate two-photon transitions is investigated. Based on the exact solution for the time-dependent density matrix of the system under consideration, negativity is calculated as a measure of the entanglement of atoms. The influence of a detuning on the dynamics of entanglement of atoms for separable and entangled initial atomic states and thermal cavity state is investigated.

Coupling distant qubits is an important goal for quantum information and for its potential applications. This kind of coupling needs the study of interaction between qubits and photons, which have been widely, studied in a cavity quantum electrodynamics (CQED) [1]. The Jaynes–Cummings model (JCM) is the simplest possible physical model of CQED [2]. JCM has been used to understand a wide variety of phenomena in quantum optics systems, such as trapped ions, superconducting circuits, optical and microwave cavity QED. In order to explore a wider range of phenomena caused by the interaction of the qubits with the quantum fields in resonators the numerous generalizations of the JCM have been proposed in recent years via using multi-levels, multi-atoms, multi-mode fields, detuning and so on. The multi-atoms JCM is usually called as Tavis-Cummings model (TCM). A lot of CQED experiments showed that JCM and TCM give good predictions for qubit-field dynamics [1]. The JCM(TCM) can be extended to nonlinear versions [2] known as multi-photon JCM(TCM). Such type of nonlinear Hamiltonian can be engineered in trapped ion domain [3], neutral atoms [4] or in superconducting circuits [5]. The interaction between the environment and quantum systems can lead to decoherence, but it may also be associated with the formation of entanglement. Thus, understanding and investigating entanglement of mixed states becomes one of the actual problems of quantum information. Kim et al. showed [6] that a chaotic field with minimal information can entangle two two-level atoms interacting with one-mode thermal field in a lossless cavity. Zhang [7] directly generalizes Kim's study to the case when the atoms are slightly detuned from the thermal field and showed that a slight detuning might cause high entanglement between the atoms. The entanglement between two two-level atoms through resonant nonlinear two-photon interaction with one-mode thermal field has been studied in [8-13]. It has been shown that entanglement induced by nonlinear interaction is larger than that induced by linear

^{*} Corresponding author: maria.guslyannikova@yandex.ru

interaction. In the present paper we investigate the influence of the detuning on atom-atom entanglement induced by thermal cavity field for separable and entangled atomic states.

We consider a system consisting of two effective two-level atoms with transition frequency ω_0 not-resonantly interacting with a single-mode thermal cavity field with frequency ω via degenerate two-photon transitions in a lossless cavity. We introduce the atom-field detuning as $\Delta = \omega_0 - 2\omega$. The Hamiltonian describing such a model in RWA is

$$H_I = \hbar\omega_0 / 2 \sigma_1^z + \hbar\omega_0 / 2 \sigma_2^z + \hbar\omega a^+ a + \hbar\gamma \sum_{i=1}^2 (a^{+2} \sigma_i^- + \sigma_i^+ a^2),$$

where a^+ and a denote the creation and annihilation operators of the field, σ_i^z are the inversion operators for i th two-level atom, $\sigma_i^+ = |e\rangle_{ii}\langle g|$, $\sigma_i^- = |g\rangle_{ii}\langle e|$ are the raising and lowering atomic operators with $|e\rangle_i$ and $|g\rangle_i$ being the excited and ground states of the i th atom ($i = 1, 2$) and γ is the effective two-photon coupling constant.

The initial atoms state is assumed to be separable such as $|\Psi(0)\rangle_A = |e, g\rangle$ or entangled $|\Psi(0)\rangle_A = \cos\theta |e, g\rangle + \sin\theta |g, e\rangle$. The initial cavity mode state is assumed to be the thermal one-mode state $\rho_F(0) = \sum_n p_n |n\rangle\langle n|$, where the weight functions are $p_n = \bar{n}^n / (1 + \bar{n})^{n+1}$. Here \bar{n} is the mean photon number in a resonator mode.

We have obtained the exact solution for the time-dependent density matrix for the model under consideration in the "dressed" states representation. Taking a trace over the field variable we obtained the reduced atomic density operator and derived the exact expression for atom-atom entanglement parameter namely negativity. Calculating the time behaviour of negativity, we have derived, firstly, that for separable initial atomic states a slight detuning can greatly enhance the degree of atom-atom entanglement induced by a thermal field. The second result is that for not-resonant two-photon atom-field interaction and entangled initial atomic states there are an appreciable decrease in the amplitudes of the negativity "Rabi" oscillations, i.e. the stabilization of entanglement takes place. In the case of a relatively intensive thermal cavity field the inclusion of detuning eliminates the effect of the sudden death of entanglement.

References

1. I. Buluta, S. Ashhab, F. Nori, Rep. Prog. Phys. **74** 104401 (2011)
2. B.W.Shore, P.L. Knight, J. Mod. Opt. **40** 1195 (1993)
3. D.M. Meekhof, C. Monroe, B.E. King et al, Phys. Rev. Lett. **77** 2346 (1996)
4. C. Hamsen, K.N.Tolazzi, T. Wilk, G. Rempe, Phys. Rev. Lett. **118** 133604 (2017)
5. L. Garziano, R. Stassi, V. Macri et al., Phys. Rev. **A92** 063830 (2015)
6. M.S. Kim, J. Lee, D. Ahn, P.L. Knight, Phys. Rev. **A65** 040101 (2002)
7. B. Zhang, Opt. Comm. **283** 4676 (2010)
8. L. Zhou, H.S. Song, J. Opt. **B4** 425 (2002)
9. E.K. Bashkurov, Laser Phys. Lett. **3** 145 (2006)
10. X.-P. Liao, M.-F. Fang, J.-W. Cai, X.-J. Zheng, Chin. Phys. **B17** 2137 (2008)
11. E.K. Bashkurov, M.P. Stupatskaya, Las. Phys. **19** 525 (2009)
12. E.K. Bashkurov, M.S. Mastuygin, Opt. Spectros. **116** 630 (2014)
13. E.K. Bashkurov, M.S. Mastuygin, Pramana-J. Phys. **84** 127 (2015)